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Precautionary Saving in the Large under Higher-Order Risk and Recursive Utility

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Abstract

The measurement of the strength of the precautionary saving motive under recursive utility (RU) has been conceptually restricted to reactions to the addition of a zero-mean risk to safe future income. This paper provides characterizations of comparative precautionary saving under RU analogous to Ross' (1981) approach to comparative risk aversion for increases in risk, also of higher order, covering the two cases of income risk and a risky saving return. The characterizations involve a comparison based on precautionary premia. I also define preference-intensity measures of the Ross-type and show how they can equivalently represent comparative precautionary saving.

Keywords: Intertemporal choice, prudence, precautionary saving, recursive preferences, higher-order risk, precautionary premium, Ross

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1 Introduction

Measuring the intensity of individual risk and time preferences is essential for inference of any risky intertemporal economic decision. In his classic paper, Kimball (1990) shows that under expected utility (EU) the coefficient formed by the negative of the third over the second utility derivative captures the intensity of the precautionary saving motive. This coefficient of absolute prudence arises when applying an analysis analogous to the one related to the Arrow-Pratt coefficient of absolute risk aversion to the negative of marginal utility, where marginal utility is a decisive element in the Euler condition for optimal saving. However, similar to the Arrow-Pratt measure, Kimball's prudence coefficient studies preference intensity only in a situation where an actuarially neutral risk is added to a context of certainty.

Eeckhoudt and Schlesinger (2008) extend the analysis of precautionary saving to risk increases, also of higher order, and consider also the alternative case with a risky return on saving. Risk increases, of course, compare two risky situations, so that, in static choice, a comparison of risk preferences appropriately refers to Ross' (1981) more risk aversion. Starting from Eeckhoudt and Schlesinger, Liu (2014) adapts Kimball's analysis of equivalent precautionary premia to the case of higher-order increases of income risk. His preference conditions to compare the strength of precautionary reactions rely on the Liu and Meyer (2013b) concept of $(n/m)^{th}$-degree Ross more risk aversion, involving the $(n+1)^{th}$ and the second EU derivatives. Heinzel (2016), then, treats the case with return risk and shows how the conditions to compare the precautionary reactions can equivalently be represented based on coefficients of precautionary intensity.

Interestingly, all of the aforementioned papers that refer to intertemporal choice stick to the EU framework, despite the well-known issue that EU does not allow to disentangle risk and intertemporal preferences (*e.g.*, Epstein and Zin 1989). To the best of my knowledge, the only work that deals thus far with a measure of precautionary intensity under recursive utility (RU) is Kimball and Weil (2009). The prudence coefficients the authors derive involve the derivatives of risk utility up to the third as well as the intensity of intertemporal substitution. However, Kimball and Weil, similar to Kimball, limit themselves to the case where an actuarially neutral risk is added to the safe future income.

In this paper, I derive preference conditions, premia, and preference-intensity measures that allow to compare the strength of the precautionary reactions to risk increases, also of higher order, under RU in the two cases of income risk and a risky saving return.

In Section 2, I introduce the two-period consumption/saving model under RU and some further basic concepts for the analysis. Section 3 deals with the case with income risk. Section 4 treats the case of return risk. Section 5 states first the impossibility to derive single-expression preference coefficients "in the small" for higher-order risks, risk changes, and return risk, and provides then a representation of the RU conditions comparing the strength of precautionary reactions based on preference-intensity coefficients "in the large." Section 6 concludes. Proofs, which do not appear in the text, are gathered in the Appendix.

2 Conceptual Foundation

Consider the following two-period consumption/saving model under RU, as in Kimball and Weil (2009) or Bostian and Heinzel (2016). Individual u chooses saving s_1 out of first-period income y_1 such as to maximize the intertemporal utility objective

$$u\left(y_1 - s_1\right) + \beta u\left(CE(\tilde{y}_2 + s_1 \tilde{R}_2)\right) \tag{1}$$

where u is the agent's per-period felicity function capturing the preference for consumption now versus later and β is the utility discount factor. Period-two consumption is composed of income y_2 and saving with interest, where R_2 represents the gross return on any amount saved. Risk may enter through either y_2 or R_2. (A tilde indicates when a variable is risky.) The certainty equivalent $CE(.)$ ranks consumption paths according to the risk preference ψ:

$$CE(\tilde{y}_2 + s_1 \tilde{R}_2) \equiv \psi^{-1}\left(E_1\left[\psi\left(\tilde{y}_2 + s_1 \tilde{R}_2\right)\right]\right)$$

In contrast to u, ψ is a von Neumann-Morgenstern utility function (Selden 1978). Well-known special cases of objective (1) are EU (for $\psi = u$) and Epstein and Zin (EZ) preferences (when relative risk aversion and the elasticity of intertemporal substitution are constant).

According to the first-order condition

$$u'(c_1) = \beta u'(CE(\tilde{c}_2))\, CE'(\tilde{c}_2) \tag{2}$$

the agent will choose saving such that the marginal utility from foregoing consumption in period 1 (*i.e.*, saving a marginal amount) is to equal to the expected discounted marginal utility from consuming instead in period 2. For the problem to be well-behaved, the increased flexibility of RU as in (1) comes with a restriction on its constituent functions. Strictly increasing and concave shapes for u and ψ – the natural extension of the EU requirements – are to be amended by concavity of ψ^{-1} in s_1, which occurs if its absolute risk tolerance (inverse of absolute risk aversion) is concave (Gollier 2001). These assumptions will be implicit throughout.

The income- and return-risk increases considered in the following refer to the nth-degree ℓ-first-moments-preserving stochastic dominance (n-ℓ-MPSD) order of Liu (2014).

