Alternative Strategies for Managing Price Risk with Options

by

Robert J. Hauser and James S. Eales

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The following statements appeared in a recent advisory newsletter: "If a producer can assess his risk and apply one of the following strategies to control it, then naturally he can be more ambitious toward his goals. An aggressive marketing plan (with options) can balance risk and reward in a tailored fashion to suit the producer's needs and objectives." (ADM Investor Services, Inc., p. 4). While it is not clear to us what comprises "ambitious" and "aggressive" marketing behavior, these types of statements reveal two points about option hedging that are often made. First, options provide an easy means to vary the "risk and reward" levels facing the hedger. Second, these risk/return levels must be assessed by the hedger. This paper addresses these two points by providing estimates of the expected outcomes of various option hedging strategies.

Considered here are price management alternatives for the commodity seller who is short hedging a fixed and known quantity. Nine option strategies were chosen for analysis on the basis of what we found being proposed by brokerage houses, advisory services, and exchanges. A target deviation model is used to describe the expected risk and return of each strategy under different assumptions about basis risk and the seller's price expectation, price-variance expectation, and risk preference. Particular attention is given to the prescriptive implications for hedging with options.

Methodology

The risk/return measurements presented in this paper represent an extension of previous work (Hauser and Andersen; Hauser and Eales) in two respects. First, additional short-hedging strategies are examined here. Second, the earlier model is extended by incorporating basis uncertainty.

The foundation of the risk/return model is Fishburn's target-deviation model. Fishburn contends that decision makers often perceive "risk" as pertaining only to those outcomes falling below a target outcome (p. 123). Holthausen extended this concept by defining "return" as a function of outcomes above the target. The following target-deviation specification has an underlying utility function that is consistent with the von-Neumann-Morgenstern axioms for expected utility (Holthausen):

$$RK = \int_{-\infty}^{G} (G-Y)^{\alpha} F'(Y)dY, \text{ and}$$

$$RT = \int_{G}^{\infty} (Y-G)^{\beta} F'(Y)dY,$$

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where RK and RT are risk and return, respectively; G is the goal or target level below which outcome Y is associated with risk and above which outcome Y is associated with return; α and β are risk preference parameters; and $F'(Y)$ is a probability density function. Empirical support for using the implied utility function is provided by Holthausen and by Fishburn and Kochenberger. They convert various utility functions from other studies in terms of α, β, and G and suggest that the flexibility offered by the α-β-G model is needed to reflect the changing risk behavior that an individual often exhibits.

Application of the α-β-G model to a particular option-hedging strategy requires the definition of each effective price that might be realized when the hedge is lifted (E_T) and the probability expected for each E_T when the hedge is placed. The strategies considered in this analysis and their E_T descriptions are presented in Table 1. The hedge is for a fixed and known quantity equal to the option contract's specification. A general description of E_T for all ending futures prices ($0<F_T<\infty$) is defined in Table 1 by strategy. Below each general description, E_T is defined for different ranges of the ending futures price outcome, F_T. For example, E_T for Strategy I (long cash and long put) is the cash price at expiration, C_T, plus the return from holding the put, P_T-P_o. This general description can be rewritten, depending on the value of F_T relative to the exercise price of the put, X^P. If $F_T<X^P$, then P_T is the intrinsic value of the option, $X-F_T$. If $F_T>X^P$, the option is worth nothing at expiration.

The E_T definitions for Strategies I-IX of Table 1 are shown in Figures I-IX under the assumption that $C_T=F_T$; i.e., the expiration basis is zero and known. One point illustrated by these diagrams is that there are many effective-price patterns that can be created by option trading. However, these types of graphical perspectives do not provide much insight to the probability or uncertainty associated with an E_T level.

Uncertainty in E_T can be thought of as arising from three sources: futures price, cash price, and basis. As described above, the effective price is defined differently for a given strategy, depending on the range in which F_T falls. One source of uncertainty is not knowing what this range will be when the hedge is placed. Additional uncertainty for an individual futures-price range is caused by either cash price variability or basis variability. Basis uncertainty exists if both C_T and F_T appear in the equation. When C_T appears without F_T, cash price uncertainty exists but not basis uncertainty. Note that, for example, when $F_T<X^P$ in Strategy I, E_T depends on the difference F_T-C_T. For $F_T>X^P$, E_T is a function of C_T but not basis.

A common assertion in advisory and pedagogical materials is that an individual faces basis risk when hedging with either options or futures. This is true. However, this type of statement can be easily misconstrued to mean that the same level is incurred in both cases. Differences in the level will be illustrated in this paper.

