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OPTION BASED ASSESSMENTS OF EXPECTED PRICE DISTRIBUTIONS*

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Bruce J. Sherrick, Scott H. Irwin and D. Lynn Forster**

Abstract

No-Arbitrage option pricing models are used to derive *ex ante* expected price distributions. The performance of the method is assessed in the context of the calibration of the derived probability density functions, evaluated at the expiration date prices. It is found that the soybean and S&P 500 option-based probability assessments display some evidence of mis-calibration very near to expiration and far from expiration.

A great deal of research on financial time series involves the forecasting or prediction of future events. Predicted, or expected values of economic variables serve as the primary inputs in business planning and decision making. Often, these variables are in the form of a mean value with an interval of possible error. Decision makers, particularly in a risk management context, behave as though they consider an entire probability weighted distribution of future events. Hence, the applicability of mean forecasts should be questioned. More useful would be a characterization of the entire expected distribution of a future event.[1] The range of possible outcomes, along with their associated probabilities would be highly valuable in a decision context to producers, processors, speculators, and other market participants. Improvements in decision making methods and risk management techniques would be facilitated with a means of accurately describing distributions of events rather than simply providing point forecasts.

In deriving estimates of values of *ex ante* variables, *ex post* data are often used. If, in fact, the distributions are non-stationary, use of *ex post* data may lead to seriously faulted conclusions. More useful would be a set of *ex ante* distribution parameters, or an ex ante description of the stochastic process governing the realizations of the random variables. Unfortunately, direct elicitation of *ex ante* parameter expectations from market participants is often difficult, if not impossible.

One particularly important uncertain variable is the price of a commodity or security at some future time. The interaction of all participants in futures and options markets results in a collective expression of their beliefs about future price distributions in current prices. This seemingly innocuous observation provides an avenue toward the recovery of the expected price distribution parameters without extensive surveys or direct elicitation.

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[1]In the case of an expected utility maximization, wherein most financial optimizations may easily be couched, a Taylor series expansion of the utility function introduces all higher relevant moments in some number of terms.

Specifically, options written on uncertain assets display valuable properties that may facilitate the examination of *ex ante* distributional parameters.

Options' payoffs are contingent upon the possible outcomes of the underlying security's price. The option price (premium) therefore implicitly contains the assessments and preferences of market participants over the distribution of the underlying security's outcomes. The parameters of those distributions are the variables of interest. The problem proposed herein is to "invert" the process of valuing options and use observed premia to recover implied or *ex ante* expected distributions. Necessarily, this requires ancillary assumptions regarding the process of valuation and of market efficiency. This paper demonstrates the application of one such technique which relies on a relatively weak set of restrictions governing valuation and makes full use of the simplifying assumptions of market efficiency.

Two specific issues regarding the "usefulness" of option premia data to assess expected future price distributions need to be delineated. First, the statistical properties of the estimates implied by option premia must be assessed to judge the appropriateness of using premia in conjunction with the valuation procedures to derive a complete probabilistic description of implied expected distributions. Fackler has drawn the necessary criteria together to assess the accuracy, and reliability or calibration of the derived probability functions. Secondly, and perhaps more importantly, an examination of the changes of the parameters of expected distributions over time provides insights into the fundamental economic forces that may be reflected in these markets. Presumably, market agents learn and update their information sets with the resolution of time and uncertainty about future events. If for no other reason than the passage of time, expected distributions will collapse to the expiration date price. Therefore, the equilibrium implied expected distributions will also potentially change as the agents' information sets change. The investigation of the equilibrium implied expected distributions would therefore be seriously suspect if, in addition to examining the static properties of the implied distributions, the issue of non-stationarity, or time-varying parameters, were not also considered.

There is a great deal of existing research, both theoretical and empirical, delving into the workings of the options markets. The pricing of options written on stocks has received the majority of the attention, but other derived assets are also well investigated. This study's primary focus is on puts and calls on futures instruments, specifically soybeans and the S&P 500 index. The reasons for choosing these instruments are for convenience of data and of interest. Specifically, an option pricing method that is less restrictive than those used in the past is employed to derive estimates of expected price distribution parameters. Then, the calibration of the estimates is examined. The empirical nature of this study then lends itself to extensions of the pricing techniques to other applications requiring a probabilistic description of future prices. Much can be learned about the scope and direction of adjustment that may be needed through examination of the calibration function through time.

The potential usefulness of deriving probabilistic descriptions of uncertain events permeates the decision making environment. Also, investigations of the forces that impact these distributions and an examination of how they change through time, provide fundamental insights into the workings of speculative markets.

OPTION PRICING:

Black-Scholes Methodology:

In the well known Black-Scholes (B-S) stock option pricing model, and the Black futures option pricing model, an option's price depends on the underlying asset's price, the strike price, time to expiration, an assumed constant risk-free rate of interest, and the instantaneous volatility of the underlying asset's return stream.[2] Of these variables, only the risk free rate and volatility (σ) are not easily observed. As a proxy for the risk-free rate, the yield on a T-Bill that expires near the option is often used. One means of estimating σ is to search for the value of σ that equates the particular model price to the observed option price, with the resulting estimate being termed the implied volatility (IV). Many studies then have related the IV to a broad range of economic variables and studied the properties of this estimate.[3] Technically, if the variation in the IV is then explained with any accuracy at all as a function of other variables, then the B-S formula is incomplete and the IV is not a meaningful measure. While this theoretical defect is important to recognize, pragmatically, the use of the B-S formula to yield instruments for *ex ante* variables appears to be quite useful. Possibly a more confounding inadequacy of B-S type approaches arises due to its near universal availability and (at times blind) application. The possibility of the circularity of testing B-S models with data that are largely generated through the application of B-S type models forces a recognition that the tests may be reduced to the point of actually testing only how accurately the data were collected.

No-Arbitrage Pricing:

A widely accepted basis for asset pricing is based on the set of no-arbitrage restrictions, first proposed by Ross. Absence of arbitrage is a necessary condition for market equilibrium, so the assumption that assets trade at equilibrium assures that there is no arbitrage. The widespread acceptance of no-arbitrage as a basis for asset pricing suggests that it may serve as a useful basis for option pricing as it does not suffer from many of the same restrictive features of the B-S formula.[4]

Ross and others (Breedon and Litzenberger, Banz and Miller, Cox and Ross) show that no-arbitrage implies the existence of a "supporting pricing function" and that the restrictions are also sufficient to insure market equilibrium. The pricing function may be interpreted as a set of supporting state prices for a particular state-space which in turn suggests a probability-like function for the outcomes in the state-space. Given reasonable statistical performance of the probability measure, derived parameters may serve as inputs for the next level of the investigation. That is, if parameter estimates are judged to be "reasonably well behaved", they will then serve as the variables of interest with respect to generating and investigating a complete probabilistic description of expected price distributions.

A few constructed examples in the context of arbitrage theory will help solidify the concepts and lead to the option pricing results. Consider a one period economy composed of two assets with known prices and suppose the state-space matrix is also known.