Definition 1 (nth-degree ℓ-first-Moments-Preserving Stochastic Dominance) *For any given integer ℓ with $1 \leq \ell \leq n-1$, $\tilde{x}_l$ dominates $\tilde{x}_h$ in the n-ℓ-MPSD if and only if $\tilde{x}_l \preceq_{nSD} \tilde{x}_h$ and $E\left(\tilde{x}_l^j\right) = E\left(\tilde{x}_h^j\right)$ for $j = 1, \ldots, \ell$.*

The n-ℓ-MPSD order is a special case of a n-concave order (*e.g.*, Denuit and Eeckhoudt 2010). While n represents in Definition 1 the stochastic-dominance order, ℓ describes how many of the first moments are invariant between the two random variables. Definition 1 covers as special cases Ekern (1980) risk increases (for $\ell = n-1$) and the nth-degree mean-preserving stochastic dominance of Denuit and Eeckhoudt (2013) (for $\ell = 1$). For example, mean-preserving spreads (Rothschild and Stiglitz 1970) arise as 2-1-MPSD, increases in downside risk (Menezes et al. 1980) as 3-2-MPSD, and increases in outer risk (Menezes and Wang

2005) as 4-3-MPSD.

Because precaution is here a property of intertemporal choice, the curvature of RU in the results below is to be interpreted as intertemporal preference, reflecting resistance to intertemporal substitution or precaution.[1] The next definition adapts the Liu and Meyer (2013b) concept of $(n/m)^{th}$-degree Ross more risk aversion to intertemporal choice. The definition refers to the class $\mathcal{U}_{k-cv}^{\mathcal{S}}$ of all k-concave functions f on $\mathcal{S}$, formally the set $\{f|(-1)^k f^{(k)}(z) \leq 0$ for some $k \in \mathbb{N}$ and $z \in \mathcal{S} \subseteq \mathbb{R}\}$.

Definition 2 (Stronger $(n/m)^{th}$-Degree Ross Intertemporal Attitude) *For two utility functions $u(x), v(x) \in \mathcal{U}_{n-cv}^{D_x} \times \mathcal{U}_{m-cv}^{D_x}$, $n > m$, $u(x)$ has a stronger $(n/m)^{th}$-degree Ross intertemporal attitude than $v(x)$ if and only if there exists a $\lambda > 0$ such that*

$$\frac{u^{(n)}(x_a)}{v^{(n)}(x_a)} \geq \lambda \geq \frac{u^{(m)}(x_b)}{v^{(m)}(x_b)} \quad \textit{for all} \ \ x_a, x_b \in D_x \subset \mathbb{R}_0^+$$

As applied to utility as in objective (1), $f^{(k)}(.)$, with $f \in \{u, v\}$, is a shorthand for the k^{th} derivative of RU with respect to saving, which contains all derivatives up to the k^{th} of f and $CE(.)$, and, thus, ψ.[2] The conditions in Definition 2 resemble formally the ones for the static case under EU as initially defined in Liu and Meyer (2013b). The higher-order Ross coefficients that have been defined in the literature for static choice involve the n^{th} and the first EU derivatives (*e.g.*, Jindapon and Neilson 2007, Denuit and Eeckhoudt 2010). The conditions in Liu (2014) describing the strength of the precautionary saving motive as associated with a given increase in income risk use the n^{th} and the second EU derivatives.

[1] As is well known, under univariate EU the inverse of the elasticity of intertemporal substitution is formally identical to the coefficient of relative risk aversion, which, in turn, corresponds in intertemporal choice to the coefficient of relative resistance to intertemporal substitution (*e.g.*, Gollier 2001).

[2] Bostian and Heinzel (2016) provide the explicit expressions of these derivatives. To simplify the presentation, I assume that intertemporal preferences u and v are associated with identical risk preferences ψ.

3 Income-Risk Increases

I study first the case of increases in income risk. Bostian and Heinzel (2016) show that an nth-order stochastic-dominance (NSD) deterioration in future income risk increases saving if and only if the agent's utility u and the certainty equivalent CE of future consumption are in the class of all k-concave functions for $k = 1, \ldots, n+1$. As an implication, in the more particular case of a deterioration from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$ in nth-degree ℓ-MPSD, that is, $\tilde{y}_{2,l} \preceq_{n-\ell-MPSD} \tilde{y}_{2,h}$ (Definition 1), u and CE are to be k-concave for $k = \ell, \ldots, n+1$.

To determine the equivalent precautionary premium θ^{y_2} associated with the change in future income from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$, consider first-order condition (2) for the case with income risk only. θ^{y_2} corresponds to the safe reduction in the original future income $\tilde{y}_{2,l}$ that has the same effect on saving as the change from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$. For individual f, the precautionary premium $\theta_f^{y_2}$ is, hence, such that saving coincides under the two conditions:[3]

$$f'(y_1 - s_1) = \beta f'(CE(\tilde{y}_{2,l} - \theta_f^{y_2} + s_1 R_2)) CE_{y_2}(\tilde{y}_{2,l} - \theta_f^{y_2} + s_1 R_2) R_2$$
$$f'(y_1 - s_1) = \beta f'(CE(\tilde{y}_{2,h} + s_1 R_2)) CE_{y_2}(\tilde{y}_{2,h} + s_1 R_2) R_2$$

The precautionary premium arises accordingly from

$$f'(CE(\tilde{y}_{2,l} - \theta_f^{y_2} + s_1 R_2)) CE_{y_2}(\tilde{y}_{2,l} - \theta_f^{y_2} + s_1 R_2) = f'(CE(\tilde{y}_{2,h} + s_1 R_2)) CE_{y_2}(\tilde{y}_{2,h} + s_1 R_2) \quad (3)$$

It is important to note that $\theta_f^{y_2}$ represents only the maximum willingness for additional saving to cope with an increase in income risk from $\tilde{y}_{2,l}$ to $\tilde{y}_{2,h}$ in the respective risk order. The equivalent precautionary premium in Kimball and Weil (2009), by contrast, refers to the maximum willingness to save more to confront the complete risk on future income.

The following theorem, stated in analogy to Ross (1981: Theorem 3), compares two individuals u, v with the same optimal saving level under the original future income $\tilde{y}_{2,l}$. It

[3] The notation CE_{y_2} stands for the first derivative of $CE(.)$ with respect to its argument.

extends Theorem 3 in Liu (2014) to RU as in objective (1).

Theorem 1 *Suppose $n \geq 2$ and $1 \leq \ell \leq n-1$. For two utility functions u, v in the class $\mathcal{U}^{D_x}_{k-cv}$ of all k-concave functions for $k = 1, 2$ and $\ell+1, \ldots, n+1$, and involving the same saving level under the future reference income $\tilde{y}_{2,l}$, the following conditions are equivalent:*

(i) $(-u')$ has a stronger k^{th}-degree Ross intertemporal attitude than $(-v')$ for all $k = \ell+1, \ldots, n$.

