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Application of discrete dynamic model for the assessment of stability of the world economy development

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Abstract: A discrete dynamic model (DDM) is used to describe the pace of the world GDP annual change rate. The Mandelbrot set of DDM was constructed for different time intervals to assess the ability of the world economic system to maintain a steady pace of development. The article shows that the world economic system is in a fundamentally non-equilibrium state. The Mandelbrot set was proposed to use as a generalized image ("pictogram") of the world economy's ability to maintain sustained development rates.

JEL Classifications: C51, C62, E32

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1. Introduction

For many years scientists have been focused on a problem of explanation of economic processes. They suggested not only various general theories describing the economy as a whole (a review of these theories can be found in Blaug, 1985), but also particular models explaining certain economic processes. In economic theory there is a generally accepted idea of the existence of four periodic processes: Kitchin and Juglar cycles, Kuznets rhythms and Kondratieff waves. It is generally accepted that they have different economic nature (Abramovitz, 1961; Åkerman, 1932; Ayres, 2006; Bernstein, 1940; Dator, 2006; Dickson, 1983; Diebolt & Doliger, 2006, 2008; Forrester, 1977; Freeman, 1987; Glazyev, 1993; Grinin, Korotayev, & Malkov, 2010; Hirooka, 2006; Juglar, 1862; Kitchin, 1923; Kondratieff, 1922; 1925; 1926; 1928; 1935; 1984; 2002; Kuznets, 1930; Maevskiy, 1997; Mensch, 1979; Modelski & Thompson, 1996; Modelski, 2001; 2006; Papenhausen, 2008; Rumyantseva, 2003; Shiode et al., 2004; Silberling, 1943; Solomou, 1989; Tylecote, 1992; Van Duijn, 1983; Yakovets, 2001).

Section 2 shows that these processes and a number of other effects can be explained in a unified manner using a discrete dynamic model (DDM). The basic assumption of DDM is that the economy has some "inertia", that is, a function that describes the rate of global GDP change in the next year, and which depends on the rate in the previous year. Using a finite number of terms of the Taylor series of this function, we can explain different effects. Moreover, the period from 1962 to 2015 can be separated on the nine non-overlapping time blocks, where the annual rate of change of the world GDP is approximated with high accuracy by Taylor polynomials of the third degree. Radii of convergence (Julia's sets) of these blocks are unique and can be used as a generalized image of the state of the world economy for these time intervals.

Section 3 examines the sustainability of the global economic system at these time intervals. Mandelbrot sets were found for these intervals. These sets can be used as an indicator of such stability. The obtained results show a weak ability of the world economy to maintain stable economic development.

2. Discrete dynamic model

Earlier the authors of the article proposed the discrete dynamical model (DDM), which explains the known economic cycles on the base of a common methodological approach (Chaldaeva & Kilyachkov, 2012, 2014; Kilyachkov & Chaldaeva, 2013; Kilyachkov A., Chaldaeva, & Kilyachkov N, 2015, 2016; 2017a; 2017b). The essence of the DDM is the following. The world economy is characterized by many factors with different nature, which interact through a large number of complex and often not obvious links. These links are constantly changing, which complicates a sophisticated scheme of their interaction even further. Therefore, one cannot talk about any functional dependence of the economic state of the world economy on any factors. It is more correct to talk about the cumulative effect of these factors, the nature of which changes over time. In other words, one cannot write a function that would describe the interdependence of some economic factors. This interdependence can be described at the level of "influence" or "impact", but not at the level of "functional dependency".

In mathematics there is a method, which allows such an impact to be described. According to this method, the process of development of the economy, which is dependent on *N* factors, represents a set of functions from *N*-dimensional arguments to *N*-dimensional functions, and is written as $f: R^n \rightarrow R^n$. In the general case it is impossible to specify a function f that would describe economic processes. However, assuming that the economy has some "inertia", one can write the dependence of an economic indicator *(*X) on all other economic parameters, as a function of itself, but in the previous time period ($f: X_n \rightarrow X_{n+1}$). One can take the world GDP annual change rate as such an economic indicator. This assumption allows describing the world GDP annual change rate in a subsequent year as a function of this rate in a previous year.