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[2]The details of these models may be found in numerous finance texts. The details are omitted here for brevity.

[3]See for example, Schmalensee and Trippi, Beckers, Chiras and Manaster, Latane and Rendleman, Park and Sears, Anderson, Jordan et al., Shastri and Tandon, and many others.

[4]In fact, the implications of the Black-Scholes approach are sometimes in direct conflict with the No-Arbitrage approach. For an interesting example, see Grinblatt and Johnson.

For example;

$$\begin{bmatrix} 2 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 2 \end{bmatrix}$$
$$\mathbf{Z} \quad \mathbf{P} \quad \mathbf{Y}$$

or, **ZP** = **Y**, where **Z** is the state-space matrix with rows referring to assets and columns referring to states and entries giving the corresponding payouts for the assets given a particular state. **Y** is the observed price vector and **P** is the "pricing function", in this case a two element vector. The condition that **ZP** = **Y** implies a set of supporting prices $\mathbf{P} = \mathbf{Z}^{-1}\mathbf{Y}$ or in this case $\mathbf{P}' = [2/3 \;\; 1/6]$. For $\mathbf{Z}^{-1}$ to exist, the state-space matrix must be of full rank implying a "complete market" or a "spanning" set of assets. The state prices may be interpreted as the price per dollar claim of a particular state, or equivalently, the cost of insuring against a particular state. The price of insuring against all states is therefore the sum of the state prices, or Σp_s and is identical to the current price of a risk-free bond that pays one dollar regardless of which state occurs. Anticipating the introduction of the common notation for a state that will be needed when we consider a continuum of states, let θ be a future state and let $r(\theta) = r_\Theta$ be the return stream for an asset in state θ. A risk-free bond, denoted b(T) or b_T promises $\mathbf{r}_\Theta = [1 \;\; 1]$ and hence $\mathbf{Y} = \mathbf{r}_\Theta\mathbf{P}$ implies, in this case, that $\mathbf{Y} = \Sigma p_s$. An identical result is obtained if combinations of asset 1 and 2 are held in proportions that promise (replicate a risk free bond) to pay 1 in either state. Example: Let w_i be the fraction of asset i held, then $\mathbf{Z'W} = \mathbf{r}_\Theta$ solves for $\mathbf{W}' = [w_1, w_2]$ via $\mathbf{r}_\Theta(\mathbf{Z}')^{-1} = \mathbf{W}$ and since the desired $\mathbf{r}_\Theta$ was [1, 1] we have $\mathbf{W}' = [1/3 \;\; 1/6]$. The required outlay for a portfolio composed of 1/3 unit asset 1 and 1/6 unit asset 2 is $\mathbf{W'Y} = (1/3)(3/2) + (1/6)2 = 5/6 = \Sigma p_s$.

If the number of independent assets is at least as great as the number of states, the above results completely generalize. If we consider the more realistic case of a finite number of assets and a continuum of possible states, bounds may be placed on the state pricing function, which we will loosely call the state price density, and denote as $Q(\theta)$ or Q_Θ. If there are multiple periods, a time argument may be added as well. If b(T) is a one dollar risk-free bond, the analogy to $b(T) = \Sigma p_s$ holds in that $b(T) = \int dQ(\theta)$. Because the emphasis is on the use of these results rather than of further exposition, the discussion is somewhat limited. A rigorous set of proofs analogous to the above assertions for continuous states across multiple time periods is given in Ingersoll.

The transition to calling $Q(\theta)$ a density rather than a set of discrete state prices is intuitively appealing and direct, but some related concepts should be discussed before accepting this proposition blindly. First, there is no guarantee that $Q(\theta)$ is unique given a continuum of states. In the discrete case, if there were more states than assets, p(s) also may not be unique and supporting state-price vectors may have been located that "implied" negative prices. While negative prices in themselves offer arbitrage opportunities, they are at least conceivable. It is, however, beyond conception to refer to a negative probability density. Ross, and Cox and Ross demonstrate the equivalence between lack of arbitrage and the existence of a strictly positive function, $Q(\theta)$. In addition, linearity (across assets) and the endogeneity of the risk-free bond price are also proved.

Given the system of asset prices, states of the asset economy, and distribution of state prices, any one of the three categories may be solved for via knowledge of the other two. The discussion above used observed prices and payoffs to determine a supporting price function, but may have as easily determined the no-arbitrage consistent asset prices given states and state prices. In the continuous framework, a return stream $\mathbf{r}_{\Theta t}$, and a distribution of state prices yields a current value consistent with no-arbitrage. It is this observation of the equivalence of the approaches that allows the option pricing methods used herein to reveal a set of parameters about an expected price distribution. Note the fact that any replicable asset in the economy may be priced via the relation:

$$V_i = \int_{t=0}^{\infty} \int_{\Theta} r_{\Theta t}\, dQ(\theta,t),$$

A bond paying one dollar at time T is therefore worth (since no intermediate time effects matter):

$$b(T) = \int_{\Theta} 1\, dQ(\theta).$$

Recall that $Q(\theta)$ is strictly positive to prevent arbitrage and therefore,

$$0 \leq Q(\theta)/b(T) \leq 1.$$

$Q(\theta)/b(T)$ behaves like a cumulative density function and in fact will serve as the proxy for the expected discounted distribution of the relevant state prices. Following Fackler's interpretation of Cox, Ross, Merton, Samuelson, and Harrison and Kreps, define $F(\theta)=Q(\theta)/b(T)$ as the artificial CDF or discounted implied pricing function. Thus $F(\theta)$ corresponds to a discounted expected cumulative density function that may be used to price options in the no-arbitrage economy. If there are other forces beyond a discount factor that affect the relation between $Q(\theta)$ and $F(\theta)$, then $F(\theta)$ may be interpreted as a utility weighted probability or as referenced in the Samuelson-Merton model, a "Util-Prob" distribution. Using $F(\theta)$, the basic valuation equation may be rewritten as:

$$V_i = \int_{t} \int_{\Theta} b(T)\, r(\theta,t)\, dF(\theta,t).$$

This equation serves as the basic valuation equation for any asset in the no-arbitrage economy. Fortunately, the application of the pricing function, $F(\theta,t)$ is simple and direct and has received previous attention. For example, in the case of calls, the Samuelson-Merton Model is:

$$V_c = \exp(r^*\tau) \int_{(S/P_s)}^{\infty} (ZP_s - S)dF(Z;\tau),$$

where V_c is the call value; r is the risk-free interest rate; τ is the time to maturity; S is the strike price; P_s is the stock price; Z is the random per dollar rate of return on the stock; and $dF(Z;\tau)$ is the risk-utility adjusted pdf of Z through τ. The impossibility of knowing the economy's $dF(Z;\tau)$ limits the direct use of the formula. However, Gastineau and Madansky have modified the model to operationalize as:

$$V_c = a1^* \exp(a_2(r^*\tau)) \int_{a_4(S/P_s)}^{\infty} a_3(ZP_s - S)dF(Z;\tau;a_5),$$

where $a_1,\ldots,a_5$ are empirical adjustment factors to take account of market frictions and $F(Z;\tau)$ is taken as an empirical distribution function, and other terms are as defined before. The models are presented here to indicate the framework in which our estimates are derived and point out that the approach has a well founded base in the literature.