(ii) There exist $\lambda > 0$ and $\phi'(x_{y_2})$ with $\phi''(x_{y_2}) \geq 0$ and $\phi'(x_{y_2}) \in \mathcal{U}^{D_x}_{(k+1)-cv}$ for $k = \ell+1, \ldots, n$ and all $x_{y_2} \in D_x \subset \mathbb{R}^+_0$ such that $u'(x_{y_2}) = \lambda v'(x_{y_2}) + \phi'(x_{y_2})$.

(iii) $\theta^{y_2}_u \geq \theta^{y_2}_v$ for all $\tilde{y}_{2,l}, \tilde{y}_{2,h}$ with $\tilde{y}_{2,l} \preceq_{n-\ell-MPSD} \tilde{y}_{2,h}$ and $\theta^{y_2}_f$ as defined in equation (3) for $f \in \{u, v\}$.

The theorem provides two ways to compare the strength of the precautionary saving reaction to a given increase in income risk. Statement (i) amounts to the same as requiring u to have a stronger $((k+1)/2)^{th}$-degree Ross intertemporal attitude than v for all $k = \ell+1, \ldots, n$. Following Definition 2, this translates into preference conditions involving the $(n+1)^{th}$ and the second RU derivatives (as stated in equation (4) below). According to Statement (iii), agent u has, moreover, a larger precautionary premium than agent v.

For $\ell = n-1$ deteriorations in the n-ℓ-MPSD order correspond to Ekern risk increases. Corollary 1 follows for this important special case from the above theorem.

Corollary 1 *Suppose $n \geq 2$ and that the two utility functions u, v are in the class $\mathcal{U}^{D_x}_{k-cv}$ of all k-concave functions for $k = 1, 2$ and $n+1$ and involve the same saving level under the future reference income $\tilde{y}_{2,l}$. Then, $\theta^{y_2}_u \geq \theta^{y_2}_v$ for all n^{th}-degree risk increases in future income if and only if $(-u')$ has a stronger n^{th}-degree Ross intertemporal attitude than $(-v')$ or, equivalently, there exists a constant $\lambda > 0$ such that*

$$\frac{u^{(n+1)}(x_{y_2,a})}{v^{(n+1)}(x_{y_2,a})} \geq \lambda \geq \frac{u''(x_{y_2,b})}{v''(x_{y_2,b})} \quad \text{for all } x_{y_2,a}, x_{y_2,b} \in D_x \subset \mathbb{R}^+_0 \tag{4}$$

4 Return-Risk Increases

Bostian and Heinzel (Proposition 3) show that a NSD deterioration in return risk makes an agent with RU as in objective (1) save more if and only if the intertemporal preference function u and the certainty equivalent $CE(.)$ are in the class $\mathcal{U}^{\mathcal{S}}_{k-cv}$ of all k-concave functions, and the analog of a partial $(k+1)^{th}$-order risk aversion coefficient for $CE(.)$ exceeds k for $k = 1, 2, \ldots, n$. This result generalizes Proposition 2 in Eeckhoudt and Schlesinger (2008) to RU. The following lemma, which is implied by the proposition in Bostian and Heinzel, adapts this result to deteriorations of return risk in the n-ℓ-MPSD order.[4]

Lemma 1 *Let $s^*_{R_{2,l}}$ and $s^*_{R_{2,h}}$ be the optimal saving choices from condition* (2) *under the return lotteries $\tilde{R}_{2,l}$ and $\tilde{R}_{2,h}$, respectively. The following statements are equivalent:*

1. $s^*_{R_{2,h}} \geq s^*_{R_{2,l}}$, *if* $\operatorname{sgn}\left[u^{(k)}(.)\right] = \operatorname{sgn}\left[CE_{y_2^k}(.)\right] = (-1)^{k+1}$ *and* $-s_1 R_2 \frac{CE_{y_2^{k+1}}(c_2)}{CE_{y_2^k}(c_2)} \geq k$ *for all* $k = \ell, \ell+1, \ldots, n$.

2. $\tilde{R}_{2,l}$ *dominates* $\tilde{R}_{2,h}$ *via n-ℓ-MPSD.*

The saving reaction to return risk reflects the interplay of a positive precautionary effect and a negative substitution effect. According to Lemma 1, saving increases after a return-risk increase when the respective level condition on the analog of the partial risk-aversion coefficient for $CE(.)$ is fulfilled. In that case, the precautionary dominates the substitution effect. Because its conditions are necessary and sufficient, the lemma similarly predicts a decrease in the agent's saving, if the level condition is not fulfilled. The simulation results in Bostian and Heinzel suggest that this can be a common case.

Measuring the intensity of risk responses is more complicated when subjects can steer risk exposure immediately by their optimal choices (as under return risk) than when the risk is fully exogenous (as under income risk). For a risk that is endogenous, in the sense that risk exposure depends directly on the choice of the endogenous variable α, Briys et al.

[4] The notation $CE_{y_2^k}$ stands for the k^{th} derivative of $CE(.)$ with respect to its argument.

(1989) define the risk premium $\bar{P}$ of individual f by:

$$f(w_0 + w(\bar{\alpha}, E\tilde{x}) - \bar{P}) = Ef(w_0 + w(\alpha^*, \tilde{x}))$$

where w_0 is the agent's initial wealth, and $w(\alpha, \tilde{x})$ is the agent's endogenous wealth component, with $\bar{\alpha}$ being the optimal choice when the risk is replaced by its expectation and α^* being the choice maximizing EU under risk $\tilde{x}$. Defined in this way, $\bar{P}$ preserves the desirable properties of the corresponding Arrow-Pratt concept for fully exogenous risk of being positive for risk averters and increasing with mean-preserving spreads.

