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APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

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Crop Insurance by Comparing Revenue Distributions  
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by

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# Optimizing Farmers' Joint Use of Forward Pricing and Crop Insurance by Comparing Revenue Distributions Estimated with Numerical Integration<sup>1</sup>

Richard Heifner and Keith Coble

Because the distributions of crop yields, prices, and revenues generally are skewed, and because the payoffs from crop insurance and put options follow censored distributions, there is no reason to expect that crop producer's incomes approach the normality needed to justify mean-variance analysis. Numerical integration is used in this paper to approximate crop revenue distributions in their entirety so that various measures of riskiness can be applied. Procedures for finding approximately optimal futures and options hedges with and without crop insurance are described. The results demonstrate that combinations of crop insurance and forward pricing are much more effective than either alone in reducing risks.

The essence of risk management is choosing among probability functions of income or wealth. In crop production, the task is complicated not only because risk preferences often are fuzzy, but also because incomes do not closely follow standard probability distributions. Crop revenues are products of random yields and prices. Yield distributions tend to be skewed to the left, price distributions to the right, and yields and prices are negatively correlated in major growing areas. Moreover, many of the risk management tools available to farmers, particularly crop insurance and put options, censor or truncate yield, price, or revenue distributions from below. The resulting joint yield-price distributions and revenue distributions are difficult or impossible to approximate analytically. Furthermore, the problem often cannot be satisfactorily circumvented by using empirical distributions because price variabilities have changed and current futures and options quotations provide information that needs to be taken into account. To deal with these problems we draw estimates of the parameters of the underlying yield-price distributions from several sources, including recent futures and options quotes, and employ numerical integration to derive estimates of farmers' revenue distributions under alternative risk management strategies. Once the revenue distributions are described, any appropriate risk or utility measure can be applied to rank the distributions. We show how strategies can be identified that are optimal under several risk measures.

Our empirical analysis starts with yield-price distribution parameters representative of farms in DeWitt County, Illinois, an area of relatively strong negative yield-price correlation and low yield variability. Sensitivity to differences in yield-price correlation and yield variability are examined by comparing these base results with results under zero yield-price correlation, doubled yield standard deviation, and the two combined. The zero yield-price correlation case illustrates the effects of removing the so called "natural hedge" afforded producers in areas where the yield-price correlation is strongly negative. The high yield variability case is included to provide a contrast to the relative stable yields of DeWitt County. The last case involving zero yield-price correlation and high yield variability approximates conditions in certain growing areas outside the Cornbelt.

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The strategies evaluated include forward pricing with both futures and options contracts combined with four types of yield and revenue insurance. The risk measures used to rank alternative strategies include measures of revenue dispersion, estimates of probabilities of revenues falling below specified levels, and certainty equivalence gains based on constant relative risk aversion (CRRA) utility functions.

Several findings with important implications for farmers' risk management decisions emerge from the analysis. Combinations of crop insurance and forward pricing part of the crop are shown to be much more effective in reducing risks than either strategy alone. Even the farmer with revenue insurance, which deals indirectly with price risk, can further reduce risk by forward pricing part of his/her crop. However, optimal forward pricing ratios generally are lower under safety first and utility maximization criteria than those estimated with traditional variance minimizing methods. The differences in risk reduction obtained through different types of insurance at the same level of coverage are not large. Without forward pricing, revenue insurance offers modestly greater risk reduction than yield insurance. With forward pricing, the difference is smaller. Replacement coverage crop and revenue insurance (basing indemnities on the higher of harvest time price and the price selected at signup time) slightly enhances risk reduction when accompanied by forward pricing. Advantages of futures hedges over options hedges, and procedures for optimizing both, are demonstrated. The sensitivity of the results to bias in price and yield expectations are explored.

#### **Risk Management Tools Evaluated**

Forward pricing involves entering a contract that sets price or a bound on price for a trade that involves delivery in the future. The forward pricing methods used by farmers to reduce risks in crop production include selling futures contracts, buying put options, and cash forward contracting. The distinction between fixed-price contracts, including futures and most cash forward contracts, and limit-price contracts, which include options and minimum-price cash forward contracts, is important. Most of this analysis pertains to fixed-price contracting, but limit-price contracting also is considered.

