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A Comparison of Analytical Approaches for Estimating Hedge Ratios for Agricultural Commodities

by

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Introduction

There is little disagreement in the literature that hedging can be an effective risk management tool for agricultural firms. However, when placing a hedge the hedger must determine the futures position to take to offset the price risk on his current or anticipated cash position. When direct hedges are placed (e.g., corn cash position hedged in corn futures etc.) the hedged quantity to cash quantity ratio or hedge ratio, is often assumed to be 1. However, in instances involving cross hedging (hedging a cash commodity in a different commodity futures market) the hedge ratio may deviate significantly from 1 because the prices of the two commodities may not change 1 for 1. Therefore, the hedge ratio should be empirically estimated. Disagreement arises on the best procedure to estimate minimum risk hedge ratios; namely, whether to use cash and futures price levels, price changes, or percentage price changes in the estimation process.

Price-level regressions have been used extensively in the estimation of cross hedging ratios. Hayenga and DiPietre (1982 a,b) have used this method to analyze the relationship between wholesale pork products and live hog futures and between wholesale beef products and live cattle futures. This procedure also has been used in the analysis of cross hedging millfeeds (Miller, 1985), rice and bran (Elam, Miller and Holder, 1984), and hay (Blake and Catlett, 1984).

However, in two recent studies, price change and percentage price change regressions were employed to estimate hedge ratios and hedging effectiveness. Wilson (1985) analyzed the cross hedging of sunflowers with soybean futures by using price changes, and Brown (1985) examined the direct hedging of corn, soybeans, and wheat by using percentage price changes.

The differences between these methods have been analyzed by using financial futures. Hill and Schneeweis (1981) calculated hedge ratios for foreign currency futures with price change regressions and compared the results with Dale's (1981), who calculated the ratios by using price levels. The two results differed significantly. Hill and Schneeweis concluded that ratios generated by price change regressions were preferred because they claim price level regressions are theoretically and statistically incorrect.

Essentially, the hedge ratio is related to the underlying objective function of the hedger, the nature of the relationship between the cash and futures prices, and whether the hedge is a storage hedge or an anticipatory hedge. These issues dictate the most appropriate technique to use to estimate the hedge ratio (i.e., using price differences, price levels, or percentage price changes). This study analyzes the theoretical and practical differences among three methods frequently used to estimate hedge ratios. Optimal hedge ratios for cross hedging sorghum and barley with corn futures are estimated by using three approaches, the results compared, and the implications for potential hedgers are analyzed. Then, the approaches that are most appropriate for several real world risk management situations are suggested.[2]

Conceptual Differences

Price Difference Model

Two equations serve as the theoretical basis for the cross hedging model utilized by Anderson and Danthine (1981), Wilson (1985), and others. The first is an equation of expected revenue from holding a commodity; the second is an equation representing the price risk associated with that commodity, ignoring brokerage and other costs. The equations are:

$$(1) \quad E[\Pi] = X_c E(C_2 - C_1) + X_f E(F_2 - F_1)$$

$$(2) \quad \text{Var } E[\Pi] = X_c^2 \sigma_c^2 + X_f^2 \sigma_f^2 + 2X_c X_f \sigma_{cf}$$

where Π = revenue from the firm's cash position and futures position

X_c = quantity of cash commodity

X_f = quantity of futures commodity

$C_{1,2}$ = cash price at the time the hedge is placed, or lifted respectively

$F_{1,2}$ = futures price at the time the hedge is placed, or lifted respectively

σ_c^2, σ_f^2, and σ_{cf} are, respectively, the variance of cash and futures

price changes and their covariance, and E refers to expectation.[3]

In this formulation, the price changes are stochastic, and the spot position, X_c, is exogenous. If the goal is to minimize the variance of returns, then the derivative of (2) with respect to X_f is set to zero and solved for $-X_f/X_c$ to derive the risk-minimizing hedge ratio:

$$(3) \quad -X_f/X_c = \frac{\sigma_{cf}}{\sigma_f^2}.$$

This hedge ratio can be estimated by regressing cash price changes on futures price changes because the slope coefficient is an estimate of (3).

