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Commodity Futures Market Reaction to Anticipated Public Reports: Frozen Pork Bellies

by

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This paper investigates the reaction of the frozen pork bellies futures market to the release of inventory information. Knight-Ridder releases their analysts' forecasts two days prior to the estimates provided by the USDA. The model provides a direct link between analysts' forecasts, the USDA estimates, and traders' beliefs on the frozen pork bellies inventories in storage. It differs from previous studies in that the price reaction depends on the information content of the difference between the USDA estimates and the analysts' forecasts, and on the dispersion among the analysts' forecasts. It is shown that empirical tests based solely only the information content of forecasts induce possible measurement error and result in the biased findings.

Introduction

The impact of public information on commodity futures markets has long been discussed. The USDA has been traditionally the most important source for public information in commodity futures markets. A steady stream of information on crop size, livestock inventories, grain exports, and a host of other statistics is provided to commodity market participants. The USDA Cold Storage Report (CSR) is one of them. The report is released monthly by the USDA and provides estimates of inventories of frozen agricultural commodities in storage, including pork bellies.

Knight-Ridder's MoneyCenter news service surveys analysts regarding their expectations of the amount of frozen pork bellies in storage and releases these expectations in the form of "pre-release estimates" two trading days prior to the release of the CSR. These pre-release estimates are the analysts' forecasts of the USDA estimates in the CSR.

The impact that the USDA reports has on commodity futures markets has been well developed empirically based on the efficient market hypothesis. There are, however, few studies that have investigated the mechanism of commodity futures market reaction to the USDA reports. Falk and Orazem (1985) construct a theoretical model of futures price determination in which they separate government forecast information from forecasts provided by other sectors. The model shows that price changes are entirely due to the unanticipated component of new information, government announcements or private forecasts. Therefore, the USDA report is valuable to market

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participants if its information content is not identical to the market's information set. This model provides a further understanding of market reaction to the government information under the existence of other information in the market. However, the framework does not consider variability in analysts' forecasts and its implication for market response (see Abarbnell, Lanen, and Verrecchia (1995)).

This paper investigates explicitly the mechanism through which market participants incorporate the analysts' forecasts and the USDA estimates of frozen pork bellies inventories in storage into their trading decisions in the pork bellies futures market. It provides a theoretical understanding of the relationship between the USDA estimates and market price movements. The behavior of market participants is examined in a two-period rational expectations equilibrium framework after the analysts' forecasts have been released and after the USDA estimates have been announced, respectively. Traders' beliefs in the market of inventories of frozen pork bellies evolves as more information becomes available in the market. The model is used to characterize traders' demand for futures contracts and equilibrium prices in terms of their information/beliefs. It illustrates how the analysts' forecasts and the USDA estimates affect traders' expectations and behavior. It also demonstrates potential measurement errors in previous studies which have used an average of forecasts as proxy for trader's beliefs or used the difference between an average of existing forecasts and the USDA forecast as unanticipated information contained in the USDA forecast.

Below, section 2 provides a description of the model. In section 3, the characterization of traders' demand for futures contracts and equilibrium prices in terms of their information/beliefs in the two rounds of trading is provided. In section 4, the theoretical constructs of trader's beliefs based on information are derived and price reactions are linked to the release of public information. Concluding comments follow.

Theoretical Model

A two-period rational expectations model of the pork bellies future market is developed. A single futures contract is traded at time 1, time 2, and matures at time 3. At time 1, Knight-Ridder (KR) releases the analysts' forecasts of the USDA estimates in the CSR and trading takes place. At time 2, the USDA releases its estimates of frozen pork bellies inventories in storage and there is another round of trading. At time 3, the spot price of the commodity becomes universally known. Deliveries and financial settlements are made on any outstanding futures contracts.

The information structure in our model bears some similarities to the structures in the previous research. Kim and Verrecchia (1991a, 1991b) assume that all traders receive a free public signal and informed traders receive a costly private signal about the risky asset return in period 1, and then a public report about the risky asset return is released in period 2. McNichols and Trueman (1994) assume that only one trader can receive a costly private signal about the risky asset return and this private signal may be positively related to the public report about the risky asset return in the second period. Demski and Feltham (1994) assume that a costly private signal received in the first

period is a forecast of the second period public report about the risky asset return. Abarbanell, Lanen, and Verrecchia (1995) assume that a costly private signal about the risky asset return and several free forecasts about the second period public report are received in the first period. Following the information structure in the model of Abarbanell, Lanen, and Verrecchia (1995), our model assumes that there are several free analysts' forecasts of the USDA estimates about frozen pork bellies inventories in the first period and the report of the USDA estimates comes out in the second period. But the costly private signal is not included in our model. The different assumptions of information structure in each model are driven by the different purposes of the studies and different characterizations of stock market and commodity futures markets.