The small percentage of options that are exercised early will be taken as sufficient evidence that the difference in value between European and the American options considered in this study is negligible. The significance of this assumed equivalence is that it allows futures options to be priced via $F(\theta,T)$ alone with no need to consider intermediate time effects. Calls and puts written on futures have clear return functions. For calls, the return at time T is simply $\max\{Y_T-x, 0\}$ where x is the exercise price and Y_T is the random futures price at time T. For puts, the function is $\max\{x-Y_T, 0\}$. The value of these contracts may be written in terms of the basic valuation equation as:

$$C = b(T) \int_0^\infty \max\{Y_T-x, 0\}\, dF(Y_T) \quad \text{and}$$

$$P = b(T) \int_0^\infty \max\{x- Y_T, 0\}\, dF(Y_T).$$

By noting that for $Y_T > x$, $P = 0$; and for $Y_T<x$, $C=0$, we can rewrite the above two equations as:

$$C = b(T) \int_x^\infty \{Y_T-x\}\, dF(Y_T) \quad \text{and}$$

$$P = b(T) \int_0^x \{x-Y_T\}\, dF(Y_T).$$

A key distinction between this and the typical option pricing approach is that no assumptions have been made about the price dynamics or in fact about anything in the interval prior to expiration. The only assumptions thus far made are that there are no arbitrage opportunities, thus guaranteeing the existence of $F(\theta)$. Even if $F(Y_T)$ were imposed (like most approaches, i.e. lognormal) the interpretation of the parameters differs from the B-S case where they describe the aspects of the price dynamics.

ESTIMATION OF EXPECTED DISTRIBUTION PARAMETERS:

For obvious reasons, we wish to choose distributions to consider for F that are as unrestrictive as possible. To be practical, a distribution is needed that does not allow negative prices (simple arbitrage), has relatively few parameters, and allows a fairly wide range of shapes to emerge for the CDF. Many studies suggest the lognormal distribution is not very descriptive of reality.[5] Studies find empirical distributions that are more leptokurtic and more or less skewed than that implied by a lognormal distribution of prices.

An important, albeit untestable point, is that there are no directly measurable or observable utility weighted expectations which are the true variables that influence current prices. It may be that market participants actually have consistently biased or inaccurate expectations and the empirical or implied distributions are in fact irrelevant to pricing. If market participants have consistently "inaccurate" expectations, the function $F(Y_T)$ may not accurately reflect those expectations. Empirical studies must make assumptions that facilitate a solution and in this case it is assumed that the mathematical sense of expectation and the normative sense of a utility weighted expectation are indistinguishable.

Based on preliminary work by Fackler, et al., this study will use the lognormal and Burr-12 or Singh-Maddala distributions as the primary candidates for $F(Y_T)$. Again, a beneficial bonus is the attention the statistics and insurance literatures have given these distributions. Both have a zero (non-negative) support and the Burr-12 may take on a wide

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[5]See Gordon or Hall, Brorsen, and Irwin and the references therein.

range of skewness and kurtosis. To operationalize, a form of the pricing function needs to be derived in which $F(Y_T)$ may be isolated. Then, an estimation criterion must be applied to the data. The typical identification problem and other feasibility and regularity conditions are, of course, assumed to be met.
The put formula may be written as:

$$P = b(T)\{ \int_0^x x dF(Y_T) - \int_0^x Y_T dF(Y_T)\}.$$

Integrating the second term by parts:

$$= b(T)\{xF(x) - [xF(x) - \int_0^x F(Y_T)dY_T]\}$$

$$= b(T) \int_0^x F(Y_T)\, dY_T.$$

In this form, it may be seen that P and b(T) are sufficient to recover $F(Y_T)$. The expression:

$$\min_{\beta} \sum_i^n \{(P_i - \int_0^{X_i} F(\beta)dY_T)^2\} + \sum_j^m \{(C_j - \int_{X_j}^{\infty} F(\beta)dY_T)^2\}$$

(Fackler, pg 29), where n is the number of observed put prices and m is the number of call prices, may be solved for β, the vector of parameters of F. If only one type of option is used, then either n or m may be zero.

As mentioned, the preferred forms for $F(\beta)$ to be investigated are the Singh-Maddala (SM) and lognormal distributions. The SM CDF is:

$$SM(y;a,b,q) = 1-(1+(y/b)^a)^{-q} \text{ for } y \geq 0,\ a,b,q > 0,$$
$$= 0 \quad \text{else.}$$

Hence, at least three data points per contract are required to recover the three parameters of the distribution. The CDF of the lognormal distribution is given by:

$$N(\ln((y)-\mu)/\sigma)$$

where $N(\bullet)$ is the cumulative normal density function. Details of the ancillary distributions that require evaluation may be found in Fackler.

The dispersion parameter in this approach has a different interpretation than the B-S implied volatility (IV). A comparison of the two distributions is made to lift up possible improvements in moving to a three parameter distribution. There is no guarantee that the parameters of $F(\beta)$ will conform to an *ex post* price distribution. Indeed, it is a simple mathematical construct with probability-like properties. Before using the derived parameters in an investigation of the economic forces reflected in the markets, an investigation of the properties of the parameters must be conducted. Only if they are judged to perform well will they become admissible candidates for decision making inputs. This evaluation is the topic of the next section.

EVALUATION OF ESTIMATED PARAMETERS:

Calibration, or reliability, refers to the correspondence between a predicted and an actual event. In terms of distributions, calibration describes how close the predicted and resulting functions are. If there were a reason for the market's aggregation of individual expectations to yield parameters that required a particular adjustment to correspond to the "true" variables, then this adjustment is the calibration function. Specifically, if the *ex ante* parameters of a distribution are truly $\phi(x)$ and the estimates are $F(x)$, then $K(F(x)) = \phi(x)$ implicitly defines a transformation $K(\bullet)$ of F to generate estimates $K(F(x))$ that are well calibrated or reliable. The function $K(\bullet)$ is called the calibration function. Equivalently, given a subjective or implied p.d.f., the process generating the subjective or implied p.d.f. is said to be well calibrated if the proportion of times the realized value lies below the r^{th} fractile of the implied p.d.f. is equal to r (Curtis, et al.). Notice that the calibration accounts for more than a simple bias in that it corrects all moments of an estimated distribution. The result of calibration is to make the long run probabilities (density) of $K(F(x)) = \phi(x)$ for any level of x. The notion of accuracy is akin to the variability about this conformity. That is, a constant forecast for $F(x)$ equal to the long run mean, regardless of the level of x is well calibrated in the mean but not very useful in a decision context. Instead, we want a $F(x)$ which is accurate across the entire range of x. If $F(x)$ is already well calibrated, then $K(\bullet)$ will be simply a uniform density. If, for example, $F(x)$ places too much weight in the lower tail, $K(\bullet)$ will be lower than a uniform density at low values of x and higher at high values reflecting the re-weighting of F that is necessary to force a correspondence to $\phi(x)$. $K(\bullet)$ therefore re-weights $F(\bullet)$ and is itself a probability measure. The test for calibration then, is equivalent to testing the uniformity of K, for if $F(\bullet)$ is calibrated, K is simply a one-to-one mapping whose CDF is a straight line. In this context, an independent sample of various $F_i(\theta)$ evaluated at their realizations, Y_T, so that $F_i(Y_T) = x_i$, gives a means of modeling a calibration function. A sample of the x_i are collected and a calibration function is fitted to the sample x_i. The calibration function is then tested for its departure from uniformity. If there is no evidence that the calibration function is not uniform, then the process generating the $F(Y_T)$ is well calibrated. If there is significant departure from uniformity, the shape of the calibration function may be examined to infer the location and dispersion biases that may exist. For the purposes of this study, the calibration function is based on the beta function with density