The general functions used to estimate risk (RK) and return (RT) are:

$$RK = \int_{LBF}^{UBF}\int_{LBC}^{UBC}(G-E_T)^{\alpha}L'(C_T|F_T)L'(F_T)dC_TdF_T$$

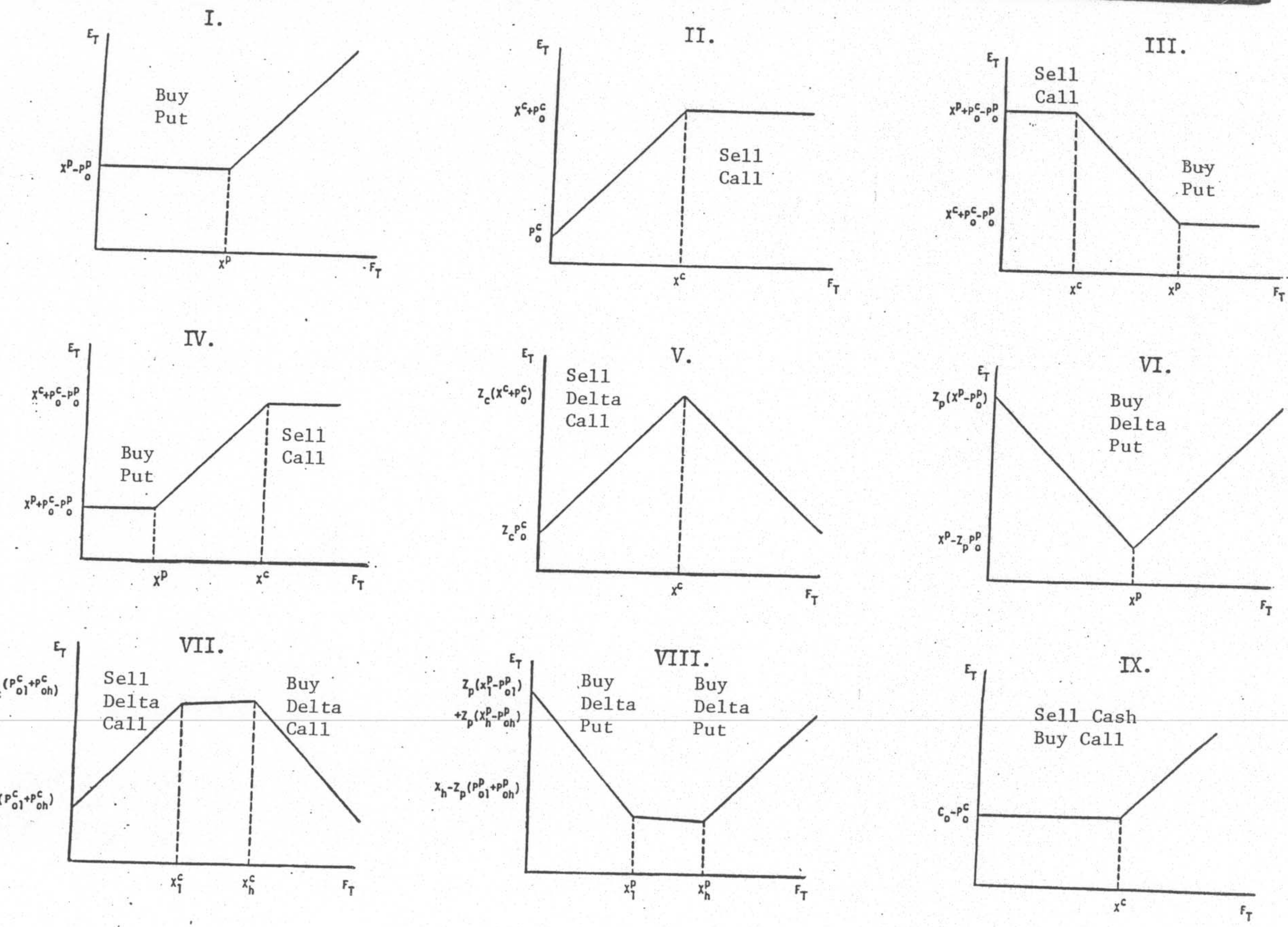
Table 1. Description of Effective Price at Expiration by Strategy.[a]

