NCCC-134
APPLIED COMMODITY PRICE ANALYSIS, FORECASTING AND MARKET RISK MANAGEMENT

Generalized Autoregressive Conditional Heteroskedasticity as a Model of the Distribution of Futures Returns

by

B. Wade Brorsen and Seung-Ryong Yang

Suggested citation format:

Brorsen, B. W., and S.-R. Yang. 1989. "Generalized Autoregressive Conditional Heteroskedasticity as a Model of the Distribution of Futures Returns." Proceedings of the NCR-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management. Chicago, IL. [http://www.farmdoc.uiuc.edu/nccc134].

GENERALIZED AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY AS A MODEL OF THE DISTRIBUTION OF FUTURES RETURNS

B. Wade Brorsen and Seung-Ryong Yang*

Knowing the correct probability distribution function for price movements is important in economic modeling. Black and Scholes' option pricing model and portfolio models of asset allocation are typically derived assuming the change in the natural logarithm of price is normally distributed with constant variance. If the distribution is not normal, the estimated option premium will be biased and statistical tests based on normality are likely to give misleading results (Fama, 1965). If the distribution is normal, but the variance is non-constant, then an adjustment for heteroskedasticity must be made when conducting statistical tests (Taylor). While the distribution of daily price changes are often assumed to be normal, research on stock prices (Fama, 1965; Officer; Teichmoeller; Barnea and Downs) and futures prices (Hudson, et al.; Cornew, et al.; Gordon; Taylor) has found that the distribution is leptokurtic rather than normal (i.e. having more values around the mean and in the extreme tails). This leptokurtosis also appears in exchange rate changes (Westerfield; McFarland, Pettit and Sung; Friedman and Vandersteel), and slightly in spot commodity price changes (Taylor).

The first hypothesis which was proposed by Mandelbrot and Fama (1967) to explain the observed leptokurtosis is that the distributions follow a symmetric stable Paretian law. A stable Paretian distribution is, by definition, invariant under addition. That is, sums of an independent stable variable will also be stable with the same form as the individual variables. This distribution has infinite second and higher moments and can model the observed leptokurtosis. Recent studies, however, show evidence against this hypothesis through tests vs. alternative distributions (Blattberg and Gonedes; Tucker and Pond) or stability-under-addition property of stable distributions (Fielitz and Rozelle; Hall, et al.).

As an alternative to the stable Paretian distribution, the student t-distribution allows the variance to change following an inverted gamma-2 distribution, and has fatter tails consistent with the observed leptokurtosis (Praetz). Praetz and Blattberg and Gonedes suggest evidence in favor of this hypothesis against the stable distribution for stock price changes.

Another hypothesis implied by the results of the stability-under-addition tests is that each sample is drawn from two or more different normal distributions with different means and variances, which lead to skewness and leptokurtosis, respectively. Kon found a discrete mixture of normal distributions more descriptive of stock market returns than the student distribution. Also, Akgiray and Booth (1989) and Tucker and Pond (1989) show the same evidence for exchange rates changes. They, however, suggest a different distribution, a mixed diffusion-jump process. This model is thoroughly discussed in Clark (1973) and Merton (1976). Tucker and Pond, and Akgiray and Booth found a mixed Brownian motion and an independent and homogeneous compound Poisson process as a more likely model of exchange rate movements than the stable, student and mixture of normal distributions.

---

* Associate Professor and Ph.D. Candidate, respectively, Department of Agricultural Economics, Purdue University, West Lafayette, IN 47907

These mixed distributions, however, should be distinguished from the processes with time-varying parameters. In the latter, the distribution itself from which an observation is drawn is time dependent while in the first the mixed distribution remains the same all the time. This difference can be also explained by different behavior of the stability-under-addition property. While, under the mixture of normal hypothesis, the characteristic exponent of the stable distribution which determines the total probability contained in the extreme tails approaches two, i.e. normal, with non-overlapping sums of individual observation, it remains well below two under the normal with time-varying parameters hypothesis. Friedman and Vandersteel examined three hypotheses of stable Paretian, mixture of two normal and normal with time-varying parameters, and found evidence in favor of the third hypothesis in foreign exchange rate movements. That is, both the trend and volatility of exchange rate movements are affected by changing economic and institutional factors over time, which might lead to serial dependence in means and variances. Taylor (1985) found serial dependence in both mean and variance for futures prices which suggests a mixed distribution is not satisfactory since observations are not independent.