$$K(x) = x^{p-1}(1-x)^{q-1}/\beta(p,q),$$

where $\beta(p,q)$ is the beta function with parameters p and q. Fackler outlines a means of using maximum likelihood estimates of the parameters of the beta distribution to explicitly model the calibration function. With the above procedure, market specific effects (non-stationarities in the process driving Y_T due to, say, drought, etc.) are empirically indistinguishable from non-calibrated estimates. Furthermore, a process describing the evolution of the parameters of $F(Y_T)$ via other variables would be more general than a point by point correction via a calibration function. Unfortunately, the options markets and applications of this technique to the options markets are too young to establish definitively whether a stable calibration function will emerge that could be used to improve estimates of price distributions.

DATA DESCRIPTION:

A strength of this study lies in its use of a fairly long time series of contemporaneous futures and option data. The data consist of all time stamped transactions of the S&P 500 futures and options and soybean futures and options from the inception of these markets on 1/28/83 and 10/31/84 respectively and ending 9/30/88. The data are provided on tape from the Chicago Mercantile Exchange and the Chicago Board of Trade and are thought to be highly accurate and free of errors. These two instruments provide some interesting comparisons and enjoy high volume and widespread acceptance as investment instruments. In addition to all trades at which a price change occurs, the data set contains bids that exceed, and asks that fall below, the previous transactions. Volume data per se are not available, but an examination of the mean trading price-change time interval was examined as a flag for possible liquidity strains. Lack of volume does not appear to be a problem per se, but the absence of data on days that the soybean futures "hit a limit" does filter out several days that would have been interesting to examine. Also, the S&P data suffered some distortions on October 19 and 20, 1987, that cause those two days to largely be dismissed.

Some exclusion criteria were considered to alleviate some induced biases. Trades that occurred more than one year prior to expiration were also excluded. Also, deep in- or out-of-the-money options were scrutinized carefully although there is no theoretical reason for exclusion. Next, the time interval between trades of matched prices should be as short as possible. Two earlier studies (Whaley, and Ogden and Tucker) require that the futures price precede the option price. Jordan et al. simply require that the put, call, and futures prices each occur within a common thirty-second interval. Since there is little reason (other than volume) to expect that one price necessarily leads or causes the other, no a priori imposition of order was made. Also, based on Bookstaber's arguments, it was felt that synchronous futures and option prices were necessary to avoid possible distortions found in closing or settlement prices. And, in order to minimize the day-to-day effects, the point in time during the day should be during a relatively stable trading interval and away from the opening and closing periods when price swings may be exaggerated. Hence, to generate the sample of trades used, a point in time near the center of the trade day (11:00) was chosen and one trade per strike price traded that day was chosen based on its proximity to 11:00. Then, the futures price nearest in time to each option was selected as the "matched" futures price. Soybeans were required to have a matched futures observation within 90 seconds of the option price and S&P options were screened at a 60 second limit. Days with less than three option trades in our time window were deleted. A description of the resulting sample is given in table 1.

To solve for the parameters of the expected price distributions, a risk-free rate was needed. The rate used is based on the daily discount-basis yield of three month T-Bills as provided by the Federal Reserve Bank of Cleveland.

METHODOLOGY:

The expression

$$\min_{\beta} \sum_{i}^{n} \{(P_i - \int_0^{X_i} F(\beta)dY_T)^2\} + \sum_{j}^{m} \{(C_j - \int_{X_j}^{\infty} F(\beta)dY_T)^2\}$$

was solved for a set of implied distribution parameters, β, using the daily sample combination of puts and calls. In the case of the SM, β is a three component vector, and in the case of the lognormal, it is a two parameter vector. As many strike prices were used for each option each day as possible (subject to the exclusion criteria discussed earlier). This expression was minimized daily for the two distributions suggested to yield the time series of implied parameters to be investigated. Specific market and contract effects dictate that each contract be treated essentially as a unique instrument, however, which limits the degree to which pooling across contracts may take place in an attempt to improve statistical power.

The literature provides clear evidence that there is an interest in explaining time varying parameters of *ex ante* distributions. However, the lack of convincing existing evidence based on IV type estimates of volatility forces a reconsideration of the techniques. Many tests encountered involved regressing IV on a time-to-maturity variable. It is suggested here that there may be a confounding of the month, year, contract month, and other time effects that renders simple univariate time-to-maturity tests weak.

An average futures price was calculated for each day as the simple average of the synchronous futures prices for the options used in the estimation. Since the futures prices do not enter into the estimation of the distributions, comparison of the first moment of the implied distribution with the average futures price gives an indication of the expected direction of futures price movements. For example, if the average futures price is consistently lower (higher) than the mean of the "pricing" distribution, it indicates that the futures price is expected to increase (decrease) at expiration. If there is a general agreement across contracts, support may be given to the hypothesis that the current futures price is, in fact, equal to the expected price at expiration. Table 2 summarizes the mean difference between average futures price and $E(Y_T)$ as reflected in the implied distributions. Note particularly the increased mean deviation over the August and September contracts in the soybean market reflecting possibly the increased uncertainties surrounding harvest. Also note that in both markets, the BR-12 method yielded estimates that were closer than the lognormal. If the connection in current futures prices to option prices is maintained through practices such as conversion-reversal, this improved fit may simply reflect the increased flexibility of the three parameter distribution rather than of specific contract effects.

To examine the usefulness of this approach to generate reliable descriptions of future distributions, the issue of calibration is examined. The 26 time series generated from various soybean contracts and the 12 series generated form the S&P contracts are used to generate independent samples of $F_i(Y_T)$ at several fixed intervals prior to expiration, evaluated at their respective expiration date prices. In other words, at fixed non-overlapping intervals prior to expiration, the $F(\theta_t)$ from each contract are evaluated at their expiration date futures prices. If the $F_i(\theta_t)$ are well calibrated, the distribution of the realizations $F_i(Y_T)$ will correspond to the probabilities assigned to that $F_i(Y_T)$. The five non-parametric tests outlined in Stephens and implemented in Fackler were used to test the resulting empirical distributions of the $F(Y_T)$ departures from uniformity. In addition, the fitted beta distribution was examined as an indication of the shape of the calibration function. Note that the Beta(p,q) distribution with $p=q=1$ corresponds to a uniform density and hence, examination of the fitted p,q values of the beta distribution reveals information about the types of biases present. Table 3 give a summary of the results of the fitted beta calibration functions. The five non-parametric tests were in general agreement with the beta test. Specific results of the non-parametric tests are available upon request.