Alternatively, if the goal is to maximize expected utility (u) using the formulation specified by Kahl (1983):

$$(4) \quad E(u) = E[\Pi] - \frac{\lambda}{2} \, \text{var} \, E(\Pi)$$

and λ is a risk-aversion parameter, the first-order condition for an optimum is:

$$(5) \quad - X_f/X_c = \frac{\sigma_{cf}}{\sigma_f^2} - \frac{E(F_2 - F_1)}{X_c \lambda \sigma_f^2}$$

Equation (5) can also be written as:

$$(6) \quad X_f = \frac{E(F_2 - F_1)}{\lambda \sigma_f^2} - X_c \frac{\sigma_{cf}}{\sigma_f^2}$$

The first portion of the right-hand side of equation (6) is a pure speculator's futures position since, in this case, $X_c = 0$ (Anderson and Danthine, 1981). The speculator is concerned with how the price will change relative to the variance of that price (i.e., relative to the riskiness of his position). If F_2, the ending futures price, is expected to be higher (lower) than the current futures price, F_1, the speculator will go long (short) futures; i.e., X_f is positive (negative) assuming $\lambda > 0$, which will be the case for a risk-averse individual.

The second portion of equation (6) is a pure hedger's futures position. Because the hedger's goal is to completely remove price variance, λ is extremely large, and the first portion becomes insignificant. When equation (3) is used to determine the hedge ratio, it is implicitly assumed that $\lambda \to \infty$; i.e., the hedger's position is not a function of the risk parameters; rather, the hedger is solely interested in reducing price risk.[4]

Percentage Change Model

When using percentage changes as advocated by Brown (1985), equations (1) and (2) are transformed by multiplying the cash position by C_1/C_1 and the futures position by F_1/F_1. This yields:[5]

$$(7) \quad E(r_p) = E[V_c r_c + V_f r_f]$$

where V_i is the total values of the cash (V_c) and futures positions (V_f) and r_i is the return from period 1 to period 2 on the values of the portfolio (r_p), cash (r_c), and futures (r_f) positions.

The variance of returns on the portfolio becomes

$$(8) \quad \text{VarE}(r_p) = V_c^2 \sigma_{r_c}^2 + V_f^2 \sigma_{r_f}^2 + 2V_c V_f \sigma_{r_c r_f}$$

where the variances and covariances are now of returns rather than prices. The hedge ratio when minimizing variance is:

$$(9) \quad -V_f/V_c = \sigma_{r_c r_f}/\sigma^2_{r_f}.$$

This hedge ratio is the slope coefficient of a regression of cash precentage price changes on futures percentage price changes.
When maximizing expected utility the hedge ratio is:

$$(10) \quad -V_f/V_c = \frac{\sigma_{r_c r_f}}{\sigma^2_{r_f}} - \frac{r_f}{V_c \lambda \sigma^2_{r_f}}.$$

which can be interpreted in much the same way as equation (6).

Price Level Model

An alternative hedging model to that given in equations (1) and (2) is appropriate when the hedger is concerned only with the variance about the expected return in an anticipatory hedge (i.e. there is no current cash position). The target or expected price:

$$(11) \quad \text{Target price} = F_1 - E(F_2 - C_2)$$

is the appropriate futures contract price observed today less the expected basis at the time of closing the hedge and completing the cash transaction. Converting this to a target value of the hedge gives:

$$(12) \quad \text{Target Value} = (-)X_f F_1 - E[(-)X_f F_2 - X_c C_2].$$

The negative sign (-) on X_f is there because a hedger will be taking an opposite futures position to the cash position. The variance of the target value (TV) is:

$$(13) \quad \text{Var (TV)} = X^2_f \sigma^2_{f2} + X^2_c \sigma^2_{c2} + 2X_f X_c \sigma_{c2f2}$$

where σ^2_{f2}, σ^2_{c2} and σ_{c2f2} are the variances and covariance of ending futures and cash prices respectively at the time the cash transaction would be completed.