A rather broad hypothesis of a time-varying distribution is further divided into several sub-hypotheses depending on how to model the return generating process. If the observed leptokurtosis is explained by a normal distribution with changing variance, theoretical models based on normality still hold if investors' horizons are short enough to avoid the variance change. Furthermore, a correction for heteroskedasticity may result in a normal distribution and permit use of familiar economic models and statistical tools. Taylor proposed that the rescaled data (original data divided by its forecast standard deviation) would give more accurate results. McCulloch (1985) tried to remove heteroskedasticity in interest rates by using an adaptive conditional heteroskedastic (ACH) model. Bollerslev (1987) suggested an extended GARCH (generalized autoregressive conditional heteroskedastic) model in which the disturbance term follows a conditional t-distribution, to fit foreign exchange rates and stock price indices. Hsieh modelled foreign exchange rate movements with an ARCH process which allowed for day-of-the-week effects in both mean and variance. Venkateswaran, et al. considered an exponentially weighted moving average of past volatility as a model of futures prices and found it was insufficient.

Voluminous research has been conducted to determine the most descriptive model of return generating process in stock and exchange markets. However, no dominant conclusion has been derived. Also relatively few studies have been performed to test hypotheses for futures price changes. This study will employ the GARCH process to model the distribution of price changes in futures markets. The GARCH model will be tested by a normality test on the rescaled residuals. If the GARCH models can not remove the observed leptokurtosis, alternative models of time varying distributions should be considered.

The GARCH Process

The GARCH model is extended from the ARCH (Autoregressive Conditional Heterroskedastic) model originally developed by Engle (1982). The ARCH model is designed to capture the effect of changing variance on the model. The time dependent conditional variance is specified as a linear function of past realization of the disturbance term. Thus, large past variation is modelled to increase the current variability. That is, this process is motivated by the conjecture that large disturbances occur together, so that a large disturbance today increases the chance of a large disturbance tomorrow. The GARCH process is generalized to allow current and lagged conditional variances

as well as past realization of the disturbance term to affect the sample generating model. Let y_t be a series of log price changes and a simple data generating model be

$$(1) \quad y_t = \mu + \epsilon_t$$

where the random shock ϵ_t is normally distributed conditionally on past information and follows the GARCH(p,q) process with the conditional variance

$$(2) \quad E(\epsilon_t^2 \mid \psi_{t-1}) = h_{t|t-1} = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j|t-j-1}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$.

This model can be further extended by imposing different distributional assumptions for the disturbance term. Bollerslev (1987) considered the GARCH-t model which assumes the disturbance term follows the student t-distribution, and found evidence that the GARCH-t fits better than the GARCH-normal process. This study employs only the GARCH-normal process. Models were also estimated for the GARCH-t process, but results were essentially the same, so only the GARCH-normal models are reported here. Even though the GARCH process has a more flexible and parsimonious lag structure than the ARCH process, as in the traditional ARMA model, a practical concern is the identification of the appropriate lag structure for the conditional variance equation. Bollerslev (1986) suggested that the simplest but often very useful GARCH model is the GARCH(1,1) process. This study follows Bollerslev's proposition.

Another advantage of the GARCH framework is that exogenous shifters of the mean and variance can easily be incorporated. The study includes ten lagged dependent variables in the mean equation (equation (1)) to determine whether serial correlation exists. Also, the day-of-the-week, holidays, maturity, and seasonality will affect the conditional variance by adding appropriate variables to equation (a). Past research into the distribution of prices has ignored these effects in spite of their likely importance. These effects, if any, directly influence the return generating process through the conditional standard deviation included in the mean equation. In short, this research tests a very general GARCH(1,1) process which includes the ten lagged dependent variables in the mean equation, and the day-of-the-week dummies, a holiday dummy, maturity and smooth seasonality variables in the variance equation under both normal and student distributions. In his article, J.C. So (1987) only considered the effect of maturity. Estimation of the GARCH models is conducted by maximum likelihood using the algorithm developed by Berndt et. al.(BHHH) with numerical gradient method.