The interpretation of the calibration function is that it serves to re-weight the estimated CDF to arrive at one that would have allowed the realizations to occur with highest probability. If the calibration functions estimated were constant, they could be used as one of the "modular" adjustments that was suggested might be needed to result in an easily used decision input. The likelihood ratio statistic is also calculated and the corresponding p-value for the null of uniformity is given. In each of the 4 cases, there is some evidence that the market's assessments were not well calibrated one week prior to expiration. The general shape of the Beta distribution for that time interval suggests that the option based assessments were overdispersed and shifted to the left, or equivalently, that the expiration futures prices were drawn from a distribution that was less dispersed and located at a higher level than that implied. Figure 1 shows a representative Beta calibration function and the corresponding $F_t(Y_T)$ and calibrated $K(F_t(Y_T))$ for the sample calibration function shown in the top panel. The specific contracts are the June 1988 S&P with the estimated calibration function for sixty days out, and the May 1988 Soybean contract at thirty-five days out along with a shorter-term calibration function. Note the general shifting upward of the S&P distribution and the tightening and shifting of the soybean distribution required for calibration.

Over the range from 10-40 days prior to expiration, there is little firm evidence in either the soybean or S&P 500 market of miscalibration. In the soybean market, the August contract was dropped from the sample from 40 days and beyond in order to avoid overlapping of intervals from which the calibration functions were estimated. Other differences in sample sizes arise due to limit days or other days on which lack of data precluded the calculations of estimated parameters at a particular interval prior to expiration. From 80 to 100 days in the soybean market and at 100 days in the S&P market, the samples are not strictly independent, so the results must be interpreted with caution. However, the portion of the interval that overlaps the adjacent interval is approximately 10% so the distortions on this account may not be large. With this condition in mind, there still seems to be some evidence of miscalibration at long intervals prior to expiration. Specific time period effects, such as a prolonged, unanticipated growth or a sudden drop could easily account for these effects in the S&P case. Unanticipated shocks in the soybean market, too, could cause estimates that appear to indicate miscalibration at long intervals. If the hypothesis holds that the current futures price is the "best guess" of the futures price at expiration, then non-stationarities in the futures price could manifest themselves as non-calibrated estimates of the expiration date price distribution.

Figure 2 shows a sample of the other calibration functions. They may be interpreted as follows. Since the PDF is given by the derivative of the CDF, the slope of the calibration function indicates the reweighting of the implied distribution needed to recover a calibrated distribution. A form of location bias is determined by examining the value of the calibration function at K(.5). If K(.5) is greater (less) than .5, then the implied distribution is located too far to the right (left). These various shapes are given as a "menu" against which to compare the p and q values given in table 3 to ascertain the approximate shape of the various calibration functions.

Although the lack of significance of the p-values for many of the intervals tested reduces the strength of the findings, a general shape that may arise for the calibration function indicates that the implied distribution is somewhat over dispersed and located to the left of a calibrated one. This could arise for example, due to a general tendency for futures prices to rise at expiration (for any number of reasons) coupled with less volatility than presumed.

To generate calibrated estimates, a calibration function may be used to transform any particular $F_i(Y_T)$ into a reliable estimate. The problem then arises in determining the appropriate calibration function. As implemented here, there does not seem to be enough agreement among the fitted beta calibration functions to suggest a single best one to use. Nor do we have a large enough sample to determine if the calibration function changes systematically as the time period to expiration varies. All that may be hoped is that repeated application of these techniques will confirm these preliminary results and shed further light on the calibration of the estimates of future price distributions.

CONCLUDING COMMENTS:

The improved data and parameter estimation techniques of this study provide an interesting backdrop for empirical study. Business risk management techniques rely on accurate descriptions of uncertainty. This study demonstrates one such technique for describing an uncertain price distribution. The tests of calibration fail to reject the notion that the method is well calibrated over intermediate time ranges in these two markets. The usefulness in this context is therefore immediate and direct although specific implementations will only slowly emerge as these techniques begin to replace the more familiar B-S procedures.

A direct application of these techniques would also be to generate a benchmark for event studies. Simple Chow tests for the stability of the estimates would indicate whether the markets actually underwent significant change at points specified as events. This appeal of taking this approach is that instruments for variables affected by "significant events" are summarized in a convenient and simple form -- the expected distribution of prices. Rather than presume to know events, and impose a structure for testing, use of *ex ante* benchmarks would conserve degrees of freedom and in some sense endogenize the switch points. Related work by the authors indicates there are substantial structural shifts in these two markets and care should therefore be used in describing the process generating the observed prices.

A possible extension of this research involves pricing and trading strategies. If low cost estimates of future price distributions are available and reliable, they may be easily incorporated in forecasted outcomes of various trading strategies. Or, if "better" descriptions of future uncertainties are available, pricing models may be developed that accurately reflect the inner workings of the particular market.

The literature is sparse in terms of ag-futures options studies. So too does it lack a good set of guidelines on pricing via no-arbitrage restrictions. The recent appearance of these topics as well as some relatively new and novel techniques will begin to fill a gap in the existing literature. Along the same vein, suggestions for new technique applications to existing problems is always a fruitful topic. The evolution of new thought and the displacement of existing mindsets is a slow and uncertain process. Hopefully the empirics herein will add fuel to the debate over the proper paths to pursue and highlight one alternative means to recovering probability distributions.

The intent of this analysis is more modest though, than to propose a solution to the range of the above problems. Instead, we simply document the features of these markets and demonstrate a technique that is in principle more appropriate than many past ones. It is thought that in the spirit of the Gastineau "modular adjustment" model, this technique could spawn a set of investigations of the adjustments necessary to be used in a wide variety of risk management situations.