The risk reducing effects of insurance which pays indemnities based on yield shortfalls, and insurance which pays indemnities based on revenue shortfalls, are compared here. We also examine the effects of replacement coverage, or market value protection (MVP), which has been offered by some private insurance companies. With MVP, yield shortfalls are indemnified either at a price selected at signup or a harvest time price, whichever is higher. This assures the farmer an indemnity large enough to replace the lost output at the market price. Such coverage complements forward selling. For example, if a farmer with 75 percent of the expected crop sold forward and 75 percent MVP coverage experienced a complete yield failure, the indemnity would be sufficient to buy the commodity needed to meet delivery obligations on the forward contract.

The four types of insurance analyzed, the acronyms assigned to each, and the formulas used to calculate indemnities for each are reported in table 1. All four types of policies are assumed to apply to the same basic insured units. This analysis does not address the issue of unit size in crop and revenue insurance design.

Table 1--Calculation of indemnities under alternative types of insurance

Type of insurance	Indemnity formula*
Multi-peril crop insurance (MPCI)	$P_0 * \text{Max}[c_* Y_h - Y, 0]$
Crop insurance with market value protection (MVP)	$\text{Max}[P_0, P_1] * \text{Max}[c_* Y_h - Y, 0]$
Revenue insurance (RI)	$\text{Max}[c_* P_0 * Y_h - P_1 * Y, 0]$
Revenue insurance w/ market value protection (CRC)	$\text{Max}[c_* \text{Max}(P_0, P_1) * Y_h - P_1 * Y, 0]$

\*Symbols are defined as follows:  $P_0$  = price at signup time,  $P_1$  = price at harvest,  $c$  = insurance coverage level (75%),  $Y_h$  = historical yield, and  $Y$  = realized yield.

Insuring or forward pricing one year's crop has a limited effect on the overall risks in farming because the well being of farmers depends on incomes in future years as well the current year. However, insuring and forward pricing consistently from year to year can make farming less risky in the longer run. The risks measured here are deviations from pre-planting expectations. Such expectations are relatively stable from year to year. To the extent that deviations from expectations can be reduced, farming is converted to a less risky business.

### Measures of Risk Employed

The method used here approximates in their entirety the revenue distributions that occur under alternative risk reduction strategies. Space does not allow so much information to be reported. Moreover, decision makers generally need help in interpreting and evaluating such detailed information. Consequently, revenue distributions are described and compared here using certain summary statistics. Included are the standard deviation (SD), the root mean squared negative deviation (RMSND), probabilities of revenue less than 60, 70, and 80 percent of the expected value of sales, and estimates of certainty equivalent gains under utility functions with CRRA coefficients of 1, 2, 4, and 8. In calculating certainty equivalent gains, utility is calculated as a function of wealth which consists of initial wealth plus the revenue deviation from its mean. Costs for commissions and interest on futures margins and options premiums were subtracted and insurance premiums are reduced by the amount of the government subsidy. Corresponding estimates of certainty equivalent gains under constant absolute risk aversion utility functions were calculated but are omitted to save space and because of their similarity to the CRRA results.

### Parameter Estimation

The analysis involves transforming observed yields and prices to obtain a joint normal distribution which is then integrated numerically to estimate revenue distributions under various risk-management strategies. Transforming into and then back out of normality preserves the skewness in the price and yield distributions and offers two major advantages. First, in multivariate normal distributions the covariability between each pair of variables is fully described by one parameter, the correlation, or covariance. In many cases data are insufficient to measure more complicated forms of covariability. Second, procedures for estimating multivariate normal probability densities are well established and easily applied.

Critical parameters are the relative variability of yield and yield-price correlation. For a crop like corn, price variability is essentially the same across the United States, except for basis risk. Relative yield variability, on the other hand, differs substantially among locations, depending on climatic conditions and the availability of irrigation. Yield coefficients of variation tend to increase with distance from major growing areas. Because price declines as supply increases and yields are spatially correlated, farm-level yield-price correlation tends to be negative in major growing areas. Yield-price correlations decline in absolute value to near zero in areas distant from the Cornbelt. When yields and prices are negatively correlated, their fluctuations tend to offset each other providing a so-called "natural hedge" that makes revenues less variable than they would be otherwise. The effectiveness of forward pricing in reducing farmers' risks is reduced by negative yield-price correlation in major growing areas and by high yield variability in fringe growing areas. Crop insurance tends to be more effective in reducing risks in fringe areas because of relatively higher yield variability.