Strategy[b]	F_T Range	Effective Price E_T	Strategy[b]	F_T Range	Effective Price E_T
I. Long Put	$0<F_T<\infty$	$C_T-P^P_o+P^P_T$	VI. Long Straddle	$0<F_T<\infty$	$C_T+Z_sP^P_T-Z_sP^P_o$
	$0<F_T<X^P$	$C_T-P^P_o+X^P-F_T$		$0<F_T<X^P$	$C_T-Z_sF_T+Z_sX^P-Z_sP^P_o$
	$X^P<F_T<\infty$	$C_T-P^P_o$		$X^P<F_T<\infty$	$C_T-Z_sP^P_o$
II. Short Call	$0<F_T<\infty$	$C_T+P^c_o-P^c_T$	VII. Short Strangle	$0<F_T<\infty$	$C_T+Z_sP^c_{o1}-Z_sP^c_{T1}+Z_sP^c_{oh}-Z_sP^c_{Th}$
	$0<F_T<X^c$	$C_T+P^c_o$		$0<F_TX^c_1$	$C_T+Z_sP^c_{o1}+Z_sP^c_{oh}$
	$X^c<F_T<\infty$	$C_T+P^c_o-F_T+X^c$		$X^c_1<F_T<X^c_h$	$C_T+Z_sP^c_{o1}-Z_sF_T+Z_sX^c_1+Z_sP^c_{oh}$
				$X^c_h<F_T<\infty$	$C_T+Z_sP^c_{o1}-Z_sF_T+Z_sX^c_1+Z_sP^c_{oh}-Z_sF_T+Z_sX^c_h$
III. Bear Spread	$0<F_T<\infty$	$C_T+P^c_o-P^c_T+P^P_T-P^P_o$			
	$0<F_T<X^c$	$C_T+P^c_o+X^P-F_T-P^P_o$	VIII. Long Strangle	$0<F_T<\infty$	$C_T+Z_sP^P_{T1}-Z_sP^P_{o1}+Z_sP^P_{Th}-Z_sP^P_{oh}$
	$X^c<F_T<X^P$	$C_T+P^c_o-F_T+X^c+X^P-F_T-P^P_o$		$0<F_T<X^P_1$	$C_T+Z_sX^P_1-Z_sF_T-Z_sP^P_{o1}+Z_sX^P_h-Z_sF_T-Z_sP^P_{oh}$
	$X^P<F_T<\infty$	$C_T+P^c_o-F_T+X^c-P^P_o$		$X^P_1<F_T<X^P_h$	$C_T-Z_sP^P_{o1}+Z_sX^P_h-Z_sF_T-Z_sP^P_{oh}$
IV. Bull Spread	$0<F_T<\infty$	$C_T+P^c_o-P^c_T-P^P_o+P^P_T$		$X^P_h<F_T<\infty$	$C_T-Z_sP^P_{o1}-Z_sP^P_{oh}$
	$0<F_T<X^P$	$C_T+P^c_o-P^P_o+X^P-F_T$	IX. Short Cash Long Call		
	$X^P<F_T<X^c$	$C_T+P^c_o-P^P_o$			
	$X^c<F_T<\infty$	$C_T+P^c_p-F_T+X^c-P^P_o$		$0<F_T<\infty$	$C_o+P^c_t-P^c_o$
V. Short Straddle	$0<F_T<\infty$	$C_T+Z_sP^c_o-Z_sP^c_T$		$0<F_T<X^c$	$C_o-P^c_o$
	$0<F_T<X^c$	$C_T+Z_sP^c_o$		$X^c<F_T<\infty$	$C_o+F_T-X^c-P^P_o$
	$X^c<F_T<\infty$	$C_T+Z_sP^c_o-Z_sF_T+Z_sX^c$			

[a] F_T, C_T, and P_T is futures price, cash price and premium at expiration T; F_o, C_o, and P_o are prices when hedge is placed at time o; X is exercise price; superscripts c and p designate call and put; subscripts l and h distinguish the low and high strike values when two options of the same type (put versus call) are used; Z_s is the number of options that make strategy s "delta neutral" when placing the hedge.

[b] I: long cash, long put; II: long cash, short call; III: long cash, short call with X^c strike, long X^P put, $X^c<X^P$; IV: long cash, long X^P put, short X^c call, $X^P<X^c$; V: long cash, short call, delta neutral; VI: long cash, long put, delta neutral; VII: long cash, short X_1 call short X_h call, $X_1<X_h$, delta neutral; VIII: long cash, long X_1 put, long X_h put, delta neutral; IX: short cash, long call.

Figures I.-IX. Effective Prices for Strategies I-IX Described in Table 1.



$$RT = \int_{LBF}^{UBF}\int_{LBC}^{UBC}(E_T-G)^{\beta}L'(C_T|F_T)L'(F_T)dC_T dF_T$$

where $L'(C_T|F_T)$ is the conditional density of cash price C_T, given futures price F_T; $L'(F_T)$ is the marginal density function for F_T. It is assumed that the target, G, is the expected price under a completely hedged position; i.e., the current futures price, F_o, minus the expected basis. The level of expected basis is set at zero and thus the target is the current futures price. The lower and upper bounds of integration over the futures price distribution (LBF and UBF) are defined for each strategy by the F_T limits given in Table 1. The limits of integration for the conditional cash distribution (LBC and UBC) are found by subtracting the relevant E_T equation (Table 1) from G. The resultant C_T range yielding a positive (negative) difference is used to find a component of RK(RT). Cash flows are discounted to expiration at an annualized rate of eight percent.