Table 1.
Descriptive Statistics of Options Samples:

SOYBEANS

Contract	Days	Total Obs.	Difll	DifFO	% Calls
Jan-85	28	192	11.6	14.2	50.5
Mar-85	57	400	16.4	18.1	61.3
May-85	72	482	16.6	21.2	64.7
Jul-85	93	684	17.2	21.8	65.8
Aug-85	32	213	21.0	28.5	65.8
Sep-85	22	169	22.4	33.2	71.0
Nov-85	138	1310	16.9	18.2	63.3
Jan-86	58	432	16.1	21.8	58.3
Mar-86	79	619	14.8	19.3	64.8
May-86	86	569	18.8	24.5	65.9
Jul-86	120	870	18.8	25.3	71.3
Aug-86	11	157	23.9	35.1	70.7
Sep-86	20	184	23.8	39.5	66.1
Nov-86	74	1338	17.2	19.7	60.4
Jan-87	44	326	19.0	26.5	61.3
Mar-87	32	310	19.9	28.9	62.0
May-87	20	242	21.9	33.4	67.4
Jul-87	87	687	18.1	25.2	68.1
Aug-87	49	420	18.6	33.0	67.2
Sep-87	42	335	19.4	35.5	58.5
Nov-87	121	1365	14.1	13.7	64.0
Jan-88	81	653	14.2	20.5	64.0
Mar-88	106	953	14.5	16.8	67.2
May-88	92	787	15.2	17.3	62.9
Jul-88	128	1058	17.6	19.1	68.5
Aug-88	23	265	23.3	28.3	66.8

Days........Total number of trade days for which option premia were sufficient to recover parameters of implied distributions.
Total Obs...Total number of observations that remained in the contract after the deletion/estimation criteria.
Difll.......Mean absolute difference in minutes of all strike prices used from 11:00 a.m.
DifFO.......Mean absolute difference in seconds between the option and futures prices.
% Calls.....Percent of the sample represented by calls.

Table 1. (Continued)

S&P 500*

Contract	Days	Total Obs.	Difll	DiffO	% Calls
Mar-85	70	536	9.3	9.4	56.9
Jun-85	79	539	9.9	11.7	59.2
Dec-85	78	651	10.1	11.5	56.5
Mar-86	87	836	8.7	9.7	52.3
Jun-86	93	977	8.8	9.4	53.5
Sep-86	93	1034	8.9	9.9	51.5
Dec-86	94	1075	7.8	9.6	56.2
Mar-87	91	1176	8.2	8.4	48.2
Jun-87	88	1365	8.8	8.0	51.2
Sep-87	99	1352	9.2	9.0	50.4
Dec-87	77	1513	11.3	8.1	53.3
Mar-88	71	888	12.2	8.4	47.7
Jun-88	72	804	12.1	8.2	39.1

Days........Total number of trade days for which option premia were sufficient to recover parameters of implied distributions.
Total Obs...Total number of observations that remained in the contract after the deletion/estimation criteria.
Difll.......Mean absolute difference in minutes of all strike prices used from 11:00 a.m.
DiffO.......Mean absolute difference in seconds between the option and futures prices.
% Calls.....Percent of the sample represented by calls.

*The Sep-85 contract data was unusable on the tape.

Table 2.
Mean differences between the implied mean futures price and the daily average futures price: (Average Price - Implied Mean)

Soybeans: Contract	Method Lognormal	BR-12
Jan-85	0.16553	0.11962
Mar-85	0.07138	0.13666
May-85	-0.06940	-0.07597
Jul-85	-0.30083	-0.14493
Aug-85	-1.25887	-0.73811
Sep-85	1.43952	-5.03158
Nov-85	-0.69442	-0.36421
Jan-86	0.02143	-0.01777
Mar-86	0.35562	0.25085
May-86	-0.07690	0.02720
Jul-86	0.22326	-0.53157
Aug-86	6.79759	1.43855
Sep-86	-1.12006	0.20227
Nov-86	6.11633	2.04194
Jan-87	-0.10032	-0.07273
Mar-87	-0.00735	-2.58517
May-87	-0.02387	-0.09380
Jul-87	1.16975	0.00814
Aug-87	2.83306	1.66205
Sep-87	0.09538	-0.36359
Nov-87	0.33831	0.26755
Jan-88	-0.06963	-0.08886
Mar-88	0.54526	0.36211
May-88	0.52561	1.31769
Jul-88	2.08270	0.79288
Aug-88	-0.91181	0.08815
Summary:		
Mean absolute Difference:	1.05439	0.72399
Mean Difference:	0.69797	-0.05356

Table 2. (Continued)

Mean differences between the implied mean futures price and the daily average futures price: (Average Price - Implied Mean)

S&P 500: Contract	Method: Lognormal	Method: BR-12
Mar-85	0.02620	0.02283
Jun-85	0.02144	0.03087
Dec-85	-0.00545	-0.02034
Mar-86	0.04073	0.00495
Jun-86	0.03168	0.07488
Sep-86	0.00032	-0.00572
Dec-86	0.01278	0.06784
Mar-87	0.01146	0.06847
Jun-87	0.08278	0.04790
Sep-87	0.03311	0.00447
Dec-87	0.49501	-0.42667
Mar-88	0.22052	-0.01997
Jun-88	0.62538	-0.55524

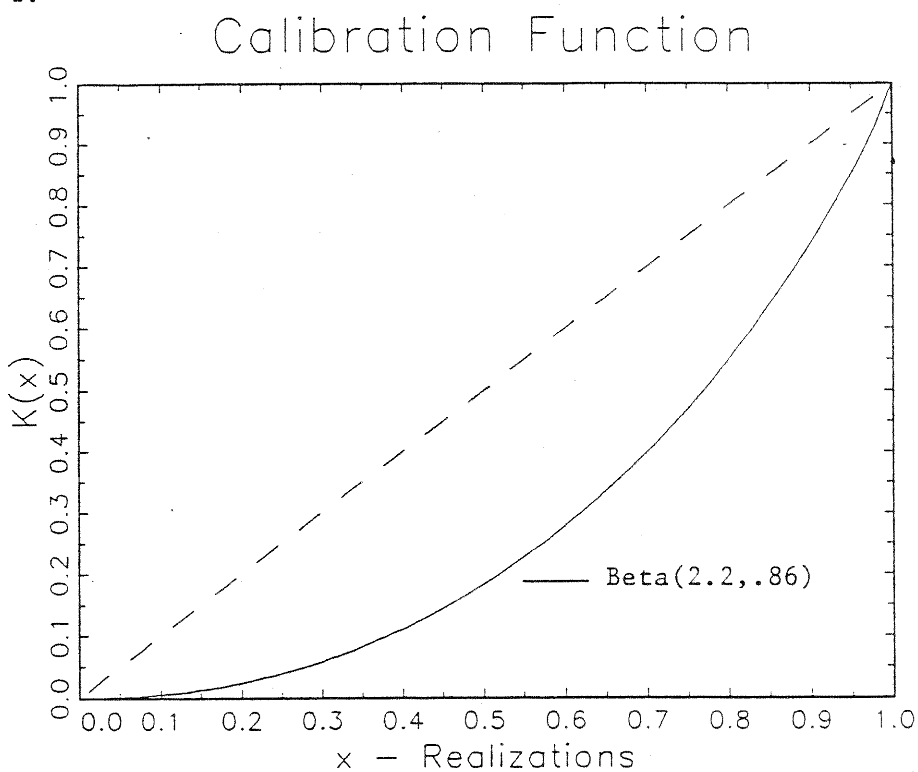
Summary:		
Mean absolute Difference:	0.12360	0.10385
Mean Difference:	0.12277	-0.05428

Table 3.