Sample Data

The data are first differences of the natural logarithms of the daily closing futures prices as recommended by Fama (1965). The data set includes 5 commodities -- corn, live cattle, deutsche mark, sugar, and gold. To maintain a continuity of data and minimize differences in the maturity of contracts, the above data set consists of the changes in the log of daily closing prices of futures contracts until the third Tuesday of the month prior to delivery,

after which the log changes in the next nearest delivery month are used and this process is continued. The time period selected is 1977-1988. The day of the week effects and holiday effects are modeled as dummy variables. For example the dummy for Monday is 1 if the day of the week is Monday and zero otherwise. Maturity is the number of days until the rollover date. Seasonality is modelled as a set of sine and cosine functions with periods one year and six months.

Results

The estimated GARCH models in Table 1 show the strong dependence in the variance. Both of the two parameters of the GARCH process are highly significant in every equation. This means that if large price changes occurred in the last few days, then a large price change is more likely today.

Several of the other parameters are significant, but not as strongly as the two GARCH parameters. Three of the five commodities have significant autocorrelation in mean. This means that the martingale efficient markets model is rejected for these three commodities. Day of the week effects are important for all commodities except live cattle. Variance tends to be highest on Monday and Friday. Variability is lowest on Tuesday. Price changes are consistently larger on days following a holiday. This is as expected since more information will come available over the longer time period. Maturity is not significant in any of the equations. Anderson also found that maturity was relatively unimportant. This might not be true for differences in maturity as large as those considered by Milonas. Seasonality in variance is important for all commodities except live cattle. Production of corn is seasonal and the seasonality in its variance is well documented (e.g., Anderson; Brorsen and Irwin). But, the finding of seasonality in variance for the two financial futures is surprising.

The GARCH model explains much of the non-normality in the futures price changes. The skewness and kurtosis of the rescaled data is about half that of the original data. The only exception is live cattle which had relatively little non-normality even in the original data. But,the rescaled data are still not normal. The null hypothesis of normality is rejected for every price series.

Concluding Comments

The GARCH model does not fully explain the non-normality in futures prices. But, the strong serial dependence in variance does mean that models that do not allow for this dependence are not correct. Tucker and Pond and Akgiray and Boothe only considered models which assumed observations were independent identically distributed (i.i.d.). The early work with stable distributions also assumed distributions were i.i.d.

We must look elsewhere for a model to fully explain the distribution of futures prices. Perry (1983), Tauchen, and Pitts (1983), and Westerfield (1977) suggested a subordinated stochastic process model for stock prices. This could be accomplished within the GARCH model by adding volume to the variance equation. We could also add other variables such as day of week to the mean equation. If a risk premium exists, we should also add the expected variance to the mean equation.

Brock has suggested deterministic chaos as a possible model of economic time series. But, tests with the gold price series using the test of Brock, et al. were not supportive of chaos. The null hypothesis of i.i.d. could not be rejected for the gold rescaled residuals. Other possibilities include combining a GARCH process with one of the past i.i.d. distributions. Possibilities are a mixed GARCH and jump process or a GARCH process with stably distributed residuals. Each alternative has different implications for option pricing models and conducting hypothesis tests.

Table 1. Estimated Parameters and Statistics of the GARCH Model of Daily Futures Price Returns, 1979-1988[a]