The futures market is a derivative market, i.e., futures contract returns depend directly upon the prices of other well-defined assets or commodities, in contrast to bonds and equities. One important feature of the model presented here is that the agents who produce the commodity to be sold on the spot market also are futures traders. These futures traders have random final period endowments which are correlated with asset returns and their decisions on futures trades are affected by their beliefs about both the spot price and their own output. The random endowments here are physical inventories which is the source of uncertainty about that traders have any information. The models used in analyses of the stock market also include random endowments, but traders do not have information about these endowments. The information that traders have in the stock market is the uncertainty of the risky asset return.

Elements and Notations

The model consists H^s pure speculators and H hedgers. Among hedgers, there are H^{hf} producers such as slaughtering plants and H^{hr} processors or stockholders. All traders are risk averse and price takers. All random variables are described by normal densities and are designated by a tilde. At time 1, all traders receive at no cost several analysts' forecasts, $\tilde{f}_j$, j=1,..,k, of the USDA estimates, $\tilde{y}$, with mean $\bar{y}$ and variance σ_y, about frozen pork bellies inventories, where $\tilde{f}_j = \tilde{y} + \tilde{\delta} + \tilde{\eta}_j$. Each forecast of $\tilde{f}_j$ is contaminated by a source of noise common to all forecasts, $\tilde{\delta}$, and a source of noise unique to each forecast, $\tilde{\eta}_j$. These two sources of noise reflect situations where the analysts acquire information from the same source and where they acquire information from their own independent sources. After traders observe these forecasts, they can weigh these forecasts differently, which are $\sum_{k} k_{ij} f_j$, i=hf, hr, s. Based on their weighting scheme, they generate their own expectations of the spot price at time 3. Then trading takes place at the market-clearing price $\tilde{p}^f_1$. Traders hold x^i_1 units of the futures, where $i = hf, hr, s$, which represent producers such as slaughtering plants, processors or stockholders, and speculators, respectively. At time 2, the USDA releases its estimates of frozen pork bellies inventories in storage, $\tilde{y}$, where $\tilde{y} = \tilde{s} + \tilde{\varepsilon}$. The $\tilde{s}$ with mean $\bar{s}$ and variance σ_s represents the actual inventories and $\tilde{\varepsilon}$ is a source of noise unique to the USDA estimates. After all traders observe $\tilde{y}$, they have the same expectations of the spot price at time 3. However, they have different risk preferences, another round of trading takes place at the market-clearing price $\tilde{p}^f_2$. After trading, traders hold x^i_2 units of the futures $(i=hf,hr,s)$. At time 3, the realization of the spot price of the commodity becomes universally known. Deliveries and financial settlements are made on any outstanding futures contracts.

It is assumed that traders have negative exponential utility with risk aversion coefficient r^{hf} for producers, r^{hr} for processors or stockholders, and r^s for speculators, and all traders have non-stochastic initial endowments which are normalized to zero.

For hedgers, the terminal wealth is the sum of net return from the spot and the futures markets. For pure speculators, the terminal wealth is the net return from the futures market. Thus, traders' terminal wealth, $\tilde{w}^i$ $(i=hf,hr,s)$, can be written as:

$$\tilde{w}^i = \theta \, \tilde{p}_3^s + (\tilde{p}_2^f - \tilde{p}_1^f)x_1^i + (\tilde{p}_3^s - \tilde{p}_2^f)x_2^i .$$

For $i=hf$, $\theta=\tilde{q}$, where $\tilde{q}$ explains the supply of a producer at time 3. For $i=hr$, $\theta=\tilde{d}$, where $\tilde{d}$ explains the commodity demand of a processor or stockholder at time 3. For $i=s$, $\theta=0$.

A perfectly competitive, rational expectations equilibrium framework requires that the futures market clears at each round of trading. This requirement is specified as

$$H^{hf}x_t^{hf} + H^{hr}x_t^{hr} + H^{S}x_t^{s} = 0, \qquad t=1,2.$$

In the spot market, it is assumed that stocks purchased by processors and stockholders represent the aggregate demand, $D(\tilde{p}_3^s)$ or $H^{hr}\tilde{d}$, at time 3. The demand function is assumed to be $D(\tilde{p}_3^s) = \tilde{a} - b\tilde{p}_3^s$, where $\tilde{a}$ is a random variable with mean E_a and variance V_a and b is a positive constant. The aggregate supply is $\tilde{s}$ at time 3, where $\tilde{s} = H^{hf}\tilde{q}$. It is also assumed that the spot market clears at time 3.

The noise terms in the model, $(\tilde{\delta}, \tilde{\eta}_j, \tilde{\varepsilon})$ are mutually independent and distributed normally with zero mean and variance, σ_δ, $\sigma_{\eta j}$, σ_ε, respectively.

Characterization of Equilibria

We now characterize the equilibrium in the model. As in most dynamic programming problems, the period 2 problem is analyzed first and folded back into the period 1 problem.