Calibration Statistics:

Days to Maturity	Number of Observations	Parameters of Beta Calibration Function p	q	LR Statistic	prob > LR
Lognormal S&P 500:					
7	12	3.246	2.294	6.285	0.043
10	12	1.010	0.851	0.354	0.838
20	12	0.813	0.770	0.558	0.757
40	12	1.758	0.969	3.455	0.178
60	12	2.116	0.795	8.335	0.015
80	12	0.639	0.429	7.613	0.022
100	12	1.032	0.641	2.997	0.223
Burr-12 S&P 500:					
7	12	3.042	2.153	5.726	0.057
10	12	0.977	0.819	0.445	0.801
20	12	0.857	0.786	0.451	0.798
40	12	1.761	1.062	2.775	0.250
60	12	2.188	0.862	7.594	0.022
80	12	0.757	0.480	5.870	0.053
100	12	1.011	0.580	4.388	0.111
Lognormal Soybeans:					
7	22	5.274	3.660	19.978	0.000
10	24	1.455	1.117	2.067	0.356
20	25	1.331	1.228	1.070	0.586
40	21	1.184	0.724	4.480	0.106
60	20	1.288	0.791	3.854	0.146
80	19	0.898	0.538	7.544	0.023
100	19	0.894	0.506	9.444	0.009
Burr-12 Soybeans:					
7	22	4.678	2.901	17.713	0.000
10	24	1.428	1.091	1.982	0.371
20	25	1.359	1.261	1.234	0.540
40	21	1.410	0.930	2.975	0.226
60	21	1.463	1.031	2.410	0.300
80	19	1.259	0.845	2.438	0.295
100	19	1.442	0.895	3.451	0.178

Figure 1.


Calibration Function
K(x)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x — Realizations
Beta(2.2,.86)

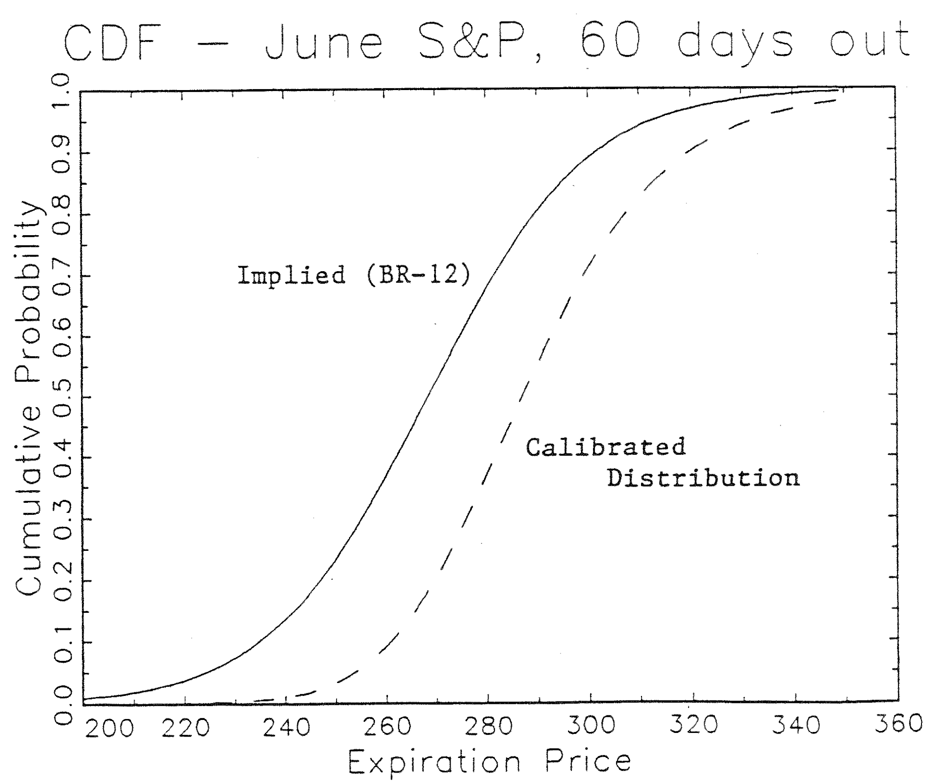
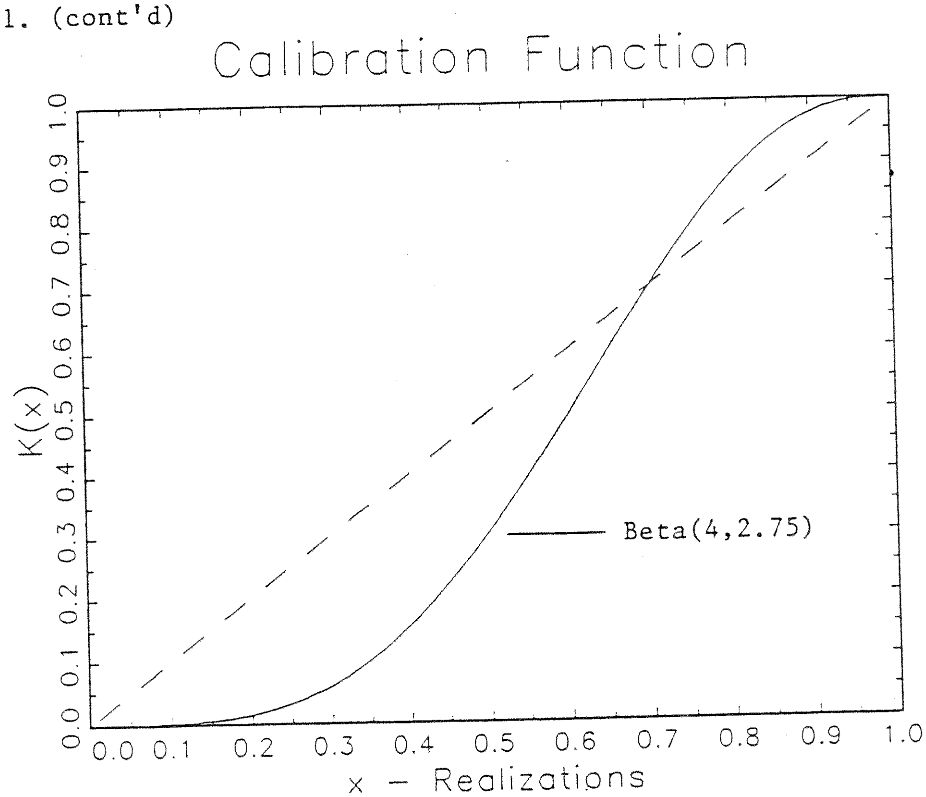

CDF — June S&P, 60 days out
Cumulative Probability
200
220
240
260
280
300
320
340
360
Expiration Price
Implied (BR-12)
Calibrated
Distribution

Figure 1. (cont'd)