Similar to Heinzel (2016), I adapt here Briys et al.'s approach to intertemporal choice in form of saving under return risk and higher-order risk increases. To determine the equivalent precautionary premium θ^{r_2} under RU for the change of the net return from $\tilde{r}_{2,l}$ to $\tilde{r}_{2,h}$, consider first-order condition (2) with return risk only. For individual f, θ^{r_2} arises by comparing this first-order condition evaluated for $\tilde{r}_{2,h}$ with that condition with present marginal utility evaluated at optimal saving s_1^{*h} under return risk $\tilde{r}_{2,h}$ but expected future marginal utility evaluated for $\tilde{r}_{2,l}$ and period-two consumption $\tilde{c}_2^{*l}$ subtracted by the safe amount θ^{r_2}, such as to compensate for the variation in expected future marginal utility due to exposure to the high return risk:

$$f'(y_1 - s_1^{*h}) = \beta f'(CE(y_2 + s_1^{*l}\tilde{R}_{2,l} - \theta_f^{r_2}))CE'(y_2 + s_1^{*l}\tilde{R}_{2,l} - \theta_f^{r_2})$$

The precautionary premium θ^{r_2} arises, thus, from the defining equation

$$f'(CE(y_2 + s_1^{*l}\tilde{R}_{2,l} - \theta_f^{r_2}))CE'(y_2 + s_1^{*l}\tilde{R}_{2,l} - \theta_f^{r_2}) = f'(CE(y_2 + s_1^{*h}\tilde{R}_{2,h}))CE'(y_2 + s_1^{*h}\tilde{R}_{2,h}) \tag{5}$$

θ^{r_2} corresponds, accordingly, to the safe variation in period-two consumption $\tilde{c}_2^*$ evaluated at optimal saving s_1^{*l} under the original return risk $\tilde{r}_{2,l}$ that has the same effect on expected future marginal utility as the change from $\tilde{r}_{2,l}$ to $\tilde{r}_{2,h}$.

Because θ^{r_2} reflects the variation in total saving, the precautionary premium under return risk is positive or negative depending on whether the precautionary effect or the substitution effect dominates. The following theorem is the analog of Theorem 1 for increases in risk on the saving return given that in period one the level of the exogenous period-two income is already known. It has two formulations depending on which of the two effects prevails.

Theorem 2 *Suppose $n \geq 2$ and $1 \leq \ell \leq n-1$. For two utility functions u, v in the class $\mathcal{U}_{k-cv}^{D_x}$ of all k-concave functions for $k = 1, 2$ and $\ell+1, \ldots, n+1$, for which saving increases (decreases) in response to a return-risk increase and involving the same saving level s_1^{*h} under the reference return $\tilde{r}_{2,h}$, the following conditions are equivalent:*

(i) $(-u')$ has a stronger k^{th}-degree Ross intertemporal attitude than $(-v')$ for all $k = \ell+1, \ldots, n$.

(ii) There exist $\lambda > 0$ and $\phi'(x_{r_2})$ with $\phi''(x_{r_2}) \geq 0$ and $\phi'(x_{r_2}) \in \mathcal{U}_{(k+1)-cv}^{D_x}$ for $k = \ell+1, \ldots, n$ and all $x_{r_2} \in D_x \subset \mathbb{R}$ such that $u'(x_{r_2}) = \lambda v'(x_{r_2}) + \phi'(x_{r_2})$.

(iii) $|\theta_u^{r_2} \geq \theta_v^{r_2}|$ for all $\tilde{r}_{2,l}, \tilde{r}_{2,h}$ with $\tilde{r}_{2,l} \preceq_{n-\ell-MPSD} \tilde{r}_{2,h}$ and $\theta_f^{r_2}$ as defined in equation (5) for $f \in \{u, v\}$, if the following condition holds:

$$\frac{u'(y_1 - s_{1,u}^{*h}) - u'(y_1 - s_{1,u}^{*l})}{v'(y_1 - s_{1,v}^{*h}) - v'(y_1 - s_{1,v}^{*l})} \geq \lambda. \tag{6}$$

The two ways to compare the strength of precautionary saving reactions to return-risk increases in Statements (i) and (iii) in Theorem 2 resemble the ones in Theorem 1 above for income-risk increases. However, while Statements (i) is identical, the comparison of the ordering of precautionary premia under return risk in Statements (iii) depends on whether the condition for increased saving in response to a risk deterioration in Lemma 1 is fulfilled or not and whether the risk reaction of agent u is sufficiently stronger than that of agent v so that condition (6) holds.

Setting $\ell = n-1$ yields the application of the above theorem for Ekern risk increases.

Corollary 2 *Suppose $n \geq 2$ and that u, v are two utility functions in the class $\mathcal{U}_{k-cv}^{D_x}$ of all k-concave functions for $k = 1, 2$ and $n+1$, for which saving increases (decreases) in response to a return-risk increase and involving the same saving level s_1^{*h} under the reference return $\tilde{r}_{2,h}$. Then, given that condition (6) holds, $|\theta_u^{r_2} \geq \theta_v^{r_2}|$ for all n^{th}-degree risk increases in the return on saving if and only if $(-u')$ has a stronger n^{th}-degree Ross intertemporal attitude than $(-v')$ or, equivalently, there exists a constant $\lambda > 0$ such that*

$$\frac{u^{(n+1)}(x_{r_2,a})}{v^{(n+1)}(x_{r_2,a})} \geq \lambda \geq \frac{u''(x_{r_2,b})}{v''(x_{r_2,b})} \quad \textit{for all } x_{r_2,a}, x_{r_2,b} \in D_x \subset \mathbb{R}_0^+.$$

5 Representation by Preference Coefficients

This section provides two complementary results on the measurement of preference intensities underlying intertemporal risk responses. The first result notes the impossibility to derive single-expression coefficients "in the small" – hence, for risks with vanishing variance – in relation to higher-order risks, risk changes, and return risk in intertemporal choice and, as far as applicable, also in static choice. The second result shows how the above theorems on comparative stronger Ross intertemporal attitude can be complemented by a representation based on single-expression preference coefficients.