The objective then is to choose the futures position (X_f) to minimize (13). This gives the optimal hedge ratio as:

$$(14) \quad -\frac{X_f}{X_c} = \frac{\sigma_{c2f2}}{\sigma^2_{f2}}$$

In this case, the hedge ratio is the regression coefficient of cash price levels regressed on futures price levels during the period when the hedger would be closing the futures position and entering the cash market.

Benninga et al. (1984) assume a formulation similar to equation (12). They state that income from hedging can be thought of as:

$$(15) \quad E[I] = X_c E(C_2) + (-) X_f E(F_1 - F_2)$$

which, if manipulated algebraically, is the same as equation (12). We have approached this problem from the standpoint of reducing basis risk, which results in the same formulation as reducing the variance of income as posed by Benninga et al.

Practical Differences

The parameters from price level and price change regressions must be interpreted differently from those of percentage price change regressions. The interpretation of the hedge ratio from price level and price change regressions is the ratio of the number of the units of futures to the number of units of the cash position that must be hedged to offset the variability of the value of the cash position. The hedge ratio derived from percentage change regressions is the ratio of the value of the futures to the value of the cash position that must be hedged to offset cash position variability.

For price changes (percentage changes), a measure of hedging effectiveness is the proportional reduction in cash price change (cash value change) variance when hedging at the optimal hedge ratio. For price levels, a measure is the average proportional reduction in variability about a mean price level. These measures of hedging effectiveness reduce to the coefficient of determination in all the regression models. However, one cannot compare the coefficients of determination in distinguishing between the different model specifications because the dependent variable is not the same in any of the models.

Brown's percentage change model is a slightly different specification than the price difference equation. The price difference equations assume a linear relationship between the cash and futures prices. The percentage change model assumes that the cash and futures prices follow a log linear relation. This specification would be useful in instances in which the cash and futures prices do not change linearly with respect to one another, as could be the case in cross hedging when different valued commodities are being compared. Ideally, one should test the degree of linearity between the cash price and the futures price. To the extent that the two series react linearly, the price difference model would be preferred to Brown's model in light of a more parsimonious model and ease of hedge ratio interpretation. However, if the relation is nonlinear, Brown's specification may provide a better statistical fit and, thus, more accurate hedge ratios, though other nonlinear functional forms could also be considered which may not be so easily interpreted.

From both a theoretical and statistical standpoint, Hill and Schneeweis (1981) state that use of price level regressions is inappropriate; Brown (1985) and Wilson (1985) have echoed this opinion in their recent articles. They claim that, theoretically, hedgers are attempting to reduce the risk of price changes in both the cash and futures markets from the time the hedge is placed until it is lifted. Benninga et al. (1984), while not arguing theoretically that price difference models are preferred, agree with the assertions that, statistically, price differences are more appropriate. Statistically, they argue that cash and futures prices can be highly correlated if corresponding trends exist between the two; if one estimates the hedging ratio by using price levels, the residuals may exhibit autocorrelation. This would result in the violation of the OLS assumptions and inefficient estimates of hedge ratios with underestimated standard errors. They state that these problems are eliminated by using price change or percentage price change data in the estimation process.

Statistically, we argue that there is no basis for claiming that price change and percentage change regressions are better than price-level

regressions. First-order autocorrelation may be reduced when first differences are used in the estimation process. However, the price change and percentage change models are not first differences. In these models, the differencing scheme used is the change in prices over the hedge interval. With this in mind, the only time the difference model would reduce first-order autocorrelation is if hedges were held only for a day, week, or month (i.e., the frequency of the analyst's data). Few if any hedges are placed for this short time period. Therefore, the problem of autocorrelation could remain a problem in the typical price difference specifications. Autocorrelation-corrected parameter estimates might be appropriate for all these alternative model specifications.

From a practical standpoint, determining the amount to hedge with price-level regressions is simpler than with differencing models. The length of time the hedge is held does not have to be defined with price-level regressions. For a given contract, the same hedge ratio can be used regardless of the length of time to maturity. With differencing models, each time the hedging period is changed, a new hedge ratio must be estimated. Therefore, although only one hedge ratio need be estimated for each contract when using price-level regressions, several may need to be estimated when using differencing models.