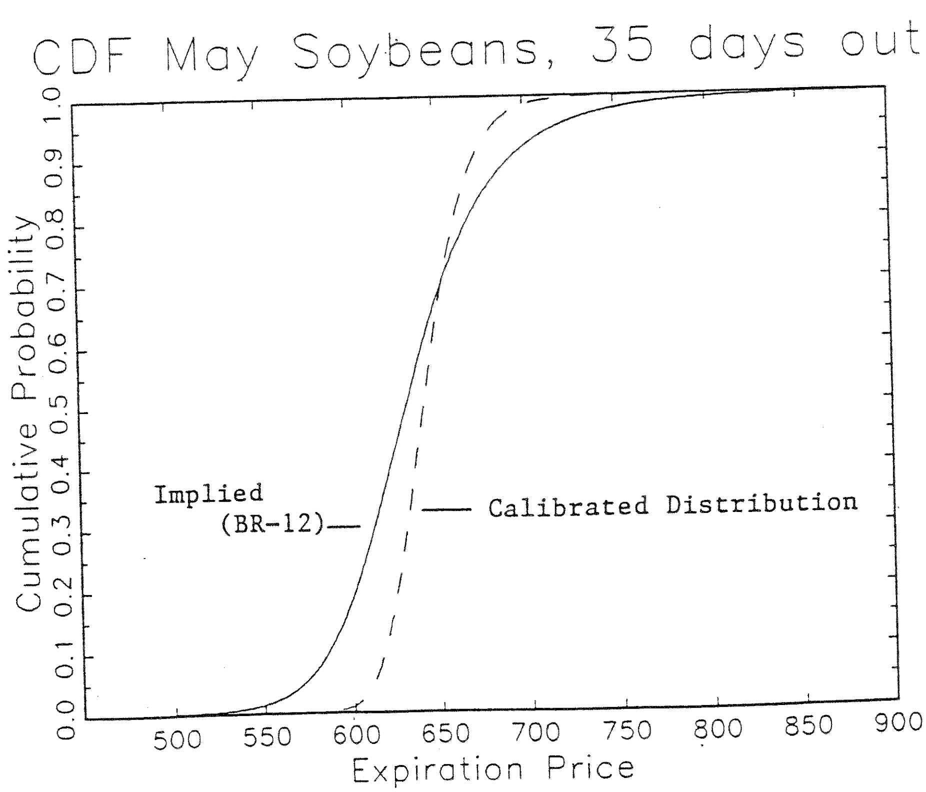
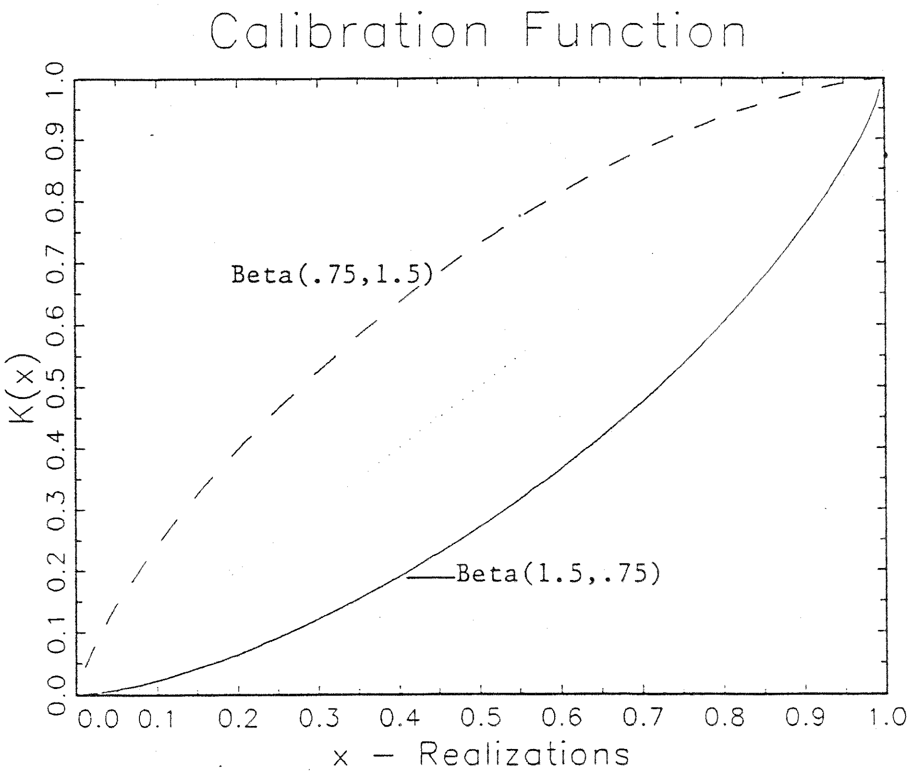
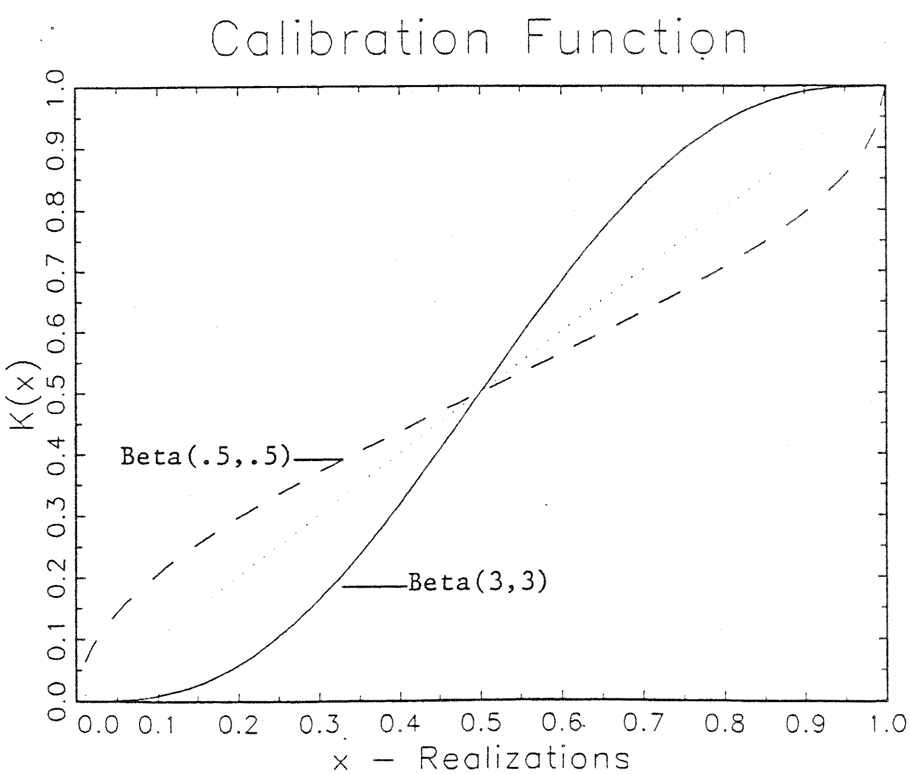


Figure 2.


Calibration Function
K(x)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Beta(.75,1.5)
Beta(1.5,.75)
x – Realizations


Calibration Function
K(x)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Beta(.5,.5)
Beta(3,3)
x – Realizations

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