Revenue deviations from expectations held in March are analyzed in this study. The base case parameters are reported in table 2. Futures prices are assumed to be log normally distributed. The December corn futures price is set at \$2.84 with a volatility to harvest of 18 percent, based on recently observed futures prices and implicit volatilities derived from recently observed options prices. Basis risk is omitted in order to focus on yield and price level risks which are of much greater magnitude.

Table 2--Parameters used for the base case, DeWitt County, Illinois

Parameter	Value
Expected futures price	\$2.84 per bushel
Price volatility to harvest	18 percent
Basis	-0.25 per bushel
Expected yield	151 bushels/acre
Standard deviation of yield	22.8 bushels/acre
Yield-price correlation	-0.43
Months to harvest	7
Acres of corn	500
Farmer's initial wealth for quantifying utility	\$500,000
Farmer's marginal rate of interest	12 percent
Margin deposit	8 percent of value of commodity
Commission rate	1 percent of value of commodity
Market rate of interest for pricing options	6 percent
Subsidy for insurance	23.5 percent of premium for 75% MPCl coverage

The expected yield is based on 1956 to 1995 county yield trends, estimated from National Agricultural Statistical Service data and adjusted for heteroskedasticity. Farm yield observations provided by the Risk Management Agency (RMA) were used to calculate the average standard deviation of yield for all farms in the county with observations for 1985 to 1994. The ratio of the average farm yield standard deviation to the county yield standard deviation for 1985-1994 was applied to the county yield series to produce a representative farm yield series. Following Taylor, these farm yields were normalized using a hyperbolic tangent transformation. The cubic function for the transformation was estimated using maximum likelihood. Yield-price correlations were estimated using transformed observations from 1975 to 1995.

For the certainty equivalence estimates, the farmer is assumed to have 500 acres of corn and a \$500,000 net worth. The farmer's marginal interest rate is assumed to be 12 percent on funds used for futures margins and options premiums. The market interest rate for determining options premiums is 6 percent. Most of the results reported here rest on assumptions that forward prices are unbiased and insurance premiums are actuarially fair. Effects of relaxing these assumptions are examined in the last sections of the paper.

#### The Numerical Integration Procedure

The numerical procedure involves calculating weighted sums of returns from crop sales, forward pricing profits or losses, and insurance indemnities minus premium costs under each risk management alternatives for a large number of possible yield-price outcomes. The procedure is implemented by establishing an  $n \times n$  grid representing possible yield-price outcomes in the space between -4 and +4 standard deviations around the mean for each normalized variable. For the results reported here,  $n$  was set at 101 giving 10,201 grid segments. The probability assigned to each grid segment and used for calculating the weighted averages and revenue distributions equals the height of the density function at the midpoint of the grid segment divided by  $n^2$ . The weighted revenue outcomes are summed by interval to approximate the probability distributions of revenues under alternative risk-management strategies.

The numerical integration is performed with a program written in the GAUSS programming language. The calculations involve the following steps:

1. For each of  $n$  equally spaced points between -4 and +4 standard deviations around the means of the normalized yield and normalized price, apply an inverse transformation to estimate corresponding untransformed yields and prices.
2. For each of the  $n^2$  grid points calculate sales, net returns from forward pricing and insurance, total revenue, and utility estimates under each different risk management strategy using the untransformed yields and prices. Assign a probability to each point by applying the bivariate normal density function to the normalized yields and prices. Calculate weighted sums, sums of squares, and sums of cubes of the variables as needed, as well as the distribution of revenues by interval.
3. Calculate statistics from the sums produced in 2. Convert utility estimates to certainty equivalents.

The ranking of strategies is sensitive to relative variability and correlations, but is not affected when means and standard deviations vary proportionately.