In the second round of trading, a trader's problem is to choose the number of futures contracts to maximize his expected utility based on the USDA estimates of frozen pork bellies inventories and the futures price, i.e.,

$$Maximize \; E[-\exp(-r^i \, \tilde{w}^i)|\Omega_2], \quad i=hf,hr,s, \quad \Omega_2=(\tilde{y}, \tilde{p}_2^f).$$

The solution and the equilibrium price, $\tilde{p}_2^f$, which must be such that the futures market clears, are characterized by the following proposition (proofs are presented in Appendix A):

Proposition 1. Each producer's demand for the futures contracts at $t=2$ is

$$x_2^{hf} = -\frac{E_a}{H^{hf}} - \left(\frac{1}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)} - \frac{2b}{H^{hf}} - \frac{r^{hf}V_a}{(H^{hf})^2}\right)\tilde{p}_2^f$$

$$+ \left(\frac{1}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)} - \frac{b}{H^{hf}}\right) E^{hf}(\tilde{p}_3^s|\Omega_2).$$

The demand of the processor or stockholder for futures at t=2 is

$$x_2^{hr} = \frac{E_a}{H^{hr}} - \left(\frac{1}{r^{hr}var^{hr}(\tilde{p}_3^s|\Omega_2)} + \frac{2b}{H^{hr}} - \frac{r^{hr}V_a}{(H^{hr})^2}\right)\tilde{p}_2^f$$

$$+ \left(\frac{1}{r^{hr}var^{hr}(\tilde{p}_3^s|\Omega_2)} + \frac{b}{H^{hr}}\right) E^{hr}(\tilde{p}_3^s|\Omega_2).$$

Each speculator's demand for futures at t=2 is

$$x_2^s = \frac{E^s(\tilde{p}_3^s|\Omega_2) - \tilde{p}_2^f}{r^s var^s(\tilde{p}_3^s|\Omega_2)} .$$

The equilibrium price is

$$\tilde{p}_2^f = \frac{\Delta}{\Delta - \Delta_1 V_a var(\tilde{p}_3^s|\Omega_2)} E(\tilde{p}_3^s|\Omega_2),$$

where

$$\Delta = \left(\frac{H^{hf}}{r^{hf}} + \frac{H^{hr}}{r^{hr}} + \frac{H^S}{r^s}\right), \qquad \Delta_1 = \left(\frac{H^{hf}}{r^{hf}} + \frac{H^{hr}}{r^{hr}}\right).$$

The second round of trading results in the futures demand for all traders that is positively related to the conditional expectation of the spot price at time 2, i.e., $E(\tilde{p}_3^s|\Omega_2)$, and is negatively related to the equilibrium price, $\tilde{p}_2^f$. In addition, the producer's demand for futures also is negatively related to the conditional expectation of physical supply, $E(\tilde{q}|\Omega_2)$, and the processor's or stockholder's futures demand is positively related to the conditional expectation of purchases of the commodity, $E(\tilde{d}|\Omega_2)$.

The equilibrium futures price, $\tilde{p}_2^f$, is a linear function of the conditional expectation of the spot price. In general, this equilibrium futures price is not a sufficient statistic for the spot price, which means that the equilibrium futures price is not equal to the expectation of the spot price, i.e., $\tilde{p}_2^f \neq E(\tilde{p}_3^s|\Omega_2)$. The reason is that the randomness of the commodity residual demand, $\tilde{a}$, serves as an additional source of uncertainty. In this situation, the risk premium is greater than zero and

81

speculation exists in the futures market. However, if the residual demand is not uncertain, i.e., $Var(\tilde{a})=0$, then the futures price is a sufficient statistic and there is no speculation in the futures market.

Now consider the traders' futures demand problem at time t=1. To simplify the model, we assume that the residual demand is not random. After the analysts' forecasts are released publicly, the traders' demands for futures depend on their own generating forecasts and the market clearing price, $\tilde{p}_1^f$. The traders' demands for futures and equilibrium futures price are given in proposition 2 (proofs are given in Appendix B).

Proposition 2. Each producer's demand for futures at t=1 is

$$x_1^{hf} = -\frac{E_a}{H^{hf}} + \frac{E^{hf}(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)}{r^{hf}var^{hf}(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)} - (\frac{A^{hf}}{r^{hf}} - \frac{(B^{hf})^2}{r^{hf}D^{hf}})\tilde{p}_1^f,$$

where $\Omega_1 = (\sum_{j=1}^{k} m_{jhf} f_j, p_1^f)$, and A^{hf}, B^{hf}, D^{hf} are given in Appendix B.

The demand of the hedger who is not a producer for futures at t=1 is

$$x_1^{hr} = \frac{E_a}{H^{hr}} + \frac{E^{hr}(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)}{r^{hr}var^{hr}(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)} - (\frac{A^{hr}}{r^{hr}} - \frac{(B^{hr})^2}{r^{hr}D^{hr}})\tilde{p}_1^f,$$

where $\Omega_1 = (\sum_{j=1}^{k} m_{jhr} f_j, p_1^f)$, and A^{hr}, B^{hr}, D^{hr} are given in Appendix B.