The essence of this type of risk/return measurement is how and why the densities and the risk-behavior parameters are defined. Assumed here is that the cash and futures price distributions are both lognormal. Lognormality is consistent with most option valuation models. Furthermore, we are not aware of any empirical evidence that clearly identifies a more appropriate distribution. The variance of the $\ln(C_T)$ distribution is assumed equal to the variance of the $\ln(F_T)$ distribution. This assumption is based on tests using the futures prices and average Illinois cash prices described in Garcia, Hauser, and Tumblin. Sample variances of log-price first differences were calculated for both the cash and March futures series for each December-February period during 1967-1983. Each of the 17 variance pairs were examined under the null hypothesis that the variances are equal versus the alternative that they are not equal. The null hypothesis at the .01 level is not rejected for any pair. The means of the two expected distributions are also assumed equal, implying that the expected basis is zero. However, this does not imply that before expiration the basis is zero. It simply means that the processes leading to the ending distributions are such that the expected ending distributions are the same. The joint distribution of cash and futures prices is determined under the assumption that the correlation coefficient, ρ, for $\ln(F_T)$ and $\ln(C_T)$ is not one. The degree of basis risk is directly related to $1-\rho$. We set ρ at .95 in the base case. The average ρ for the 1967-83 series defined above is about .92. However, the base-case ρ of .95 is used because the five ρ estimates during 1979-83 ranged from .962 to .975.

It is assumed that 5,000 bushels of soybeans are hedged at the Chicago Board of Trade. The futures price is $6.00 per bushel when the hedge is placed. The hedge is lifted after three months at option expiration. Most of the option hedging strategies examined here were drawn from advisory newsletters or from discussion with market advisers. Three strategies (IV, V, and VII) were not found in our search but are considered because they offer the counterpart to other strategies (III, VI, VIII). Each strategy is defined in footnote b of Table 1. Unless "delta neutral" is specified, one option is sold or purchased. "Delta neutral" means that the number of options either purchased or sold is the inverse of the first derivative of the option portfolio value with respect to the futures price, given Black's solution. Continuous use of this strategy over time is the foundation on which the option valuation model is based. However, it is assumed here that all hedges are routine and thus new positions in the option market are not acquired during the three-month hedging period.

The option is priced with Black's model using an annualized interest rate of eight percent and five strike prices. The volatility used is 23 when expressed as the annualized standard deviation percentage. This volatility is the average of the 17 volatilities found using the Garica et al. data for the three-month pre-March hedging periods during 1967-1983. A base case is defined in which the seller's expected volatility is also 23 and expected price is the current futures price, \$6.00. Under these assumptions, the ending lognormal distribution expected when the hedge is placed has prices one standard deviation on either side of the mean equal to \$5.28 and \$6.65. These prices as well as \$5.70 and \$6.30 are used to illustrate the effects on risk and return when the seller's price expectation deviates from the current price. Deviations from the annualized volatility of 23 are based on Hauser and Andersen's estimates of representative forecast errors in volatility. The alternative volatilities used are 19, 21, 25, and 27.

The base case is also defined by $\alpha=\beta=1$. Although this implies risk neutrality in a risk preference framework (Holthausen), we believe it is the most useful perspective insofar as it reveals the market's estimation of expected deviation from the target. The extent to which this is so depends on whether F_0 is the market's forecast of spot price and whether the premium's implied volatility (23) is the market's forecast of variance. In other words, it depends on whether the price and variance parameters expected by the market can be observed. Using exponential weights different than one places the "objective" market estimates of target deviation into a subjective framework. However, cases are considered in which α and β are not one. When α is greater (less) than one, risk aversion (seeking) is implied for outcomes below the target. When β is greater (less) than one, risk seeking (aversion) is implied for outcomes above the target.

Results

The risk/return estimates for each strategy and case are presented in Table 2. They are summarized and discussed below by four scenario types: the base case; the base case with deviations in price expectation and variance expectation; the base case with deviations in risk preference parameters; and the base case with deviations in basis risk level.

Base

The base-case results emphasize that there is no free lunch. For each strategy, as return increases, risk increases. Risk and return are always equal here for three reasons. First, $\alpha=\beta=1$. Second, the density function used to price the option is also used in forming expected deviations above and below the target. Third, the target is the expected hedged price (current futures price minus expected basis). For cases discussed later, we will maintain the third condition but not the first two.

The unhedged or open position of the base has a risk and return level of .28 while the completely hedged level is .09. The array of risk/return

Table 2. Risks and Returns in Dollars per Bushel.