The Arrow-Pratt coefficients of risk aversion are most widely used in the literature to measure the subjective desire of EU agents to avoid risky situations.[5] Kimball (1990) and Kimball and Weil use a procedure similar to the Arrow-Pratt approximation to derive their coefficients of prudence, respectively, under EU and RU. The following proposition states that corresponding coefficients "in the small" can generally not be obtained in relation to higher-order risks, risk changes, or return risk. The proposition is, in part, based on results for

[5]Moreover, because of its formal identity to the inverse of the elasticity of intertemporal substitution, their coefficient of relative risk aversion is, in abuse of its original meaning, also often used to represent consumption-smoothing preference in the univariate additive EU model of intertemporal choice.

static choice, for example, in Ross (1981), Denuit and Eeckhoudt (2010), and Liu and Meyer (2013b), but emphasizes especially that these results also hold for intertemporal choice.

Proposition 1 *Neither under EU, nor under RU single-expression preference coefficients "in the small" can be derived in static or intertemporal settings for higher-order risks, risk changes, or return risk using a procedure analogous to the Arrow-Pratt approximation.*

The reason for this negative result lies in the nature of the Arrow-Pratt approximation and the considered problems. Regarding higher-order risks under EU, Proposition 1 is notwithstanding the many economically meaningful applications higher-order Arrow-Pratt coefficients have found in the literature (*e.g.*, Caballé and Pomansky 1996, Gollier and Pratt 1996, Lajeri-Chaherli 2004, Eeckhoudt and Schlesinger 2008). None of the latter applications, however, falls in one of the categories the proposition covers. Regarding RU, Proposition 1 implies notably that the Kimball and Weil prudence coefficients do not have an extension to higher orders and apply neither for risk changes nor under return risk.

The following theorem shows how comparative stronger Ross intertemporal attitude can be represented using single-expression preference coefficients "in the large." Definition 2 of stronger $(n/m)^{th}$-degree Ross intertemporal attitude refers to curvature properties of two utility functions, but it does not entail a concept to immediately measure and compare associated preference intensities between them. To fill this gap, the following definition adapts to RU the corresponding Definition 3 in Heinzel (2016) under EU, which relies on two concepts from Liu and Meyer (2013a,b).

Definition 3 (Generalized Liu and Meyer Measure) *For an n-times differentiable utility function $f(x) \in \mathcal{U}_{m-cv}^{[a,b]}$, with $n > m \geq 1$ and $[a,b] \subset D_x \subset \mathbb{R}_0^+$, the generalized Liu and Meyer measure of $(n/m)^{th}$-degree Ross intertemporal attitude is defined as*

$$C_{(n/m)_f}(x;a) = (-1)^{n-m} \frac{f^{(n)}(x)}{f^{(m)}(a)} \tag{7}$$

Liu and Meyer (2013a) introduce this measure for the case $(n/m) = (2/1)$ and show that it provides another way to equivalently represent Ross (1981) more risk aversion. Heinzel (2016) proves that, under slightly stronger conditions, the case $(n/m) = ((k+1)/2)$ allows to measure and compare the strengths of the precautionary saving reactions of two EU agents u, v to a return-risk increase in the n-ℓ-MPSD order, when u is $((k+1)/2)^{th}$-degree Ross more risk averse than v, for $k = \ell + 1, \ldots, n$.

Theorem 3 provides a similar representation of the comparison of the relative precautionary responses of two agents to a given risk increase in Theorems 1 and 2 above, referring to generalized Liu and Meyer measures as in Definition 3.

Theorem 3 *Given that u is more prudent than v,* i.e., $-\frac{u'''(x)}{u''(x)} \geq -\frac{v'''(x)}{v''(x)}$ *for all $x \in [a,b] \subset D_x$, then the following condition is equivalent, respectively, for $x = x_{y_2}$, to Statements (i)–(iii) in Theorem 1 and, for $x = x_{r_2}$, to Statements (i)–(iii) in Theorem 2:*

$$C_{((k+1)/2)_u}(x;a) \geq C_{((k+1)/2)_v}(x;a) \quad \textit{for all} \quad k = \ell + 1, \ldots, n \tag{8}$$

with $C_{((k+1)/2)_f}(x;a)$, for $f \in \{u, v\}$, as defined in equation (7).

For $\ell = n - 1$, Theorem 3 applies to the case of Ekern risk increases, as in Corollaries 1 and 2. Theorem 2 in Heinzel (2016) (which applies similarly for income-risk increases) and Theorem 3 here show together that coefficients "in the large" in the form of generalized Liu and Meyer measures can cover all cases addressed in Proposition 1, under EU and RU. Theorem 3, in particular, complements the Kimball and Weil prudence coefficients by measures which capture the precautionary intensity of an agent's reaction to risk increases of higher order, in the two cases of income and return risk.

6 Conclusion

Measuring the strength of the precautionary saving motive under RU has been conceptually restricted to reactions to the addition of a zero-mean risk to safe future income. This paper provides characterizations of comparative precautionary saving under RU analogous to Ross' (1981) approach to comparative risk aversion for increases in risk on future income or the saving return, also of higher order. The characterizations involve a comparison based on precautionary premia generalizing the respective concept in Kimball and Weil (2009).

I define, moreover, preference-intensity measures and show how they can equivalently represent the comparative precautionary-saving statements in the main theorems. Similar to the more special concepts in Liu and Meyer (2013a) and Heinzel (2016), these normalized Ross-type coefficients are measures 'in the large' and quantify preference intensity using a single expression. The paper notes the similarity of these concepts for the two cases of income- and return-risk increases. I show further that the conventional measures 'in the small' – such as Arrow-Pratt's for risk aversion or Kimball's and Kimball and Weil's for precautionary saving – do not have single-expression analogs for reactions to higher-order risks, risk increases, or return risk, under either EU or RU.

The paper complements the contributions by Liu (2014) and Heinzel (2016), who develop corresponding characterizations of comparative precautionary saving for income- or return-risk increases under EU. Also, the paper starts from a formulation of RU for the two-period case, which is more general than the standard implementation of Epstein and Zin (1991) preferences. In further work, it will be interesting to introduce the different measures of precautionary-saving intensity to empirical applications and to compare the results under different RU and EU specifications. Naturally, applications should account for the strengths and weaknesses of such Ross-type measures as discussed, for example, in Pratt (1990) and Liu and Meyer (2013a). Furthermore, extensions of the analysis to other choice contexts, such as production decisions under output or prices risks or consumption/leisure choices under wage-rate risk, promiss to be valuable.