Independent Variable	Commodity				
	Corn	Live Cattle	D. Mark	Sugar	Gold
Intercept (Mean)	-0.002 (-0.12)	0.029 (1.38)	-0.023 (-1.90)	-0.049 (-0.92)	0.101 (3.45)
Intercept (Variance)	0.087 (2.06)	0.102 (1.48)	0.063* (2.62)	0.551 (1.52)	0.383* (4.52)
e_t^2	0.086 (7.91)	0.047* (4.97)	0.089* (9.11)	0.048* (7.11)	0.066* (8.33)
h_t	0.888* (64.62)	0.941* (73.13)	0.883* (69.74)	0.938* (110.02)	0.911* (97.23)
Monday	0.190* (2.39)	-0.042 (-0.38)	-0.002 (-0.04)	1.077 (1.93)	-0.177 (-1.20)
Tuesday	-0.284* (-3.91)	-0.175 (-1.61)	-0.127* (-3.42)	-2.003* (-3.24)	-0.812* (-6.01)
Wednesday	-0.067 (-1.10)	-0.132 (-1.20)	-0.086* (-2.73)	-0.850 (-1.47)	-0.371* (-3.67)
Thursday	-0.141 (-1.72)	-0.085 (-0.68)	-0.043 (-1.21)	-0.751 (-1.16)	-0.488* (-3.81)
Holiday	0.285* (4.14)	0.226* (2.24)	0.203* (5.65)	2.930* (5.55)	0.822* (7.11)
Maturity	0.00007 (0.40)	-0.0003 (-0.88)	-0.00004 (-0.41)	0.0001 (0.07)	0.0004 (1.62)
Autocorrelation F-statistic	2.95*	1.439	1.88*	1.649	12.37*
Seasonality F-statistic	18.47*	0.699	9.35*	7.887*	3.58*

[a] Asterisks denote significance at the 5% level. The numbers in parentheses are t-values.

Table 2. Test Statistics for the Null Hypothesis that Futures Price Returns are Normally Distributed

	Raw Data			Rescaled Residuals		
Commodity	Kolmogorov-Smirnov D-Statistic	Skewness	Relative Kurtosis	Kolmogorov-Smirnov D-Statistic	Skewness	Relative Kurtosis
Corn	0.06*	0.61*	1.96*	0.02*	0.20*	0.87*
Live Cattle	0.02*	-0.09	-0.20*	0.02*	-0.13*	-0.28*
Deutsch Mark	0.06*	0.30*	2.16*	0.05*	0.15*	0.99*
Sugar	0.03*	-0.05	1.59*	0.02*	0.06	0.87*
Gold	0.12*	1.32*	2.99*	0.07*	0.74*	1.46*

[a] Asterisks denote significance at the 5% level.

References

Akgiray, Vedat and G. Geoffrey Booth, "Mixed Diffusion-Jump Process Modeling of Exchange Rate Movements," The Review of Economics and Statistics (Feb. 1988):631-637.

Anderson, R.W., "Some Determinants of the Volatility of Futures Prices," Journal of Futures Markets 5(1985):332-48.

Barnea, Amir and Davis H. Downes, "A Re-examination of the Empirical Distribution of Stock Price Change," Journal of the American Statistical Association 168(June 1973):348-50.

Berndt, E.K., B. Hall, R.E. Hall, and J.A. Hausman, "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement 3(1974):653-65.

Black, F. and M. Scholes, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy 81(1973):637-59.

Blattberg, Robert C. and Nicholas J. Gonedes, "A Comparision of the Stable and Student Distributions as Statistical Models for Stock Prices," Journal of Business 47(1974):244-80.

Bollerslev, Tim, "A Conditionally Heteroskedastic Time Series Model for Speculative Prices and Rates of Return," The Review of Economics and Statistics 69(Aug. 1987):542-47.

Brock, W. A., "Distinguishing Random and Deterministic Systems: Abridged Version," Journal of Economic Theory 40(1986):168-95.

Brock, W.A., W.D. Dechert, J. Scheinkman, and B. Lebaron, "A Test for Independence Based on the Correlation Dimension," Social Systems Research Institute Working Paper #8816, University of Wisconsin, Madison, Aug. 1988.

Brorsen, B. Wade and Scott H. Irwin, "Futures Funds and Price Volatility," Review of Research in Futures Markets 6(2)(1987):118-35.

Clark, P.K., "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," Econometrica 41(Jan. 1973):135-55.

Cornew, Ronald W., Donald E. Town, and Lawrence D. Crowson, "Stable Distributions, Futures Prices, and the Measurement of Trading Performance," The Journal of Futures Markets 4(1984):531-57.

Dunn & Hargitt, Inc., The Dunn & Hargitt Commodity Data Bank, P.O. Box 1100, Lafayette, IN 47902.

Engle, Robert F., "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation," Econometrica 50(1982):987-1007.