Each speculator's demand for futures at t=1 is

$$x_1^s = \frac{1}{r^s var^s(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)} (E^s(E(\tilde{p}_3^s|\Omega_2)|\Omega_1) - \tilde{p}_1^f),$$

where $\Omega_1 = (\sum_{j=1}^{k} m_{js} f_j, p_1^f)$.

The relation between the futures purchases by each trader and expectation of the spot price conditional on the analysts' forecasts and the USDA estimates, i.e., $E^i(E(\tilde{p}_3^s|\Omega_2)|\Omega_1)$ is positive, and the relation between the futures purchases for each trader and $\tilde{p}_1^f$ is negative. Also, E_a is negatively related to the producers' futures purchases and positively related to the futures purchases of the processors and stockholders. These results are consistent with the results in the second round of trading.

Moreover, the equilibrium price is

$$\tilde{p}_1^f = \frac{\sum\limits_{i=hf,hr}^{s} \Delta_3^i \, E^{\,i}(E(\tilde{p}_3^{\,s}|\Omega_2)|\Omega_1)}{\sum\limits_{i=hf,hr}^{s} \Delta_3^i},$$

$$\Delta_3^i = \frac{H^i}{r^i} \; \frac{\lambda - \dfrac{\Theta\sigma_s^2}{\sigma_s^2 + \sigma_\varepsilon^2}}{\dfrac{\Theta\sigma_s}{\sigma_s + \sigma_\varepsilon} - \dfrac{\sigma_s^2 Var^{\,i}(\tilde{y}|\Omega_1)}{\beta^2(\sigma_s + \sigma_\varepsilon)^2}}, \quad for \ i=s, \ \Theta=0 \ and \ \lambda=-1, \ for \ i=hf,hr, \ \Theta=\lambda=1.$$

where

In general, the futures price can not be a sufficient statistic for the spot price after the analysts' forecasts are released, even though the residual demand is assumed as non-random. The reason is that traders have varied expectations of the spot price. Traders receive the same analysts' forecasts, but they use these forecasts in the different ways, generating their own forecasts. Based on their own forecasts, they have their own expectations of supply. These expectations of supply directly transfer to the expectations of the spot price. If traders use the analysts' forecasts in a similar manner, then they have the same expectations of the spot price. In this case, the futures price is a sufficient statistic for the spot price and risk premium will be zero, and there is no speculation in the futures market. These results are consistent with the results in Bray (1986). If there is one source of uncertainty about which traders have information and if they also have the same expectations, the futures price is a sufficient statistic for the spot price. When there are two sources of uncertainty and traders have no information on the one of two sources, the futures price is not a sufficient statistic.

The Linkage Between Trader's Beliefs and Forecast Information

From last section, we obtain the relationship between futures prices and traders' beliefs about the spot price when the analysts' forecasts are released and the USDA report is released. To investigate the price reaction to the USDA report, we have to link traders' beliefs to forecast information.

The studies that examine market reactions to the USDA reports follow the market efficiency hypothesis that price will respond only to unanticipated information contained in the USDA reports. There is a tradition of using the average of existing forecasts as market/traders' beliefs or the difference between the average of existing forecasts and the USDA forecast as unanticipated information contained in the USDA reports. See, for example, Falk and Orazem (1985), Grunewald, McNulty, and Biere (1993), Colling, Irwin, and Zulauf (1994), and Garcia, Irwin, Leuthold, and

Yang (1994). However, this measure could be associated with possible measurement error because the trader's beliefs are not, in general, equal to the average of existing forecasts.

In this section, we relate trader's beliefs to the analysts' forecasts and the USDA estimates, and investigate the possible measurement error. A trader's belief about the spot price at time 3 is defined as the expectation of the spot price conditional on information available at that time. At the time that the USDA estimates are released, traders form their expectations of supply based on the USDA estimates of frozen pork bellies inventories. The expectations[2] of supply are

$$E^{i}(\tilde{s} \mid \tilde{y}) = \frac{\sigma_{\varepsilon}}{\sigma_{s} + \sigma_{\varepsilon}}\bar{S} + \frac{\sigma_{s}}{\sigma_{s} + \sigma_{\varepsilon}}\tilde{y}, \qquad i=hf,\ hr,\ s.$$

The difference between expectation of supply and the USDA estimates is

$$\tilde{y} - E^{i}(\tilde{s} \mid \tilde{y}) = \frac{\sigma_{\varepsilon}}{\sigma_{\varepsilon} + \sigma_{s}}(\tilde{y} - \bar{y}).$$

This difference is not equal to zero except when the USDA estimates are equal to the mean of the USDA estimates, i.e., $\tilde{y} = \bar{y}$.