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Appendix

A Proof of Theorem 1

(i) ⇒ (ii). By Definition 2, there exist λ_k for all $k = \ell + 1, \ldots, n$ such that

$$\frac{(-u')^{(k)}(x_{y_2,a})}{(-v')^{(k)}(x_{y_2,a})} \geq \lambda_k \geq \frac{(-u')'(x_{y_2,b})}{(-v')'(x_{y_2,b})} \quad \text{for all } x_{y_2,a}, x_{y_2,b} \in D_x$$

Define $\lambda \equiv \min\{\lambda_k\}_{\ell+1}^{n} > 0$. Then, for $k = \ell + 1, \ldots, n$, $\frac{u^{(k+1)}(x_{y_2,a})}{v^{(k+1)}(x_{y_2,a})} \geq \lambda \geq \frac{u''(x_{y_2,b})}{v''(x_{y_2,b})}$ for all $x_{y_2,a}, x_{y_2,b} \in D_x$. Define $\phi'(x_{y_2})$ by $u'(x_{y_2}) = \lambda v'(x_{y_2}) + \phi'(x_{y_2})$. Thus, $\phi''(x_{y_2}) = u''(x_{y_2}) - \lambda v''(x_{y_2}) \geq 0$ and $(-1)^k \phi^{(k+1)}(x_{y_2}) = (-1)^k \left[u^{(k+1)}(x_{y_2}) - \lambda v^{(k+1)}(x_{y_2})\right] \geq 0$ for all $x_{y_2} \in D_x$ and $k = \ell + 1, \ldots, n$.

(ii) ⇒ (iii). With $u'(x_{y_2}) = \lambda v'(x_{y_2}) + \phi'(x_{y_2})$, $\phi'(x_{y_2}) \in \mathcal{U}^{D_x}_{(k+1)-cv}$ for $k = \ell + 1, \ldots, n$, and $\phi''(x_{y_2}) \geq 0$, $\theta_u^{y_2} \geq \theta_v^{y_2}$ is implied by

$$\begin{aligned}
u'(CE(\tilde{y}_{2,l} - \theta_u^{y_2} + s_1R_2))CE_{y_2}(\tilde{y}_{2,l} - \theta_u^{y_2} + s_1R_2) &= u'(CE(s_1, \tilde{y}_{2,h}))CE_{y_2}(s_1, \tilde{y}_{2,h}) \\
&= [\lambda v'(CE(s_1, \tilde{y}_{2,h})) + \phi'(CE(s_1, \tilde{y}_{2,h}))]\, CE_{y_2}(s_1, \tilde{y}_{2,h}) \\
&\geq \lambda v'(CE(s_1, \tilde{y}_{2,h}))CE_{y_2}(s_1, \tilde{y}_{2,h}) + \phi'(CE(s_1, \tilde{y}_{2,l}))CE_{y_2}(s_1, \tilde{y}_{2,l}) \\
&= \lambda v'(CE(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2))CE_{y_2}(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2) + \phi'(CE(s_1, \tilde{y}_{2,l}))CE_{y_2}(s_1, \tilde{y}_{2,l}) \\
&\geq [\lambda v'(CE(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2)) + \phi'(CE(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2))]\, CE_{y_2}(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2) \\
&= u'(CE(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2))CE_{y_2}(\tilde{y}_{2,l} - \theta_v^{y_2} + s_1R_2)\,,
\end{aligned}$$

where the first inequality follows, because Lemma 1 in Bostian and Heinzel (2016) holds in an analogous way for some function $f'(.) \in \mathcal{U}^{D_x}_{(k+1)-cv}$ for $k = \ell+1, \ldots, n$, when $\tilde{y}_{2,l}$ dominates $\tilde{y}_{2,h}$ in the n-ℓ-MPSD order, and the second inequality holds because $\phi''(x_{y_2}) \geq 0$.

(iii) ⇒ (i). From $\theta_u^{y_2} \geq \theta_v^{y_2}$ for all $\tilde{y}_{2,l}, \tilde{y}_{2,h}$ such that $\tilde{y}_{2,l} \preceq_{n-\ell-MPSD} \tilde{y}_{2,h}$ and $\theta_f^{y_2}$ defined as in (3) for $f \in \{u, v\}$, it follows that $\theta_u^{y_2} \geq \theta_v^{y_2}$ for all $\tilde{y}_{2,l}, \tilde{y}_{2,h}$ such that $\tilde{y}_{2,h}$ is a kth-degree Ekern risk increase over $\tilde{y}_{2,l}$ for all $k = \ell + 1, \ldots, n$. Based on an argument analogous to the one in Liu (2014) and on Definition 2, this implies that $(-u')$ has a stronger kth-degree Ross intertemporal attitude than $(-v')$ for all $k = \ell + 1, \ldots, n$. ■

B Proof of Theorem 2

(i) ⇒ (ii). By Definition 2, there exist λ_k for all $k = \ell + 1, \ldots, n$ such that

$$\frac{(-u')^{(k)}(x_{r_2,a})}{(-v')^{(k)}(x_{r_2,a})} \geq \lambda_k \geq \frac{(-u')'(x_{r_2,b})}{(-v')'(x_{r_2,b})} \quad \text{for all } x_{r_2,a}, x_{r_2,b} \in D_x$$

Define $\lambda \equiv \min\{\lambda_k\}_{\ell+1}^{n} > 0$. Then, for $k = \ell + 1, \ldots, n$, $\frac{u^{(k+1)}(x_{r_2,a})}{v^{(k+1)}(x_{r_2,a})} \geq \lambda \geq \frac{u''(x_{r_2,b})}{v''(x_{r_2,b})}$ for all $x_{r_2,a}, x_{r_2,b} \in D_x$. Define $\phi'(x_{r_2})$ by $u'(x_{r_2}) = \lambda v'(x_{r_2}) + \phi'(x_{r_2})$. Thus, $\phi''(x_{r_2}) = u''(x_{r_2}) - \lambda v''(x_{r_2}) \geq 0$ and $(-1)^k \phi^{(k+1)}(x_{r_2}) = (-1)^k \left[u^{(k+1)}(x_{r_2}) - \lambda v^{(k+1)}(x_{r_2})\right] \geq 0$ for all $x_{r_2} \in D_x$ and $k = \ell + 1, \ldots, n$.