Fama, Eugene F., "Mandelbrot and the Stable Paretian Hypothesis," Journal of Business 36(Oct. 1963):420-29.

_____, "The Behavior of Stock Market Prices," Journal of Business 38(Jan. 1965):34-105.

Fielitz, Bruce D. and James P. Rozelle, "Stable Distributions and the Mixtures of Distributions Hypotheses for Common Stock Returns," Journal of the American Statistical Association 78(March 1983):28-36.

Friedman, Daniel and Stoddard Vandersteel, "Short-run Fluctuations in Foreign Exchange Rates" Journal of International Economics 13(Aug. 1982):171-86.

Gordon, J. Douglas, "The Distribution of Daily Changes in Commodity Futures Prices," Technical Bulletin No.1702, ERS, USDA, July 1985.

Hall, Joyce A., B. Wade Brorsen and Scott H. Irwin, "The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypotheses," Journal of Financial and Quantitative Analysis 24 (March 1989):105-116.

Harris, L., "Transaction Data Tests of the Mixture of Distributions Hypothesis," Journal of Financial and Quantitative Analysis 22(June 1987):127-41.

Hsieh, David A., "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," Journal of International Economics 24(1988):129-145.

Hudson, Michael A., Raymond M. Reuthold and Gboroton F. Sarassoro, "Commodity Futures Price Changes: Recent Evidence for Wheat, Soybean and Live Cattle," Journal of Futures Markets 7(June 1987):287-301.

Hull, J. and A. White, "The Pricing of Options on Assets with Stochastic Volatilities," Journal of Finance 62(June 1987):281-300.

Johnson, H. and Shanno, "Option Pricing When Variance is Changing," Journal of Financial and Quantitative Analysis 22(June 1987):143-51.

Kon, S., "Models of Stock Returns - A Comparision" Journal of Finance 39(1984):147-65.

Mandelbrot, Benoit, "The Variation of Certain Speculative Prices," Journal of Business 36(Oct. 1963):394-419.

McCulloch, J. Huston, "Interest-Risk Sensitive Deposit Insurance Premia: Stable ACH Estimates" Journal of Banking and Finance 9(March 1985):137-56.

_____, "The Pricing of Short-lived Options When Price Uncertainty is Log-Symmetric Stable," National Bureau of Economics Research, Working Paper No. 264, 1978.

McFarland, James, Richardson Pettit and Sam Sung, "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement," Journal of Finance 37(June 1982):693-715.

Merton, Robert, "Option Pricing when Underlying Stock Returns Are Discontinuous," Journal of Financial Economics 3(Jan. 1976):125-44.

Officer, R., "The Distribution of Stock Returns," Journal of the American Statistical Association 67(Dec. 1972):807-12.

Perry, P.R. "More Evidence on the Nature of the Distribution of Security Returns," Journal of Financial and Quantitative Analysis 18(June 1983):211-21.

So, J.C. "The Sub-Gaussian Distribution of Currency Futures: Stable Paretian or Nonstationary?" Review of Economics and Statistics 69(Feb. 1987):100-07.

Tauchen, G. and M. Pitts, "The Price Variability-Volume Relationship on Speculative Markets," Econometrica 51(March 1983):485-505.

Taylor, S.J., "The Behavior of Futures Prices Over Time" Applied Economics 17(1985):713-34.

Teichmoeller, John, "A Note on the Distribution of Stock Price Changes," Journal of the American Statistical Association 66(June 1971):282-84.

Tucker, Alan L. and Lallon Pond, "The Probability Distribution of Foreign Exchange Changes: Tests of Candidate Processes" The Review of Economics and Statistics (Feb. 1988):638-47.

Venkateswaran, M., B.W. Brorsen, and J.A. Hall, "The Distribution of Standardized Futures Price Changes," AAEA Selected Paper, Aug. 1988.

Westerfield, R., "The Distribution of Common Stock Price Changes: An Application of Transactions Time and Subordinated Stochastic Models," Journal of Financial and Quantitative Analysis 12(Dec. 1977):743-65.

Westerfield, Janice, "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes" Journal of International Economics 7(May 1977):181-200.