The remaining question then, is which of these methods (differences or levels) is theoretically more appropriate? If the decision maker has a current inventory in storage as a seller or if he has storage available as a buyer, then the price difference model would seem to have some merit. However, for a pork producer whose produce is not immediately marketable or a corn producer whose crop is yet in the field, the current cash price is irrelevant. Their concern is the current futures price and the ending basis, not the change in cash and futures prices between now and when the commodity is sold. For anticipatory hedges the price level model seems appropriate.

Empirical Results
     To illustrate the differences among these alternative hedge ratio estimation approaches, each was estimated by using the same data to estimate cross-hedging relationships between barley and sorghum cash prices and nearby corn futures prices.[6] The prices of Minneapolis barley (no. 3) and Kansas City sorghum (no. 2) are Thursday closing prices reported by the U.S. Department of Agriculture, Agricultural Marketing Service, provided to us by Sparks Commodities, Inc. Barley prices are quoted in dollars per bushel while sorghum prices are in dollars per hundredweight. Corn futures prices are also Thursday closing prices at the Chicago Board of Trade. The data encompassed 1975 to 1984.

Price change and percentage change regressions were estimated for an arbitrarily selected 3-month hedging period. Price-level regression parameters were estimated by regressing the current cash price on the nearby futures contract price, since that is the critical relationship in anticipatory hedges.

In defining the five contract periods, cross-hedges were assumed to be terminated before threat of making or taking delivery occurred. Thus, hedges were assumed to be lifted by the 15th of the contract month. In the price-level model, the time when the hedge is placed is irrelevant because the

only concern is the cash-futures price relationships at the time of the cash transaction. With price change and percentage change models, the placement of the hedge is arbitrarily assumed to be 3 months before the contract month (different equations would have to be estimated for other hedging intervals).

The estimated equations are summarized in Table 1. All slope coefficients are different from zero at the .05 level of significance. All the sorghum hedge ratios (slope coefficients) from the price change regression analysis are smaller than those from the price level regressions. Conversely, all but one of the barley hedge ratios generated by the price change regressions are larger than those from the price level regressions. A smaller (larger) hedge ratio indicates that one futures contract will establish the price of a larger (smaller) amount of the cash commodity.[7]

In testing which model is statistically preferred, one cannot perform the standard F-tests because the dependent variables differ across models. Rather, to compare the price-level model with the price difference model, one should consider the statistical assumption being made in the differencing process. When taking k-th order differences of both the dependent and independent variables in a regression model, one implicitly assumes a k-th order autocorrelation coefficient of one. The probability of every k-th order (k = 1, 2, 3 ...) autocorrelation coefficient being one is extremely low inasmuch as k, the length of the hedging period, will change as the time horizon changes. Thus, unless the order of differencing matches an order of autocorrelation with a rho coefficient near one, the differencing will not be statistically sound. Only under these circumstances would the differencing model be clearly preferred to the price- level model.

In comparing the price difference models with the percentage change models, the gauge is the degree of linearity between the cash price and futures price differences. If the cash price of the commodity to be (cross) hedged responds linearly with the futures price, the price difference model would be preferred because a goal is to keep the model as simple as possible. If a definite nonlinear relationship exists between the prices, the percentage change model may be preferred.

The Durbin-Watson statistics indicate high levels of first-order autocorrelation for all the regressions. As a result, it is likely that the statistical significance of the hedge ratios is somewhat overstated. Estimating the hedge ratios for the specific contracts individually by using generalized least-squares (GLS) autocorrelation models is not useful in this case because the only parameter of concern is the hedge ratio (i.e., the slope coefficient). Theoretically, this ratio should not change under a GLS model. The estimated autocorrelation coefficient will not add any useful information to aid the decision maker when placing a hedge.