(ii) ⇒ (iii). Consider first the case where saving under u and v increases in response to a given return-risk increase (*i.e.*, the saving-increase condition in Lemma 1 is fulfilled for all $k = \ell, \ell + 1, \ldots, n$), so that $\theta_u^{r_2}, \theta_v^{r_2} \geq 0$ and $s_{1,u}^{*l} \leq s_{1,v}^{*l} \leq s_{1,u}^{*h} \equiv s_{1,v}^{*h}$. With $u'(x_{r_2}) = \lambda v'(x_{r_2}) + \phi'(x_{r_2})$, $\phi'(x_{r_2}) \in \mathcal{U}_{(k+1)-cv}^{D_x}$ for $k = \ell + 1, \ldots, n$, and $\phi''(x_{r_2}) \geq 0$, $\theta_u^{r_2} \geq \theta_v^{r_2}$ is implied by

$$\begin{aligned}
& u'(CE(y_2 + s_{1,u}^{*l}\tilde{R}_{2,l} - \theta_u^{r_2}))CE'(y_2 + s_{1,u}^{*l}\tilde{R}_{2,l} - \theta_u^{r_2}) \;=\; u'(CE(s_{1,u}^{*h}, r_{2,h}))CE'(s_{1,u}^{*h}, r_{2,h}) \\
&= \left[\lambda v'(CE(s_{1,u}^{*h}, r_{2,h})) + \phi'(CE(s_{1,u}^{*h}, r_{2,h}))\right] CE'(s_{1,u}^{*h}, r_{2,h}) \\
&\geq \lambda v'(CE(s_{1,u}^{*h}, r_{2,h}))CE'(s_{1,u}^{*h}, r_{2,h}) + \phi'(CE(s_{1,u}^{*l}, r_{2,l}))CE'(s_{1,u}^{*l}, r_{2,l}) \qquad (9) \\
&= \lambda v'(CE(s_{1,v}^{*h}, r_{2,h}))CE'(s_{1,v}^{*h}, r_{2,h}) + \phi'(CE(s_{1,u}^{*l}, r_{2,l}))CE'(s_{1,u}^{*l}, r_{2,l}) \\
&= \lambda v'(CE(y_2 + s_{1,v}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2}))CE'(y_2 + s_{1,v}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2}) + \phi'(CE(s_{1,u}^{*l}, r_{2,l}))CE'(s_{1,u}^{*l}, r_{2,l}) \\
&\geq \lambda v'(CE(y_2 + s_{1,v}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2}))CE'(y_2 + s_{1,v}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2}) \\
&\quad + \phi'(CE(y_2 + s_{1,u}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2}))CE'(y_2 + s_{1,u}^{*l}\tilde{R}_{2,l} - \theta_v^{r_2})
\end{aligned}$$

$$\begin{aligned}
&\leq \ \lambda v'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v) \\
&\quad + \phi'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v) \\
&= \ u'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v),
\end{aligned}$$

if condition (6) holds. In system (9), the first inequality follows, because Lemma 1 and Proposition 3 in Bostian and Heinzel (2016) hold in an analogous way for some function $f'(.) \in \mathcal{U}^{D_x}_{(k+1)-cv}$ for $k = \ell + 1, \ldots, n$, when $\tilde{r}_l$ dominates $\tilde{r}_h$ in the n-ℓ-MPSD order, the second inequality is true due to $\phi''(x_{r_2}) \geq 0$ and $\theta^{r_2}_v \geq 0$, and the third inequality is implied by $v''(x_{r_2}) \leq 0$ and $s^{*l}_{1,v} \geq s^{*l}_{1,u}$. Condition (6) arises by comparing in system (9) the difference between the two sides of the first inequality with the supremum for $\theta^{r_2}_v \to 0$ of the difference between the two sides of the third inequality, noting that, by definition, $\phi'(x_{r_2}) = u'(x_{r_2}) - \lambda v'(x_{r_2})$ and substituting finally in each case present for future marginal utility according to the appropriate version of first-order condition (2).

In the other case, where saving under u and v decreases in response to a given return-risk increase ($i.e.$, the saving-increase condition in Lemma 1 is violated for all $k = \ell, \ell+1, \ldots, n$), so that $\theta^{r_2}_u, \theta^{r_2}_v \leq 0$ and $s^{*h}_{1,u} \equiv s^{*h}_{1,v} \leq s^{*l}_{1,v} \leq s^{*l}_{1,u}$, $\theta^{r_2}_u \leq \theta^{r_2}_v$ is implied by

$$\begin{aligned}
&u'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_u))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_u) \ = \ u'(CE(s^{*h}_{1,u}, r_{2,h}))CE'(s^{*h}_{1,u}, r_{2,h}) \\
&= \ \left[\lambda v'(CE(s^{*h}_{1,u}, r_{2,h})) + \phi'(CE(s^{*h}_{1,u}, r_{2,h}))\right] CE'(s^{*h}_{1,u}, r_{2,h}) \\
&\leq \ \lambda v'(CE(s^{*h}_{1,u}, r_{2,h}))CE'(s^{*h}_{1,u}, r_{2,h}) + \phi'(CE(s^{*l}_{1,u}, r_{2,l}))CE'(s^{*l}_{1,u}, r_{2,l}) \\
&= \ \lambda v'(CE(s^{*h}_{1,v}, r_{2,h}))CE'(s^{*h}_{1,v}, r_{2,h}) + \phi'(CE(s^{*l}_{1,u}, r_{2,l}))CE'(s^{*l}_{1,u}, r_{2,l}) \\
&= \ \lambda v'(CE(y_2 + s^{*l}_{1,v}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,v}\tilde{R}_{2,l} - \theta^{r_2}_v) + \phi'(CE(s^{*l}_{1,u}, r_{2,l}))CE'(s^{*l}_{1,u}, r_{2,l}) \\
&\leq \ \lambda v'(CE(y_2 + s^{*l}_{1,v}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,v}\tilde{R}_{2,l} - \theta^{r_2}_v) \\
&\quad + \phi'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v)
\end{aligned}$$

$$
\begin{aligned}
&\geq \lambda v'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v) \\
&\quad + \phi'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v) \\
&= u'(CE(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v))CE'(y_2 + s^{*l}_{1,u}\tilde{R}_{2,l} - \theta^{r_2}_v),
\end{aligned}
$$

if condition (6) holds. The inequalities as well as the sufficient condition (6) follow in an analogous way to before, only the direction of the inequalities is reversed because of the violation of the saving-increase condition in Lemma 1.