The trader's expectations of supply at the time the analysts' forecasts are released public are

$$E^{i}\left(\tilde{y} \mid \sum_{j=1}^{k} k_{ji}\tilde{f}_{j}\right) = \frac{\left(\sigma_{\delta} + \sum_{j=1}^{k} k_{ji}^{2}\sigma_{\eta j}\right)\sum_{j=1}^{k} k_{ji}\bar{f}_{j} + (\sigma_{s} + \sigma_{\varepsilon})\sum_{j=1}^{k} k_{ji}\tilde{f}_{j}}{(\sigma_{s} + \sigma_{\varepsilon} + \sigma_{\delta}) + \sum_{j=1}^{k} k_{ji}^{2}\sigma_{\eta j}}, \qquad i=hf,\ hr,\ s.$$

The differences between expectations of supply and the USDA estimates are

$$E^{i}\left(\tilde{y} \mid \sum_{j=1}^{k} k_{ji}\tilde{f}_{j}\right) - \sum_{j=1}^{k} k_{ji}\tilde{f}_{j} = \frac{\left(\sigma_{\delta} + \sum_{j=1}^{k} k_{ji}^{2}\sigma_{\eta j}\right)\sum_{j=1}^{k} k_{ji}(\bar{f}_{j} - \tilde{f}_{j})}{(\sigma_{s} + \sigma_{\varepsilon} + \sigma_{\delta}) + \sum_{j=1}^{k} k_{ji}^{2}\sigma_{\eta j}}, \qquad i=hf,\ hr,\ s.$$

The differences are not equal to zero except when each analyst's forecast is equal to its mean of forecast, i.e., $\tilde{f}_{j} = \bar{f}_{j}$. These differences can be viewed as measurement errors associated with using the existing forecast or the average of forecasts as the market/trader's belief.

Since it is assumed that the spot market clears, the expectation of the spot price at the time

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[2]The formula for calculating the conditional expectation is given in Anderson (1958, p. 27).

that the USDA estimates and the analysts' forecasts are released can be calculated and are given as follow, respectively:

$$E^{\,i}(\tilde{p}_3^{\,S}|\Omega_2) = \frac{E_a}{b} - \frac{E^{\,i}(\tilde{s}|\tilde{y})}{b}, \ and$$

$$E^{\,i}(E(\tilde{p}_3^{\,s}|\Omega_2)|\Omega_1) = \frac{E_a}{b} - \frac{\sigma_\varepsilon}{b(\sigma_\varepsilon+\sigma_s)}\bar{s} - \frac{\sigma_s}{b(\sigma_\varepsilon+\sigma_s)}E^{\,i}(\tilde{y}\ |\sum_{j=1}^{k}k_{ji}\tilde{f}_j), \ \ i=hf,hr,s.$$

Finally, price changes at the time the USDA estimates are released are given by

$$\tilde{p}_2^{\,f}-\tilde{p}_1^{\,f} = \frac{\sigma_s}{b(\sigma_s+\sigma_\varepsilon)}\left(\frac{\sum\limits_{i=hf,hr}\Delta_3^{\,i}\dfrac{(\sigma_\delta+\sum\limits_{j=1}^{k}k_{ji}^2\sigma_{\eta j})\sum\limits_{j=1}^{k}k_{ji}\bar{f}_j + (\sigma_s+\sigma_\varepsilon)\sum\limits_{j=1}^{k}k_{ji}\tilde{f}_j}{\sigma_s+\sigma_\delta+\sigma_\varepsilon+\sum\limits_{j=1}^{k}k_{ji}^2\sigma_{\eta j}}}{\sum\limits_{i=hf,hr}\Delta_3^{\,i}} - \tilde{y}\right).$$

It is difficult to measure how traders use the analysts' forecasts. To implement the price reaction function, we simplify the model by assuming that all traders weigh the analysts' forecasts equally, i.e., $k_{ji}=k_{qi}=\dfrac{1}{K}$ for $j\neq q$. Hence, the expectation of the USDA estimates at time 1 for all traders is

$$E(\tilde{y}\ |\frac{1}{K}\sum_{j=1}^{k}\tilde{f}_j) = \frac{(\sigma_\delta+\dfrac{1}{K^2}\sum\limits_{j=1}^{k}\sigma_{\eta j}\ )\dfrac{1}{K}\sum\limits_{j=1}^{k}\bar{f}_j + (\sigma_s+\sigma_\varepsilon)\dfrac{1}{K}\sum\limits_{j=1}^{k}\tilde{f}_j}{(\sigma_s+\sigma_\varepsilon+\sigma_\delta)+\dfrac{1}{K^2}\sum\limits_{j=1}^{k}\sigma_{\eta j}}.$$