Strategy	Strike	Base		Expected Price								Expected Variance								α and β				ρ					
				5.28		5.70		6.30		6.65		19		21		25		27		0.8		1.5		1.0		0.9		0.8	
		RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT	RK	RT
I. Long Put	5.50	.24	.24	.47	.03	.34	.12	.15	.41	.08	.66	.22	.19	.23	.22	.24	.26	.25	.29	.11	.10	1.69	2.20	.24	.24	.24	.24	.24	.24
	5.75	.21	.21	.36	.03	.28	.10	.14	.36	.08	.60	.20	.16	.20	.18	.21	.26	.10	.21	.10	.09	1.34	1.89	.21	.21	.21	.21	.22	.22
	6.00	.17	.17	.26	.03	.21	.08	.12	.31	.07	.53	.17	.13	.17	.15	.17	.20	.17	.22	.08	.08	1.01	1.51	.16	.16	.18	.18	.20	.20
	6.25	.14	.14	.18	.03	.16	.07	.11	.25	.07	.44	.14	.09	.14	.12	.14	.16	.14	.19	.07	.07	.76	1.14	.12	.12	.16	.16	.19	.19
	6.50	.12	.12	.14	.04	.13	.07	.10	.20	.07	.35	.11	.08	.11	.10	.12	.14	.12	.17	.06	.05	.61	.86	.08	.08	.14	.14	.18	.18
II. Short Call	5.50	.11	.11	.33	.05	.17	.09	.08	.13	.07	.14	.07	.10	.09	.11	.13	.11	.15	.12	.05	.05	.67	.59	.07	.07	.14	.14	.18	.18
	5.75	.13	.13	.44	.05	.22	.10	.08	.16	.05	.18	.08	.13	.11	.13	.15	.13	.18	.13	.06	.07	.90	.75	.11	.11	.15	.15	.19	.19
	6.00	.16	.16	.53	.05	.28	.11	.09	.21	.05	.25	.11	.16	.14	.16	.18	.16	.21	.16	.07	.08	1.22	1.00	.15	.15	.17	.17	.20	.20
	6.25	.19	.19	.61	.05	.33	.13	.10	.27	.05	.33	.15	.19	.17	.20	.22	.20	.24	.20	.09	.09	1.55	1.32	.19	.19	.20	.20	.21	.21
	6.50	.22	.22	.67	.05	.37	.14	.12	.32	.05	.43	.17	.22	.20	.22	.25	.23	.27	.23	.10	.10	1.83	1.68	.22	.22	.23	.23	.23	.23
III. Bear Spread	--[a]	.13	.13	.04	.22	.09	.18	.18	.10	.23	.07	.13	.13	.13	.13	.15	.14	.15	.15	.07	.07	.89	.82	.11	.11	.17	.17	.22	.22
IV. Bull Spread	--	.12	.12	.21	.03	.16	.08	.08	.17	.05	.21	.11	.11	.11	.12	.14	.14	.13	.13	.06	.06	.66	.72	.11	.11	.14	.14	.17	.17
V. Short Straddle	5.50	.12	.12	.24	.08	.13	.11	.14	.11	.19	.09	.08	.12	.10	.12	.14	.12	.16	.12	.06	.06	.75	.67	.08	.08	.15	.15	.20	.20
	5.75	.15	.15	.30	.09	.17	.14	.18	.14	.26	.11	.09	.16	.12	.15	.18	.14	.21	.14	.07	.07	1.04	.89	.12	.12	.17	.17	.21	.21
	6.00	.18	.18	.36	.09	.20	.16	.24	.18	.36	.15	.11	.21	.15	.19	.22	.17	.26	.17	.08	.09	1.49	1.21	.16	.16	.20	.20	.24	.24
	6.25	.23	.23	.41	.10	.24	.18	.31	.23	.53	.21	.14	.26	.18	.24	.27	.21	.33	.20	.10	.10	2.15	1.64	.21	.21	.24	.24	.27	.27
	6.50	.27	.27	.45	.10	.27	.20	.40	.30	.73	.28	.16	.30	.21	.28	.22	.25	.40	.24	.11	.12	3.07	2.10	.26	.26	.28	.28	.30	.30
VI. Long Straddle	5.50	.29	.29	.26	.93	.31	.41	.23	.31	.15	.45	.33	.17	.31	.22	.28	.39	.26	.43	.13	.13	2.32	3.27	.28	.28	.30	.30	.32	.32
	5.75	.24	.24	.17	.64	.24	.32	.21	.27	.15	.41	.27	.14	.26	.19	.23	.29	.22	.35	.11	.11	1.79	2.28	.23	.23	.25	.25	.28	.28
	6.00	.20	.20	.12	.44	.18	.24	.18	.23	.14	.36	.22	.12	.21	.16	.19	.24	.18	.28	.09	.09	1.34	1.61	.18	.18	.21	.21	.24	.24
	6.25	.16	.16	.08	.30	.14	.18	.16	.19	.13	.31	.17	.10	.17	.13	.15	.19	.15	.22	.08	.07	1.00	1.16	.13	.13	.18	.18	.22	.22
	6.50	.13	.13	.07	.21	.11	.14	.13	.16	.12	.26	.14	.08	.13	.11	.13	.16	.13	.18	.07	.06	.77	.86	.09	.09	.16	.16	.20	.20
VII. Short Strangle	--	.17	.17	.34	.09	.19	.15	.22	.17	.35	.14	.10	.19	.13	.18	.21	.16	.24	.16	.08	.08	1.36	1.06	.15	.15	.19	.19	.23	.23
VIII. Long Strangle	--	.18	.18	.11	.41	.17	.22	.17	.21	.13	.34	.20	.11	.19	.14	.17	.22	.16	.26	.09	.08	1.16	1.47	.16	.16	.20	.20	.23	.23
IX. Short Cash Long Call	5.50	.24	.24	.47	.03	.34	.12	.15	.40	.08	.66	.22	.19	.23	.21	.24	.26	.25	.28	.11	.10	1.66	2.17	.24	.24	.24	.24	.24	.24
	5.75	.21	.21	.36	.02	.28	.10	.14	.36	.08	.60	.20	.16	.20	.18	.21	.23	.21	.25	.10	.09	1.25	1.85	.21	.21	.21	.21	.21	.21
	6.00	.16	.16	.25	.02	.21	.08	.12	.30	.07	.52	.17	.12	.17	.14	.16	.19	.16	.21	.09	.07	.84	1.46	.16	.16	.16	.16	.16	.16
	6.25	.12	.12	.16	.01	.14	.05	.09	.23	.06	.42	.13	.08	.12	.10	.12	.14	.12	.16	.07	.05	.50	1.05	.12	.12	.12	.12	.12	.12
	6.50	.08	.08	.10	.01	.09	.03	.07	.17	.05	.32	.09	.05	.08	.06	.08	.10	.08	.12	.05	.04	.26	.69	.08	.08	.08	.08	.08	.08
X. Open		.28	.28	.76	.04	.44	.15	.15	.46	.07	.73	.22	.23	.25	.25	.30	.30	.32	.22	.12	.12	2.33	2.60	.28	.28	.28	.28	.28	.28
XI. Hedge		.09	.09	.08	.08	.08	.08	.09	.09	.10	.10	.07	.07	.08	.08	.09	.09	.10	.10	.05	.05	.44	.44	.00	.00	-.12	.12	.17	.17