Consider, for example, a feeder knows in March that sorghum purchases will have to be made in August. To hedge those anticipated August purchases in March, the hedge ratio estimates must be used corresponding based on the historical relationship between sorghum cash and corn futures prices in August. The most recent residual will be from the previous year's August price relationship. The basis this August may be quite different from the previous year's basis because of changes in factors such as local supply and demand for the commodity, transportation costs, interest rates, and so forth. Thus, the standard autocorrelation model would not significantly improve our

TABLE 1: Estimated Hedge Ratios for Cross Hedging Sorghum
and Barley in Corn Futures, 1975-1984.

Sorghum

	March	May	July	Sept.	Dec.
Price Levels					
Intercept	.31	.018	-.31	-.09	.12
(t-statistic)	(2.29)	(.113)	(-1.76)	(-.548)	(1.06)
Slope	1.45	1.54	1.67	1.61	1.53
(t-statistic)	(29.37)	(27.61)	(27.32)	(25.87)	(36.79)
R^2	.82	.85	.84	.83	.86
D.W.	.069	.117	.155	.112	.088
N	163	119	131	125	180
Price Differences					
Intercept	.19	.08	.09	-.01	.12
(t-statistic)	(8.95)	(3.38)	(2.92)	(-.271)	(4.31)
Slope	1.19	1.17	1.36	1.19	1.14
(t-statistic)	(11.25)	(12.07)	(11.33)	(13.23)	(9.97)
R^2	.62	.72	.67	.74	.53
D.W.	.301	.582	.282	.200	.155
N	60	52	56	54	68
Percentage Changes					
Intercept	.02	.018	.019	-.005	.022
(t-statistic)	(6.06)	(3.14)	(2.55)	(-.51)	(3.24)
Slope	.82	.78	.97	.75	1.04
(t-statistic)	(12.08)	(12.41)	(12.04)	(12.60)	(11.28)
R^2	.66	.72	.70	.72	.60
D.W.	.543	.586	.228	.169	.200
N	60	52	56	54	68

Barley

	March	May	July	Sept.	Dec.
Price Levels					
Intercept	-.10	.24	.86	.47	.27
(t-statistic)	(-.986)	(1.56)	(5.11)	(3.18)	(2.67)
Slope	.81	.67	.44	.59	.68
(t-statistic)	(21.89)	(12.39)	(7.52)	(11.46)	(19.35)
R^2	.73	.54	.29	.49	.64
D.W.	.125	.136	.205	.187	.161
N	163	119	131	125	180
Price Differences					
Intercept	-.017	.015	-.07	.13	-.08
(t-statistic)	(-.843)	(.623)	(-2.92)	(3.29)	(-2.18)
Slope	.58	.97	1.18	.81	.89
(t-statistic)	(6.26)	(10.06)	(11.84)	(9.77)	(10.37)
R^2	.33	.64	.69	.61	.55
D.W.	.341	.420	.615	.313	.406
N	60	52	56	54	68
Percentage Changes					
Intercept	.01	.02	-.028	.077	.027
(t-statistic)	(1.48)	(2.17)	(-2.39)	(3.52)	(2.53)
Slope	.58	1.22	1.51	1.21	1.38
(t-statistic)	(4.23)	(11.17)	(12.22)	(9.56)	(9.24)
R^2	.20	.68	.71	.60	.50
D.W.	.471	.519	.634	.446	.597
N	60	52	56	54	68

estimate of the cash price corresponding to the futures price we could lock in today.

Alternative adjustments for autocorrelation may prove to be more useful. A simple method would involve transforming the data by taking first differences (not the same as price difference model) of the data before estimating the hedge ratio. This, however, assumes a first-order autocorrelation coefficient of 1.0, which is unlikely. However, it may be more desirable to assume an autocorrelation coefficient of 1.0 than to completely ignore it and assume it to be zero.

A better approach would involve estimating the cash price-nearby futures price relationships using an unconstrained stacked multiple regression model, allowing both different intercepts and slope coefficients for each contract. This would result in the same parameter estimates as separate simple regression models (of the cash price regressed on the nearby futures price) for each contract month. This allows the hedger to use the most recent errors in translating futures prices into cash price equivalents, rather than the errors a year earlier. However there are tradeoffs. Separate contract regression models can be useful in analyzing the error distribution of the hedging or cross-hedging effectiveness (or risk of adverse consequences) in an individual contract. For example, the harvest times for the two commodities may not be the same, and/or the two commodities may not follow the same seasonal price trend, in which case certain contract months may not offer as much price variability risk reduction through cross hedging as other contract months.