(iii) $\Rightarrow$ (i). Consider first the case where saving under u and v increases in response to a given return-risk increase. From $\theta^{r_2}_u \geq \theta^{r_2}_v$ for all $\tilde{r}_{2,l}, \tilde{r}_{2,h}$ such that $\tilde{r}_{2,l} \preceq_{n-\ell-MPSD} \tilde{r}_{2,h}$ and $\theta^{r_2}_f$ defined as in (5) for $f \in \{u, v\}$, it follows that $\theta^{r_2}_u \geq \theta^{r_2}_v$ for all $\tilde{r}_{2,l}, \tilde{r}_{2,h}$ such that $\tilde{r}_{2,h}$ is a kth-degree Ekern risk increase over $\tilde{r}_{2,l}$ for all $k = \ell + 1, \dots, n$. Based on an argument analogous to the one in Liu (2014) and on Definition 2, this implies that $(-u')$ has a stronger kth-degree Ross intertemporal attitude than $(-v')$ for all $k = \ell+1, \dots, n$. In the second case, where saving under u and v decreases in response to a given return-risk increase, $\theta^{r_2}_u \leq \theta^{r_2}_v$ follows analogously. ■

C Proof of Proposition 1

The Arrow-Pratt approximation, or any analogous application, starts from the (implicit) definition of the risk, or precautionary, premium (*cf.* Pratt 1964, Arrow 1965, Kimball 1990, Kimball and Weil 2009). Necessary to derive single-expression preference coefficients is, then, that the first element of the Taylor developments of the two sides are identical and, for the (more) risky side, that the second element vanishes (because of the zero mean the considered risk has by definition) and that beyond the second only one further element is relevant. Higher-order risks violate the latter condition, because as soon as a moment beyond the second of the distribution is relevant, the variance of the risk, while vanishing, cannot be identically zero. The result regarding risk increases follows from Ross (1981). For return risk, following the Briys et al. (1989) approach, the first elements of the Taylor developments of the two sides will differ (*cf.* also Capéraà and Eeckhoudt 1975). ■

D Proof of Theorem 3

Income risk: It is sufficient to prove the equivalence between the ordering of the generalized Liu and Meyer measures as in (8) and Statement (i) in Theorem 1, given that, for all $x_{y_2} \in [a, b] \subset D_x$,

$$-\frac{u'''(x_{y_2})}{u''(x_{y_2})} \geq -\frac{v'''(x_{y_2})}{v''(x_{y_2})}. \tag{10a}$$

(i) ⇒ (8). Given (i), there exists, for all $x_{y_2,a}, x_{y_2,a} \in [a, b]$ and $k = \ell + 1, \ldots, n$, a $\lambda > 0$ such that,

$$\frac{u^{k+1}(x_{y_2,a})}{v^{k+1}(x_{y_2,a})} \geq \lambda \geq \frac{u''(x_{y_2,b})}{v''(x_{y_2,b})}. \tag{10b}$$

Let $x_{y_2,b} = a$. Then, for all $x_{y_2} \in [a, b]$ and $k = \ell + 1, \ldots, n$,

$$\frac{u^{k+1}(x_{y_2})}{v^{k+1}(x_{y_2})} \geq \frac{u''(a)}{v''(a)} \quad \Leftrightarrow \quad (-1)^{k-1}\frac{u^{k+1}(x_{y_2})}{u''(a)} \geq (-1)^{k-1}\frac{v^{k+1}(x_{y_2})}{v''(a)}.$$

(8) ⇒ (i). Assume $C_{((k+1)/2)_u}(x_{y_2}; a) \geq C_{((k+1)/2)_v}(x_{y_2}; a)$ for all $x_{y_2} \in [a, b]$ and $k = \ell + 1, \ldots, n$, so that, for $x_{y_2} = x_{y_2,a}$, equivalently,

$$(-1)^{k-1}\frac{u^{k+1}(x_{y_2,a})}{u''(a)} \geq (-1)^{k-1}\frac{v^{k+1}(x_{y_2,a})}{v''(a)} \quad \Leftrightarrow \quad \frac{u^{k+1}(x_{y_2,a})}{v^{k+1}(x_{y_2,a})} \geq \frac{u''(a)}{v''(a)}.$$

By setting $\lambda \equiv \frac{u''(a)}{v''(a)}$, the first inequality in (10b) arises. The second inequality in (10b) holds because $\frac{u''(a)}{v''(a)}$ decreases in its argument under condition (10a).

Return risk: As to return-risk increases, the proof refers to Statement (i) in Theorem 2 and to x_{r_2} in lieu of x_{y_2}, and but is otherwise analogous. ■

The FOODSECURE project in a nutshell

Title	FOODSECURE – Exploring the future of global food and nutrition security
Funding scheme	7th framework program, theme Socioeconomic sciences and the humanities
Type of project	Large-scale collaborative research project
Project Coordinator	Hans van Meijl (LEI Wageningen UR)
Scientific Coordinator	Joachim von Braun (ZEF, Center for Development Research, University of Bonn)
Duration	2012 - 2017 (60 months)

Short description

In the future, excessively high food prices may frequently reoccur, with severe impact on the poor and vulnerable. Given the long lead time of the social and technological solutions for a more stable food system, a long-term policy framework on global food and nutrition security is urgently needed.

The general objective of the FOODSECURE project is to design effective and sustainable strategies for assessing and addressing the challenges of food and nutrition security.

FOODSECURE provides a set of analytical instruments to experiment, analyse, and coordinate the effects of short and long term policies related to achieving food security.

FOODSECURE impact lies in the knowledge base to support EU policy makers and other stakeholders in the design of consistent, coherent, long-term policy strategies for improving food and nutrition security.

EU Contribution	€ 8 million
Research team	19 partners from 13 countries

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