With this simplification, we can obtain the following price reaction function:

$$\tilde{p}_2^{\,f}-\tilde{p}_2^{\,f} = \Phi_1 + \Phi_2 \ (surprise),$$

where

$$surprise = (\tilde{y} - \frac{(\sigma_s + \sigma_\varepsilon)\frac{1}{k}\sum_{j=1}^{k}\tilde{f}_j}{(\sigma_s + \sigma_\delta + \sigma_\varepsilon) + \frac{1}{k^2}\sum_{j=1}^{k}\sigma_{\eta j}}),$$

$$\Phi_1 = \frac{\sigma_s}{b(\sigma_s + \sigma_\varepsilon)}(\frac{(k\sigma_\delta + \frac{1}{k}\sum_{j=1}^{k}\sigma_{\eta j})\sum_{j=1}^{k}\bar{f}_j}{k^2(\sigma_s + \sigma_\delta + \sigma_\varepsilon) + \sum_{j=1}^{k}\sigma_{\eta j}}), \quad and \quad \Phi_2 = -\frac{\sigma_s}{b(\sigma_s + \sigma_\varepsilon)}.$$

The price reaction to the USDA report is proportional to the weighted mean of the analysts' forecasts and the surprise contained in the USDA report. The sign of coefficient for the surprise is negative, which is consistent with what it is expected and also is consistent with empirical finding by Colling, Irwin, and Zulauf (1994). When stocks of pork bellies are higher than expected, the futures price is expected to drop to reflect that larger-than-expected supply.

When the variation of the common noise, σ_δ, of the analysts' forecasts at the time they are released is very large or the dispersion among the analysts' forecasts is very large, $\frac{1}{k}\sum_{i=1}^{k}\sigma_{\eta j}$, traders ignore the analysts's forecasts. Therefore, the price reaction to the USDA report is proportional to the difference between the USDA estimates and the mean of the USDA estimates, i.e.,

$$\tilde{p}_2^f - \tilde{p}_1^f = -\frac{\sigma_s}{b(\sigma_s + \sigma_\varepsilon)}(\tilde{y} - \bar{y}).$$

When the variation of the common noise of the analysts' forecasts and the dispersion among the analysts' forecasts are very small (approach zero), the price reaction to the USDA report is proportional to the difference between the average of the analysts' forecasts and the USDA estimates, i.e.,

$$\tilde{p}_2^f - \tilde{p}_1^f = -\frac{\sigma_s}{b(\sigma_s + \sigma_\varepsilon)}(\tilde{y} - \frac{1}{k}\sum_{j=1}^{k}\tilde{f}_j).$$

In this case, the price reaction function is the same as the price reaction function used in previous empirical studies except that no constant term is included.

Comparing the theoretical price reaction function to the empirical price reaction function in

the previous studies, if the variation of common noise and the dispersion are not very small[3], then the previous empirical studies of price reaction to the USDA report always underestimate the unanticipated information (surprise) contained in the USDA report. The reason is that the weighted average of the analysts' forecasts in the our theoretical model is always less than the average of analysts' forecasts used in the previous empirical studies, i.e.,

$$\frac{(\sigma_s+\sigma_\varepsilon)\frac{1}{k}\sum_{j=1}^{k}\tilde{f}_j}{(\sigma_s+\sigma_\delta+\sigma_\varepsilon)+\frac{1}{k^2}\sum_{j=1}^{k}\sigma_{\eta j}} < \frac{1}{k}\sum_{j=1}^{k}\tilde{f}_j \; .$$

This underestimated unanticipated information contained in the USDA report could generate a measurement error, and either underestimate or overestimate the effect of the USDA report on the price movements.

Overall, the theoretical price reaction function has important empirical implications for market reaction to public information: (1) it suggests that previous studies of price reaction to the USDA report would underestimate the unanticipated information in the USDA report if the dispersion among the analysts' forecasts is not very small, therefore, the effect of the USDA report on the price movements could be either underestimated or overestimated; (2) it provides an explicit regression specification for testing market reaction to the USDA report.

Conclusion

In this paper, theoretical reaction of commodity futures prices to the USDA Cold Storage Reports in a two-period rational expectations equilibrium model of pork bellies futures markets is studied. The analysis provides the relation between price reaction to the USDA estimates in Cold Storage reports and the relation between forecasts and trader's beliefs.

There are three contributions of this study. First, it provides an explicit characterization of trader uncertainty when the analysts' forecasts of pork bellies inventories are provided prior to the USDA estimates. This increases our understanding of how commodity futures markets react to the release of new information from government agencies. Second, it demonstrates the possible measurement error in the empirical studies of price reaction to the USDA reports. The measurement error is the use of the forecast or the average of forecasts as proxy for trader's belief/market expectation. Third, it provides insight into information structure of commodity futures markets.

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[3] Colling, Irwin, and Zulauf (1994) show that the mean analysts' forecasts are generally unbiased. But there is no empirical evidence on the variation and dispersion among the analysts's forecasts.

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Appendix A

Equilibrium futures demand and price after the USDA announcement.