[a] The two strikes used are 5.75 and 6.25.

combinations for an option strategy is usually bounded by these levels. This range of risk/return levels between the unhedged and hedged position is perhaps the most important characteristic of options as a hedging tool. If, for instance, only 5,000 bushels are available to be hedged at the Board, then the hedger can choose between two levels of risk/return when considering a routine hedge with futures--the unhedged level (.28) and the hedged level (.09). The availability of options expands this choice set considerably. When the potential hedging quantity increases, more than two income risk/return combinations are available with futures since total quantity need not be hedged. Therefore, the practical importance of the contract divisibility constraint is greatest at small hedging levels. Even at large finite levels, however, there are always more combinations available with options than with futures.

Two of the strategies have risk/return levels outside the hedged versus unhedged bounds. Strategy IX (short cash, long call) has a risk/return level that is less than the hedged level. This is because no basis risk is incurred when using this strategy. Indeed, this strategy replicates Strategy I when there is no basis risk. In our search for recommended option strategies, we found that the two most frequently advised strategies were I and IX; that is, it was often recommended that the producer either buy a put or sell the commodity and buy a call. Although we suspect that the perceived difference between the strategies is often exaggerated, there are at least two important reasons why they can differ. The first reason is illustrated in the results shown here. Basis risk causes the risk/return level of the short-cash strategy to be less than the risk/return level of the long-put strategy for a given strike price. This difference increases as the strike price increases. The second reason is not illustrated here but concerns the hedger's belief about what the return to storage will be relative to the interest rate.

The second strategy not bounded by the unhedged and hedged levels is the long straddle (Strategy VI). The strike of 5.50 has a risk/return level of .29, as opposed to the unhedged level of .28. The long straddle is the same as the long put strategy except that more than one put per 5,000 bushels are purchased. The number of puts in the straddle is determined such that incomes from the option and underlying commodity positions are offsetting when the underlying commodity price changes. When the number of puts is adjusted continuously, this offsetting effect causes the hedge portfolio to be riskless. However, if the delta-neutral position is established only once and then held for three months, the resulting risk level is greater than other option positions. For instance, "delta neutral" Strategy VI yields larger risk/return levels than the long-put position (I) for a given strike, begging the question of how many puts are needed to reach the risk/return minimum level. We suspect that there is an analytic solution to this question but have not tried to find it. We did, however, vary delta from .10 to 2 by increments of .10, meaning that the number of puts (one divided by delta) was varied from 10 to .5. For both the short straddle and long straddle, the delta at each strike that yields the lowest risk/return level in this static framework is greater than the delta used in the continuous trading scenario on which the option-valuation model is based.

Finally, with respect to the base-case results, note that the risk/return level of the unhedged position (Strategy X) is reduced by about 70 percent through hedging (Strategy XI). This reduction is consistent with the results of hedging effectiveness studies that measure the return variance of a hedged position relative to the variance of an unhedged position. For example, Ederington estimates that the unhedged return variance can be reduced by about 60 percent when hedging corn over two-six month periods given the minimum variance ratio of futures to cash positions. Using the three-month March soybean data referred to earlier, the average reduction is estimated at 75 percent when the basis expectation model is based on the market's storage return forecast (see Garcia et al., Method II).