### Empirical Results

The results reported here are based on pre-planting yield-price distribution parameters estimated for the representative DeWitt County, Illinois farm. Results also are reported for three variations from the DeWitt County base to illustrate the effects of lower yield-price correlation and higher yield variability.

### Optimal Amounts to Price Forward

The farmer's optimal forward sale is less than 100 percent of his/her expected crop so long as yield uncertainty remains unresolved. It becomes smaller as the yield-price correlation becomes more negative. The problem of determining how much to price forward when both price and yield are uncertain has been addressed most often within a mean-variance format. Early conceptual work was done by McKinnon and by Grant (1985). Grant (1989) and Miller and Kahl provided estimates of minimum-risk price hedge levels and hedge effectiveness when yields are uncertain. Risk minimizing hedges that include yield as well as price futures have recently been estimated by Li and Vukina for corn in North Carolina, Tirupattur et al. for soybeans in Illinois, and by Heifner and Coble for corn in counties across the United States. McNew has examined the effectiveness of yield options in reducing crop grower's risk. Lapan and Moschini have addressed the problem of price hedging with both price and yield risk within the more general framework of utility maximization.

The optimal amount to price forward occurs at the point where the marginal value of risk reduction attained equals the cost of contracting an additional unit. The cost of contracting is low in highly competitive markets. For futures hedging, it includes commissions and interest foregone on margin deposits. For buying options, it consists of commissions and the interest foregone on the premium during the interval until the option matures or is sold. The premium itself is not a direct cost because, on average, it is returned when the option is sold or exercised.

The determination of optimal futures hedge ratios for corn grown in DeWitt County with 75 percent crop insurance is illustrated in table 3. Each row in the table shows how one risk measure varies as the hedge ratio changes from 0 to 1 in increments of 0.1. In each row, the minimum-risk level is printed in bold. For example, the minimum RMSND attainable is \$35.37 per acre at a hedge ratio of 0.5 while the probability of revenue less than 70 percent of expected sales is minimized at 0.04 with a 0.6 hedge ratio.

Six of the nine criteria give optimal hedge ratios in the 0.4 to 0.6 range. More precise hedge ratio estimates do not seem to be needed because risk changes little as hedge ratios vary by  $\pm 0.1$  from the minimum-risk level.

At 0.7, the minimum SD hedge, which can be calculated directly from yield and price variances and covariances, is the largest of those shown. This suggests that minimum-variance hedge ratios may be too high for many farmers. Among the safety first measures, the probability of revenues below 70 percent of expected value of sales has been found to be most revealing based on this and other cases. Probabilities of revenues below 60 percent of expectation are quite small in the Cornbelt, even without insurance or forward pricing, giving little room to quantify gains with the 60 percent criterion. The 60 percent point is of more

Table 3--Measures of risk at different futures hedge ratios with 75% MPCI, DeWitt County, Illinois

Risk measure	Futures hedge ratio										
	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
SD, \$/ acre	72.60	66.78	61.38	56.56	52.44	49.23	47.10	<b>46.20</b>	46.60	48.27	51.09
RMSND, \$/acre	46.94	43.50	40.47	37.99	36.24	<b>35.37</b>	35.47	37.52	38.44	41.07	44.24
100 x Prob [R<.6E(R)]	.42	.15	.04	.01	0	<b>0</b>	0	.02	.10	.38	.94
100 x Prob [R<.7E(R)]	3.40	2.14	1.23	.62	.26	.07	<b>.04</b>	.30	1.18	2.83	4.33
100 x Prob [R<.8E(R)]	13.04	10.98	9.05	7.57	<b>6.79</b>	8.17	10.19	10.78	10.71	11.38	11.74
CE gain \$/ac. CRRA=1	<b>1.35</b>	1.13	.86	.53	.14	-.29	-.83	-1.41	-2.05	-2.75	-3.51
CE gain \$/ac. CRRA=2	1.71	1.89	<b>1.94</b>	1.87	1.69	1.38	.95	.39	-.29	-1.10	-2.04
CE gain \$/ac. CRRA=4	2.49	3.41	4.09	4.54	<b>4.75</b>	4.71	4.42	3.87	3.07	1.99	.63
CE gain \$/ac. CRRA=8	4.29	6.62	8.48	9.86	10.77	<b>11.17</b>	11.06	10.41	9.19	7.37	4.89

interest where yield risks are relatively higher, such as for wheat in the Great Plains. On the other side, the probability of revenue below 80 percent of its expectation is a somewhat ambiguous measure of risk because many risk-reduction strategies shift probabilities from below 70 percent to within the 70-100 percent range.