The stacked model, while not allowing detailed analysis of individual contract month cross-hedging effectiveness (i.e., the only useful information will be the t-statistic for the coefficient on the respective contract month), will allow the decision maker to utilize the information about the autoregressive error properites. If positive first-order autocorrelated errors are present and if cash prices were high relative to typical futures prices 1 month ago, the same phenomenon might be expected to persist this month. Therefore, the current basis would be expected to remain narrow, and a smaller than normal basis would be expected. However, considering hedges further into the future, i.e., 2, 3, or more contract months away, the usefulness of the autoregressive parameter progressively declines.

Table 2 summarizes the results of the stacked, generalized least-squares regressions for sorghum and for barley. The hedge ratios in the autocorrelation adjusted model change only slightly from the values in table 1 and not in a systematic way. The first-order autocorrelation coefficients for each model are positive and highly significant, as was suspected by observing the Durbin-Watson statistics from the individual price level equations in table 1. The information gained in going from the OLS models to the GLS models is the adjustment coefficient (i.e., coefficient on lagged residuals), which will aid the decision maker only in the very short run, and the slight changes in the hedge ratios. The information lost in this transition is the detailed comparisons on hedging effectiveness of the various contract months.

TABLE 2: Generalized Least-Squares Estimated Hedge Ratios
for Cross-Hedging Sorghum and Barley in Corn Futures, 1975-1984

			Contract Month		
	March	May	July	Sept.	Dec.
Sorghum					
Intercept	.19	.35	-.16	.37	.42
(t-statistic)	(.90)	(1.48)	(-.62)	(1.95)	(2.33)
Slope	1.57	1.41	1.52	1.42	1.44
(t-statistic)	(21.54)	(19.53)	(18.47)	(24.16)	(23.64)

Lagged
Residual coefficient   +.905
(t-statistic)   (60.29)

$$R^2 = .76, \quad N = 792$$

	March	May	July	Sept.	Dec.
Barley					
Intercept	-.02	.04	1.17	.71	.25
(t-statistic)	(-.08)	(.18)	(4.90)	(4.06)	(1.51)
Slope	.73	.75	.35	.57	.69
(t-statistic)	(10.57)	(10.94)	(4.43)	(10.07)	(11.95)

Lagged
Residual coefficient   +.880
(t-statistic)   (52.41)

$$R^2 = .41, \quad N = 792$$

Conclusion

Three procedures--price level regressions, price change regressions, and percentage change regressions--have been used to estimate optimal hedge ratios. Debate has arisen over which method is correct. Proponents of price change and percentage price change regressions claim that these methods are statistically superior to price-level regressions, which supposedly exhibit significant degrees of autocorrelation.

Comparing all three procedures, hedge ratios based on price change models were not necessarily statistically superior. First-order autocorrelation can be reduced sometimes with first differencing, but the price change models do not transform the data this way because few hedges are held for a day, week or month (i.e., the frequency of the data). The only reason one would statistically prefer the price difference model to the price-level model would be if the order of the difference corresponded precisely to the estimated order of (high) autocorrelation (i.e., the lag structures have to be matched).

Consequently, optimal hedge ratios generated by price-level regressions are as statistically correct as those by the other two procedures. It may be desirable to estimate the hedge ratios in a GLS framework by using a stacked price level model to increase the efficiency of the estimates and to enhance information for short-run decisions. However, the estimated hedge ratios would not be expected to change appreciably in going from the several individual contract month equations to the single equation GLS stacked model.