Price and Variance Expectations

When price and variance expectations deviate from the market's expectations, risk is not equal to return for a given hedging alternative and it is therefore possible for one alternative to dominate another. A hedging alternative, A_1, dominates another alternative, A_2, if the risk of A_1 is not greater than the risk of A_2, and the return of A_1 is not less than the return of A_2, and at least one of these inequalities is strict.

Hedging alternatives that have risks greater than returns are not necessarily dominated by others that have returns greater than risks. For instance, when the seller expects a price of $5.70, the long straddle (VI) yields a return that is larger than risk at each strike. The short cash, long call strategy (IX) has returns less than risks. None of the strikes of VI dominates the 6.50 strike of IX because risk levels of the long straddle do not fall below the .09 level. However, the bear spread strategy does dominate this alternative. In this risk-neutral context ($\alpha=\beta=1$), it is reasonable to assume that the preferred strategy will not be one that has returns less than risks. In the few cases here where nothing dominates an alternative in which risks are less than returns, it is usually because we have not defined other strategies which would dominate. Therefore, we wish to focus first on the effects of price and variance expectations on whether they cause returns to be greater or less than risks.

Table 3 presents summary information comparing the results for Strategies I, II, V, and VI. These comparisons reveal some basic tradeoffs involved when (a) using puts and calls at the rate of one option per 5,000 bushels (I and II) versus the rate implied by delta (VI and V) and (b) the use of puts versus calls. The top half of the table designates whether the price or variance expectation (relative to the base case) causes the returns to be greater or less than the risk for a given strike price. One set of parentheses indicates that the put (call) strategy is usually not dominated by the other put (call) strategy. Two sets of parentheses indicate that the strategy is usually not dominated by any of the other three strategies.

Consider the inequality signs between RT and RK, keeping in mind that $\alpha=\beta=1$. When price expectation varies from the current futures price, the direction of inequality is the same for the simple put and call strategies (I and II). That is, return is greater than risk for both I and II when high prices are expected, whereas return is less than risk when low prices are expected. In contrast, the signs change between the delta strategies

Table 3. Summary Information on Strategies I, II, V, and VI.

	Long Put I	Delta Long Put VI	Short Call II	Delta Short Call V
High Price Expectation[a]	(RT>RK)[b]	RT>RK	(RT>RK)	RT<RK
Low Price Expectation	RT<RK	((RT>RK))[c]	RT<RK	(RT<RK)
High Variance Expectation	RT>RK	((RT>RK))	(RT<RK)	RT<RK
Low Variance Expectation	(RT<RK)	RT<RK	RT>RK	((RT>RK))
$\alpha>1$	X[d]			
$\alpha<1$			X	
$\beta>1$	X			
$\beta<1$			X	

[a] High and low are in relation to the price and variance used in the base case.

[b] One set of parentheses means that a strike of the put(call) strategy is usually not dominated by any of the strikes from the other put(call) strategy.

[c] Two sets of parentheses means that a strike of that strategy is usually not dominated by any of the other strategies.

[d] If X is under I, put strategies not dominated; if under II, call strategies not dominated.

(VI and V) for a given price expectation. When variance expectations vary from the base case, the signs change between the simple put and call strategies and between the delta put and call strategies.

The signs of the variance-expectation cases are perhaps more intuitive than those for the price-expectation cases. (As will be discussed below, this is probably because our intuition is biased toward risk-aversion results.) When variance expectation is high (low) relative to the implied volatility, returns are greater (less) than risks when using puts but returns are less (greater) than risks when using calls. Common advice for using puts versus calls is that if the hedger believes that prices will be "stable", then calls are recommended relative to puts since one receives the premium. This type of advice is particularly prevalent for investors considering the use of a long straddle (VI) versus a short straddle (V). If the investor foresees only "sideways movement" in prices, then a short straddle should be acquired. Long straddles should be used when the investor believes that large price changes are likely but that direction is unknown. This analysis shows that these types of descriptions should be better qualified in terms of the relationship between the investor's or hedger's variance expectation and the implied volatility. The hedger may, for instance, believe that price volatility will be "low" but if this expectation is greater than the market's variance forecast (i.e., the implied volatility) then hedging with calls yields risks that are greater than returns.

For the same reason that long straddles are recommended when variance expectations are high, this strategy yields returns greater than risks when expected price is either much higher or lower than current price. Examination of the long straddle diagram (VI) reveals that high effective prices are realized only when the futures price deviates from the current price by a minimum level. The price expectations used here meet this minimum level but other expectations closer to the current price would not cause the returns to be greater than risks. Also, since the deviations are larger than the critical minimum, the short strangle (V) has returns that are less than risks in both the high and low price-expectation cases.

When using either the simple put or call strategy (I and II), return is greater than risk in the high-price expectation case but not in the low-price case. This result might not have as much intuitive appeal as other results because it does not suggest that a short put position is preferred to a long call when prices are expected to fall. However, these cases reflect perceptions of a risk-neutral decision maker. We now consider the effects of changing these preferences.