In contrast to the measures of dispersion and probabilities of revenues below specified levels shown at the top of the table, the measures of certainty equivalence gains in the last four rows increase as risk declines. They show only modest dollar gains from hedging with crop insurance for farmers at this level of wealth. For example, hedging at the optimal level compared to not hedging is worth only \$1.94 per acre to the farmer with CRRA = 2 and \$4.75 per acre for the farmer with CRRA = 4. It should be emphasized that these numbers represent pure gains after commissions and interest on money invested in premiums have been subtracted. Nonetheless, they may not be considered worthwhile by many farmers.

The certainty equivalence results show optimal hedge ratios increasing with risk aversion, as expected. The hedge ratios implied by RMSND and the safety first measures correspond to relatively high levels of CRRA.

### Optimal Options Hedges

Although fixed forward pricing reduces risk more than limit pricing according to the widely used risk measures employed here, some farmers may prefer limit pricing due either to a preference for positive skewness in revenues or a preference for exchange-traded contracts

and desire to avoid margin calls on futures. To hedge in options or minimum-price contracts, one must choose not only the size of the contact, but also the minimum price (strike price). The optimal option position occurs at the coverage level and strike price where the marginal value of the risk reduction obtained equals the hedger's marginal costs for interest and commissions. Thus, the farmer with abundant capital and a low interest rate for borrowing will prefer to use deeper strike price options than the farmer with higher marginal interest rates, other things equal.

Table 4 illustrates the determination of minimum-risk options positions using most of the risk measures used in table 3. The first column shows risk without either insurance or hedging; the second with 75 percent crop insurance and no hedging, and the third with the optimal futures hedge from table 3. The last four columns show risk with optimal options coverages for strike price ratios ranging from 1 to 1.3. The criterion of optimality used in

Table 4--Measures of risk under minimum-risk futures and options hedges at different strike prices with 75% MPCI, DeWitt County, Illinois

Risk measure	No insurance or hedge	75% crop insurance with alternative minimum-risk hedges					
		No hedge	Futures	Strike price relative to futures price for put hedge			
				1	1.1	1.2	1.3
SD, \$/acre	76.95	72.60	<b>46.20</b> (.7)	54.25 (.8)	49.52 (1.0)	47.40 (.8)	46.40 (.8)
RMSND, \$/acre	51.77	46.94	<b>35.37</b> (.5)	37.09 (.8)	35.97 (.7)	35.81 (.6)	36.07 (.6)
100 x Prob [R<.6E(R)]	1.41	.42	<b>0</b> (.5)	0 (.6)	0 (.6)	0 (.5)	0 (.5)
100 x Prob [R<.7E(R)]	5.46	3.40	.04 (.6)	.58 (.7)	<b>0</b> (.8)	0 (.7)	.01 (.6)
100 x Prob [R<.8E(R)]	15.22	13.04	<b>6.79</b> (.4)	7.75 (.6)	7.90 (.5)	7.90 (.4)	7.56 (.4)
CE gain \$/ac. CRRA=2	0	1.71	<b>1.94</b> (.2)	1.71 (0)	1.71 (0)	1.71 (0)	1.71 (0)
CE gain \$/ac. CRRA=4	0	2.49	<b>4.75</b> (.4)	3.22 (.4)	3.75 (.4)	3.85 (.4)	3.65 (.3)
CE gain \$/ac. CRRA=8	0	4.29	<b>11.17</b> (.5)	8.47 (.7)	9.65 (.6)	9.93 (.6)	9.74 (.5)

\* Minimum-risk hedge ratios are shown in parentheses.

each case is the hedge ratio that minimizes the risk measure for that strike ratio. Only at-the-money and in-the-money puts are shown because they out perform out-of-the-money puts for the risk measures and interest rates used here.