A.1 Calculation of traders' futures demand

The objective for the producers is to find x^{hf}_2 which

$$Maximize \ V_2^{hf} = Max \ E[-\exp(-r^{hf} \tilde{w}^{hf})|\Omega_2],$$

where

$$W^{hf} = \tilde{q}\tilde{p}_3^{s} + (\tilde{p}_2^{f} - \tilde{p}_1^{f})x_1^{hf} + (\tilde{p}_3^{s} - \tilde{p}_2^{f})x_2^{hf}, \ and \ \Omega_2 = (\tilde{y}, \ \tilde{p}_2^{f}).$$

There are two random variables, $\tilde{q}$ and $\tilde{p}_3^s$. Assume that the joint distribution of $\tilde{q}$ and $\tilde{p}_3^s$ is bivariate normal. So, $E^{hf}(\tilde{q}, \tilde{p}_3^s | \Omega_2)) = (E^{hf}(\tilde{q}|\Omega_2), E^{hf}(\tilde{p}_3^s|\Omega_2))$ and

$$Var^{hf}(\tilde{q}, \tilde{p}_3^s|\Omega_2) = \begin{vmatrix} var^{hf}(\tilde{q}|\Omega_2) & cov^{hf}(\tilde{q}, \tilde{p}_3^s|\Omega_2) \\ cov^{hf}(\tilde{q}, \tilde{p}_3^s|\Omega_2) & var^{hf}(\tilde{p}_3^{\tilde{s}}|\Omega_2) \end{vmatrix} \equiv \Sigma,$$

where $Var^{hf}(\tilde{q}, \tilde{p}_3^s |\Omega_2)$ is positive semi-definite (see Anderson 1958, p.29). Although all random variables are normal, the product of $\tilde{q}$ and $\tilde{p}_3^s$ is not normal, indicating that V^{hf}_2 does not have a standard moment generating function. In this case,

$$V_2^{hf} = -2\pi|\Pi|^{1/2}\int_{\tilde{p}_3^s=-\infty}^{\infty}\int_{\tilde{q}=-\infty}^{\infty} \exp(-Z)d\tilde{q} \ d\tilde{p}_3^s,$$

where

$$Z = r^{hf}W^{hf} + \frac{1}{2}(\tilde{q}-E^{hf}(\tilde{q}|\Omega_2), \ \tilde{p}_3^s-E^{hf}(\tilde{p}_3^s))\Sigma^{-1}(\tilde{q}-E^{hf}(\tilde{q}|\Omega_2), \ \tilde{p}_3^s-E^{hff}(\tilde{p}_3^s|\Omega_2))'.$$

Rearranging the above equation, we obtain:

$$Z = \frac{1}{2}x'Ax + b',$$

where $x = (\tilde{q} - E^{hf}(\tilde{q}|\Omega_2), \ \tilde{p}_3^s - E^{hf}(\tilde{p}_3^s|\Omega_2))$,

$$A = \begin{vmatrix} \dfrac{var^{hf}(\tilde{p}_3^s|\Omega_2)}{var^{hf}(\tilde{p}_3^s|\Omega_2)var^{hf}(\tilde{q}|\Omega_2)-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)^2} & \dfrac{-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)}{var^{hf}(\tilde{p}_3^s|\Omega_2)var^{hf}(\tilde{q}|\Omega_2)-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)^2}+r^{hf} \\[3em] \dfrac{-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)}{var^{hf}(\tilde{p}_3^s|\Omega_2)var^{hf}(\tilde{q}|\Omega_2)-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)^2}+r^{hf} & \dfrac{var^{hf}(\tilde{q}|\Omega_2)}{var^{hf}(\tilde{p}_3^s|\Omega_2)var^{hf}(\tilde{q}|\Omega_2)-cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)^2} \end{vmatrix}$$

$$b = r^{hf}(\,E^{hf}(\tilde{q}\,|\Omega_2),\ E^{hf}(\tilde{p}_3^s|\Omega_2) + x_2^{hf}\,),$$
$$c = r^{hf}(\,E^{hf}(\tilde{q}\,|\Omega_2)\,E^{hf}(\tilde{p}_3^s|\Omega_2) - \tilde{p}_3^s x_2^{hf} + (\tilde{p}_2^f - \tilde{p}_1^f)\,x_1^{hf}.$$

If A is positive definite, V^{hf}_2 is finite. Thus,

$$V_2^{hf} = -|\Pi|^{1/2}\det|A|^{-1/2}\exp(\frac{1}{2}b'A^{-1}b - c).$$

Therefore, V^{hf}_2 is maximized when $\frac{1}{2}\,b'A^{-1}b$ - c is minimized, that is, when

$$(\frac{\partial b}{\partial x_2^{hf}})'A^{-1}b = \frac{\partial c}{\partial x_2^{hf}}.$$

Solve this equation and obtain

$$x_2^{hf} = -\frac{[1+r^{hf}cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)]^2-(r^{hf})^2var^{hf}(\tilde{q}|\Omega_2)var^{hf}(\tilde{p}_3^s|\Omega_2)}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)}\tilde{p}_2^f$$

$$+ \frac{1+r^{hf}cov^{hf}(\tilde{q},\tilde{p}_3^s|\Omega_2)}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)}\,E^{hf}(\tilde{p}_3^s|\Omega_2)\ -\ E^{hf}(\tilde{q}|\Omega_2)\ .$$