Risk Preference

As mentioned earlier, there is evidence that risk behavior can change markedly at a critical target. Fishburn and Kochenberger find behavior that reflects both risk seeking and averse behavior for outcomes above as well as below the target. However, among the four combinations of behavior, the most prevalent combination found was risk aversion above the target and risk seeking below the target (i.e., in this study's framework, $\alpha<1$ and $\beta<1$).

The risk/return results for α=.8, β=.8, α=1.5, and β=1.5 are shown in Table 2. All other conditions of these cases are the same as in the base case. The effects of changing the risk parameters are summarized in the bottom half of Table 3 for Strategies I, II, V, and VI. An "X" under the put column means that I is never dominated by II and that VI is never dominated by V. Likewise, an "X" in the call column means that II is never dominated by I and that V is never dominated by VI. All results for the non-base case preferences (as well as others tried but not shown) yield risks and returns that are not comparable between I and VI and between II and V. That is, no domination occurs between these two pairs.

The general implication of Table 3 is that puts are more likely to be preferred when there is either risk aversion below the target (α>1) or risk seeking above the target (β>1). The opposite is true for calls. As highlighted by Hauser and Eales, this result implies that risk aversion below the target encourages the use of put options. This type of risk aversion might be thought of as being directly related to how much value the hedger places on the put's floor effect. In contrast, risk aversion above the target does not encourage the use of puts. The most conducive preference conditions for put use is when α>1 and β>1. Given Fishburn and Kochenberger's frequent finding of α<1 and β<1, and given Holthausen's estimation of α=.4 and β=.8 for grain farmers, perhaps we should not be surprised if farmers do not use puts. These results, however, certainly do not imply that farmers would not use calls.

Now consider the implications of combining the information presented in the top and bottom half of Table 3. Increasing price expectation and α individually both imply an increased propensity toward using the long put strategy. To check whether these types of individual effects can be interpreted in an additive fashion, the results from 16 combinations of price or variance expectation and α or β levels were examined. It was found that the additive effects implied in Table 2 almost always hold.

The overall results in Table 3 indicate that the use of puts in short hedging is most likely when the seller is risk averse over outcomes below the target, risk seeking for outcomes above the target, expects the variance to be higher than the implied volatility, and does not believe that futures price is the correct unbiased forecast. The direction in which the forecast is biased determines which put strategy will be used (i.e., I versus VI). Furthermore, if a high price is expected, some of the short-call strikes (II) are not dominated. The conditions most conducive to selling calls are when the hedger is risk seeking below the target, risk averse above the target, and expects a variance that is less than the implied volatility.

<u>Basis Risk</u>

The last six columns of Table 2 are cases in which the base-case ρ of .95 is changed to 1.0 (no basis risk), and to .9 and .8 (increased basis risk over the base case). The resulting risk/return changes from the base case are only in level; risk is still equal to return for a given strike and therefore strategies are non-comparable in a dominance sense.

The effects of increasing basis risk are different, depending on strategy and strike. For the long put and short call strategies (I and II), there is virtually no basis-risk effect for options that are out of the money by 50 cents. This is because the probability that the option will be exercised (or offset) at expiration is small; therefore, the probability that the effective price is a function of the basis is also small. When the options are 50 cents in the money, risk/return increases by about twofold when changing ρ from 1.0 to .9. Our ρ calculatins for 1967-83 indicate that the .9 level is common. Therefore, the effect of basis risk when using in-the-money options should not be ignored in practice. The effect on (deep) out-of-the-money options seems minimal.

The delta counterparts to Strategies I and II (VI and V) exhibit larger absolute responses in risk/return to the changes in ρ because, in concept, more options are exercised when exercise is desirable. Thus, the effect of basis risk depends generally on three factors: the strike, the number of options used in the strategy, and ρ.

When ρ is .8, .9, .95, and 1, the risk/return level when hedging with futures (XI) is, respectively, .17, .12, .09, and .00, indicating a non-linear relationship between ρ and risk/return level. However, varying ρ from .95 to 1.0 by increments of .01 reveals a nearly linear relationship. The rate of change in risk/return level decreases considerably below .95.

Concluding Remarks

We conclude this paper by simply highlighting six select points illustrated in the analysis. (1) Hedging with options is not a free lunch. Regardless of strategy, as return increases, risk increases. (2) The short cash, long call strategy is the same as the long cash, long put strategy <u>if</u> there is no basis risk. (3) "Delta-neutral" positions do not minimize expected risk if the position is held over an extended time period. The delta should be increased if the objective is to minimize the risk. (4) Marketing advice, particularly on the use of such strategies as long or short straddles, should emphasize the hedger's variance expectation relative to the implied volatility of the premium. (5) The use of puts is most likely when the hedger is risk averse over outcomes below the expected hedge price, risk seeking above the price, expects the variance of log-prices to be greater than the premium's implied volatility, and believes that the futures price is a biased forecast. The direction of bias influences the type of put strategy preferred. (6) The effects of basis risk on expected risk/return levels can be quite different, depending on the option strategy and strike price.

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