Risks are lower with futures hedges than options hedges in all but one case where the difference is negligible. This illustrates the basic advantage of fixed price contracting over limit price contracting for those hedgers who can arrange to meet margin calls. The risks under put hedges generally decline, up to a point, as the strike price is increased. The certainty equivalent measures provide a basis for determining an optimal option hedge, which occurs at the strike price and hedge ratio where the certainty equivalent gain is greatest. The optimal strike price under the certainty equivalence criteria depends on the farmer's marginal interest rate which, along with commissions, determines the cost of the hedge.

### Effect of Type of Insurance on Risk

We turn next to how risk differs between various types of insurance with and without forward pricing. To limit the number and size of tables, only results for futures hedges and only two measures of risk are shown. Table 5 reports the percent of the time that net revenue will be less than 70 percent of the expected value of sales for four types of insurance with and without hedging for DeWitt County as a base, and for the three variations from the DeWitt County yield and price distribution. The price distribution parameters, mean yield, and the shape of the yield distribution are the same for all four cases. Corresponding estimates of certainty equivalent gains under  $CRRA=4$  are reported in table 6.

We note first that the risk reductions obtained by combining insurance and forward pricing are substantially larger than for either strategy alone. For example, in the upper left corner of table 5, the percent of revenues less than 70 percent of expected sales declines from 5.46 percent with neither insurance or forward pricing to 3.40 percent with insurance alone, 4.40 percent with forward pricing alone, and 0.04 percent with the two combined. The substantial advantage of combining forward pricing with insurance holds for all four types of insurance, all four yield-price parameter sets, and for both risk measures.

Without forward pricing, revenue insurance reduces risk more than crop insurance for all four combinations of yield-price correlation and yield variability. The certainty equivalent advantages of revenue insurance range from about \$0.50 to about \$2.65 per acre. With forward pricing, revenue insurance performs better than yield insurance more often than not, but the advantage is less than \$1.00 per acre in all cases but one. Minimum-risk hedge ratios with revenue insurance are only 10 to 20 percent under the safety first criterion (table 5), but almost as high as minimum-risk hedge ratios with yield insurance under  $CRRA=4$  (table 6).

The MVP supplement for crop insurance generally reduces risk slightly if the crop is forward priced and has little effect if the crop is not forward priced.

### Effects of Bias in Price Expectations

The results reported so far rest on assumptions that futures prices and the yields used to determine insurance premiums are unbiased. We now consider the effects of bias in price and yield estimates.

Keynes suggested that an excess of short hedging over long hedging would require short hedgers to pay a premium to long speculators for carrying risk and result in futures prices being

Table 5--Effect of yield-price correlation and yield variability on percent of time that revenue is less than 70 percent of the expected value of sales, alternative types of crop insurance, with and without forward pricing\*

Yield-price distribution	Forward priced	Alternative types of insurance				
		No ins.	MPCI	MVP	RI	CRC
Base: DeWitt County, IL	No	5.46	3.40	3.46	.11	.41
	Yes	4.40 (.2)	.04 (.6)	0 (.7)	0 (.1)	0 (.1)
Zero yield-price correlation	No	10.54	7.93	8.00	.65	1.00
	Yes	6.11 (.7)	.07 (.7)	0 (.7)	0 (.1)	0 (.1)
Yield variability doubled	No	13.36	4.72	4.97	.74	1.74
	Yes	12.44 (.6)	1.41 (.4)	0 (.7)	.01 (.1)	0 (.2)
Zero correlation & doubled variability	No	17.41	9.97	10.35	3.97	5.91
	Yes	13.63 (1.0)	1.80 (.6)	.17 (.7)	.38 (.1)	0 (.2)

\* Minimum-risk hedge ratios are shown in parentheses.