Since we assume that $D(\tilde{p}_3^s) = \bar{a} - b\tilde{p}_3^s$ and $H^{hr}\ \tilde{d}^{..}$, we can simplify the above equation and obtain

$$x_2^{hf} = -\frac{E_a}{H^{hf}} - (\frac{1}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)} - \frac{2b}{H^{hf}} - \frac{r^{hf}V_a}{(H^{hf})^2})\tilde{p}_2^f$$

$$+ (\frac{1}{r^{hf}var^{hf}(\tilde{p}_3^s|\Omega_2)} - \frac{b}{H^{hf}})\,E^{hf}(\tilde{p}_3^s|\Omega_2)\ .$$

Similarly, the demands of processors or stockholders for the futures are

$$x_2^{hr} = -\frac{[1-r^{hr}cov^{hr}(\tilde{d},\tilde{p}_3^{s}|\Omega_2)]^2-(r^{hr})^2var^{hr}(\tilde{d}|\Omega_2)var^{hr}(\tilde{p}_3^{s}|\Omega_2)}{r^{hr}var^{hr}(\tilde{p}_3^{s}|\Omega_2)}\tilde{p}_2^{f}$$

$$+\frac{1-r^{hr}cov^{hr}(\tilde{d},\tilde{p}_3^{s}|\Omega_2)}{r^{hr}var^{hr}(\tilde{p}_3^{s}|\Omega_2)}E^{hr}(\tilde{p}_3^{s}|\Omega_2)+E^{hr}(\tilde{d}|\Omega_2)$$

$$=\frac{E_a}{H^{hr}}-\left(\frac{1}{r^{hr}var^{hr}(\tilde{p}_3^{s}|\Omega_2)}+\frac{2b}{H^{hr}}-\frac{r^{hr}V_a}{(H^{hr})^2}\right)\tilde{p}_2^{f}$$

$$+\left(\frac{1}{r^{hr}var^{hr}(\tilde{p}_3^{s}|\Omega_2)}+\frac{b}{H^{hr}}\right)E^{hr}(\tilde{p}_3^{s}|\Omega_2).$$

Speculator's demand for the futures at t=2 is

$$x_2^{s}=\frac{E^{s}(\tilde{p}_3^{s}|\Omega_2)-\tilde{p}_2^{f}}{r^{s}var^{s}(\tilde{p}_3^{s}|\Omega_2)}.$$

A.2 Calculation of equilibrium price

We assume the futures market clears: $H^{hf}x_2^{hf}+H^{hf}rx_2^{hr}+Sx_2^{s}=0$. Based on this condition, we obtain

$$\sum\frac{H^{i}}{r^{i}var^{i}(\tilde{p}_3^{s}|\Omega_2)}\tilde{p}_2^{f}-\sum\frac{r^{i}}{H^{i}}Var(\tilde{a})\tilde{p}_2^{f}=\sum\frac{H^{i}E^{i}(\tilde{p}_3^{f}|\Omega_2)}{r^{i}Var^{i}(\tilde{p}_3^{s}|\Omega_2)},\qquad i=hf,\ hr,\ s.$$

In period 2, the only information traders receive is the report of the USDA estimates. Therefore, it is reasonable to assume that traders have the same expectations and variations of the spot price at time 3. Hence, the equilibrium futures price is

$$\tilde{p}_2^{f}=\frac{\Delta}{\Delta-\Delta_1 V_a var(\tilde{p}_3^{s}|\Omega_2)}E(\tilde{p}_3^{s}|\Omega_2),$$

where

$$\Delta=\left(\frac{H^{hf}}{r^{hf}}+\frac{H^{hr}}{r^{hr}}+\frac{S}{r^{s}}\right),\qquad\Delta_1=\left(\frac{H^{hf}}{r^{hf}}+\frac{H^{hr}}{r^{hr}}\right).$$

Appendix B

Using the same procedure as in appendix A, we obtain the futures demand for traders at t=1, where

$$A^i = - \frac{var^i(\tilde{p}_3^s | \Omega_1)}{var^i(E(\tilde{p}_3^s | \Omega_2) | \Omega_1)(var^i(E(\tilde{p}_3^s | \Omega_2) | \Omega_1) - var^i(\tilde{p}_3^s | \Omega_1))} - \frac{4r^{ib}\theta}{H^i},$$

$$B^i = \frac{1}{var^i(E(\tilde{p}_3^s | \Omega_2 | \Omega_1) - var^i(\tilde{p}_3^s | \Omega_1)} + \frac{2r^i b\theta}{H^i}, \ and$$

$$D^i = - \frac{1}{var^i(E(\tilde{p}_3^s | \Omega_2 | \Omega_1) - var^i(\tilde{p}_3^S | \Omega_1)} - \frac{2r^i b\theta}{H^i},$$

where i =hf, hr. For i =hf, θ =1 and for i =hr, θ = -1.