Theoretically, the proper hedge ratio estimation technique depends upon the objective function of the hedger and the type of hedge being considered. If the hedge is purely anticipatory the current cash price is irrelevant; for a highly risk averse hedger the hedge ratio can be appropriately estimated by a price-level regression. If the hedge is a storage hedge the current cash price is relevant to the hedging decision because there is an opportunity cost of hedging and not entering the cash market immediately--the change in cash price over time reflects this cost. Therefore, for a pure storage hedge, the price change model seems appropriate. Finally, if the hedger's objective is to maximize expected utility as opposed to minimizing the variance of returns then none of these estimation techniques will provide the appropriate hedge ratio. Under this situation the hedge ratio will be a function of more than simply the variability in the cash and futures prices (see equation (6) for example).

In summary, for anticipatory hedges, for highly risk averse hedgers, the price-level model is theoretically sound and preferred to the change models except when: 1) the cash-futures price relationship is nonlinear as opposed to linear in the levels, in which case one should consider a nonlinear transformation on the cash and futures prices; 2) the price-level equation exhibits strong k-th order positive autocorrelation (where k is the length of the hedging horizon), in which case the price (k-th order) differences model may be preferred; 3) first-order (and similar low orders) autocorrelation occurs in the price level model, one should consider using a stacked generalized least-squares estimation technique on price level relationships. When one is dealing with a carrying-charge hedge for a storable commodity, the price difference model would be preferred theoretically. The exception would be when cash and future price changes are log-linear; then the percentage change model would be preferred. Consequently, anticipatory hedgers may likely take different futures positions than storage hedgers.

FOOTNOTES

[1] The variability in cash and futures prices may not be identical and therefore the optimal hedge ratio is not always represented by a unit-for-unit hedge (hedge ratio of 1) between cash and futures. This is particularly true when cross hedging is considered because the cash and futures prices are for different commodities, with different price levels and variabilities. The minimum risk hedge ratio thus represents the futures position for a given cash position in order to minimize the price risks associated with unequal price changes in the cash and futures markets during the duration of the hedge.

[2] The goal of this analysis is to determine the optimal futures position for each unit of the cash commodity, given that the individual has decided to hedge. The decision of whether to hedge and the timing of the hedge is ignored because that is a different issue.

[3] A positive sign preceeding the cash (futures) quantity indicates a long cash (futures) position and a negative sign indicates a short postion.

[4] Bond and Thompson (1985) point out that, if one assumes a nonlinear storage cost function in the expected profit function, equation (1), then the risk parameter does become relevant to the hedger, and the hedge ratio is dependent upon the risk parameter.

[5] The procedure is as follows: (1) $E(\pi) = X_c E(C_2 - C_1) + X_f E(F_2 - F_1)$

multiply X_c by C_1/C_1 and X_f by F_1/F_1 and regroup gives

$$E(\pi) = X_c C_1 \frac{E(C_2 - C_1)}{C_1} + X_f F_1 \frac{E(F_2 - F_1)}{F_1}$$

defining $X_c C_1 = r_c$; $X_f F_1 = r_f$; $(C_2 - C_1)/C_1 = V_c$; and $(F_2 - F_1)/F_1 = V_f$

gives (7) $E(r_p) = E[V_c r_c + V_f r_f]$.

[6] The regressions to estimate the respective hedge ratios are as follows:

Price Levels

$C_t = a + bF_t + \varepsilon_\tau$ where ε_t is a random error, a and b are parameters to be estimated, and C and F are the cash and futures prices respectively during the period when the hedge would be lifted. The estimate of b will be an estimate of the hedge ratio in equation (14).

Price Differences

$(C_1 - C_2) = d + h(F_1 - F_2) + \alpha_t$ where α_t is a random error, d and h are parameters to be estimated, and other variables are as previously defined. The estimate of h will be an estimate of the hedge ratio in equation (13).

6(cont.) Percentage Changes

$$\frac{(C_1 - C_2)}{C_1} = k + s \frac{(F_1 - F_2)}{F_1} + \delta_t$$ where δ_t is a random error, k and s

are parameters to be estimated, and other variables are as previously defined. The estmiate of s will be an estimate of the hedge ratio in equation (9).

7The dependent and independent variables across equations are different, therefore no standard statistical comparison tests among parameters across equations could be performed.

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