Table 6--Effect of yield-price correlation and yield variability on certainty equivalent gain under CRRA=4, alternative types of crop insurance, with and without forward pricing\*

Yield-price distribution	Forward priced	Minimum risk option hedge with alternative types of insurance				
		No ins.	MPCI	MVP	RI	CRC
Base: DeWitt County, IL	No	0	2.49	2.42	3.02	3.25
	Yes	1.45 (.3)	4.75 (.4)	4.91 (.5)	4.36 (.3)	5.39 (.4)
Zero yield-price correlation	No	0	3.65	3.65	5.78	5.98
	Yes	5.58 (.7)	9.40 (.7)	9.53 (.7)	9.72 (.6)	10.42 (.6)
Yield variability doubled	No	0	23.94	23.86	24.32	24.93
	Yes	0 (0)	24.78 (.3)	25.45 (.4)	24.55 (.1)	26.22 (.3)
Zero correlation & doubled variability	No	0	26.93	26.98	29.11	29.63
	Yes	3.13 (.5)	31.88 (.6)	32.65 (.7)	32.08 (.5)	33.86 (.6)

\* Minimum-risk hedge ratios are shown in parentheses.

less than expected cash prices at delivery. Later authors have pointed out that speculators with diversified portfolios could be expected to carry the risks at very low cost. The question of whether such bias exists has been the subject of many studies, the majority of which conclude that bias is small or absent, particularly for the grains where substantial long hedging exists to help balance short hedging. However, the possibility of bias cannot be fully ruled out empirically. Short hedging does exceed long hedging and transactions costs exist that may prevent risk premiums from being fully eliminated.

Table 7 illustrates the effects of futures price biases of -1 and -2 percent on the certainty equivalent gains from hedging and insurance for DeWitt County farmers with CRRA=4. It shows that a bias of -2 percent essentially eliminates the gains from forward pricing. Although a 2 percent bias seems small, it would if it persisted result in attractive profits for long speculators, whose trading costs are very small, and would tend to be bid away in a competitive market. The point is that the effectiveness of forward pricing is quite sensitive to bias in the market and requires low transaction costs.

Table 7—Certainty equivalent gains under CRRA=4 with negatively biased futures prices, alternative types of insurance, with and without forward pricing, DeWitt County, IL\*

Price bias	Forward priced	Alternative types of insurance				
		No ins.	MPCI	MVP	RI	CRC
Unbiased	No	0	2.49	2.42	3.02	3.25
	Yes	1.45 (.3)	4.75 (.4)	4.91 (.5)	4.36 (.3)	5.39 (.4)
1% negative bias in price	No	0	2.54	2.51	2.64	3.03
	Yes	.32 (.2)	3.26 (.3)	2.39 (.3)	2.89 (.2)	3.70 (.2)
2% negative bias in yields	No	0	2.58	2.60	2.28	2.84
	Yes	0	2.62 (.2)	2.68 (.2)	2.28 (0)	2.86 (0)

\* Minimum-risk hedge ratios are shown in parentheses.

### Effects of Negative Bias in the Yields Used to Determine Insurance Premiums

Some farmers do not buy insurance because they expect yields higher than the historical yields used to determine their premiums. Such differences in yield expectations reflect differences in information rather than risk premiums, but the implications are similar. A major difference is that government subsidies make crop insurance advantageous to the farmer even in the presence of considerable downward bias in yield estimates. For example, table 8 shows that insurance provides a small increase in certainty equivalent return for a farmer with CRRA=4 under a 5 percent negative yield bias, but none under a 10 percent bias.

Table 8—Certainty equivalent gains under CRRA = 4 with negatively biased yields, alternative types of insurance, with and without forward pricing, DeWitt County, IL\*

Yield bias	Forward priced	Minimum risk option hedge with alternative types of insurance				
		No ins.	MPCI	MVP	RI	CRC
Unbiased	No	0	2.49	2.42	3.02	3.25
	Yes	1.45 (.3)	4.75 (.4)	4.91 (.5)	4.36 (.3)	5.39 (.4)
5% negative bias in yield	No	0	.43	.13	.90	.25
	Yes	1.93 (.4)	2.96 (.5)	2.86 (.5)	2.71 (.4)	2.72 (.5)
10% negative bias in yield	No	0	-1.12	-1.61	-.51	-1.98
	Yes	2.43 (.4)	1.78 (.5)	1.42 (.5)	1.82 (.5)	.88 (.5)

\* Minimum-risk hedge ratios are shown in parentheses.

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