This dependency is recorded in the form of an iterative function $X_{n+1} = F(X_n, t_n)$, a specific type of which is unknown. This function can be expanded in a Taylor series (Korn G. & Korn T., 1968) up to the m-th order inclusive. (In this article we will not consider the question of its convergence.)

$$X_{n+1} = F(X_n, t_n) = \sum_{k=0}^{m} a_k(X_n, t_n) \cdot X_n^k , \tag{1}$$

where, X_{n+1} - the world GDP annual change rate in $(n+1)$ year; X_n - the world GDP annual change rate in a previous n-*th* year; t_n - time corresponding to the n-*th* year; $a_k(X_{n,t})$ - the coefficients of the expansion of the function in Taylor series, which are k-

th derivative of $F(X_n, t_n)$ with respect to X_n evaluated at the point X_n=*0* with accuracy up to a multiplier.

The coefficients $\{a_k(X_n, t_n)\}$ reflect the influence of all factors that determine the world GDP annual change rate. These factors are of a different nature (resource, technological, financial, etc.) and in the general case they are functions of time. However, for small time intervals, not accompanied by economic shocks, these coefficients can be considered as constant $\{a_k(X_n, t_n) = a_k\}$.

Let us consider what economic processes can be described by approximation of a function $F(X_n, t_{n+1})$ by using a finite number of terms of its Taylor series. Cases of zero and first degree polynomials are trivial. Expansion up to the second-degree polynomial allowed describing the emergence of all known economic cycles in a unified manner. Chaldaeva & Kilyachkov (2012) showed that this approximation explains the emergence of Kitchin and Juglar economic cycles (Kitchin, 1923; Juglar, 1862), Kuznets rhythms (Kuznets, 1930) and Kondratieff waves (Kondratieff, 1922; 1925; 1926; 1935) as a bifurcation of some basic cycle ($T \approx 3$ years). In addition, the proposed model described the cycle of $T \approx 25$ years term which was detected by spectral analysis of the world GDP annual change rate covering the period between 1871 and 2007 (Korotayev & Tsirel, 2010a; 2010b).

Representation of a function $F(X_n, t_n)$ by the Taylor polynomial of degree 3 (Kilyachkov & Chaldaeva, 2013; Chaldayeva & Kilyachkov, 2014), which has the form

$$X_{n+1} = F(X_n, t_n) = a_0 + a_1 \cdot X_n + a_2 \cdot X_n^2 + a_3 \cdot X_n^3 , \qquad (2)$$

explained a complex power spectra of the world GDP annual change rate, revealed in the works of Korotayev & Tsirel (2010a; 2010b). For this Taylor polynomial, various attractors were constructed for the world GDP annual change rate (Kilyachkov A., Chaldaeva & Kilyachkov N., 2015; 2016). That gave an opportunity to move from a qualitative explanation of the observed effects to their quantitative analysis. Julia sets (radii of convergence), which were found for different attractors, had a self-similar pattern, thus forming quasi-fractals. For fixed points and attractive fixed sets their Julia sets were conjunct, while for strange attractors they were disjunct (Kilyachkov A., Chaldaeva, & Kilyachkov N., 2015; 2016).

For the period from 1962 to 2015, nine non-overlapping time intervals (blocks) were identified: (1961-1967), (1968-1974), (1975-1981), (1982-1986), (1987-1991), (1992-1997), (1998-2003), (2004-2009) and (2010-2015). These intervals were made not less than 5 years long, which is long enough to calculate reliably the coefficients of polynomial (2) and yet short enough to believe that these coefficients remain constant. In these blocks, Taylor polynomials of degree 3 approximated the world GDP annual change rate with high accuracy (Kilyachkov A., Chaldaeva, & Kilyachkov N., 2017a; 2017b). The Julia sets corresponding to these blocks were unique for each time interval. This makes it possible to describe the state of the world economy for different time intervals not as a set of numbers or graphs, but as a generalized image, a "pictogram".

3. Mandelbrot set as an indicator of stability of the economic system

DDM can be used to assess the ability of the economy to maintain in a stable state during the time intervals mentioned above. To do this, we used the Mandelbrot set. The Mandelbrot set is a set of complex values of the coefficient C for which the value of the iterated function

$$Zn + 1 = Z_n^k + C \tag{3}$$

is limited (see, for example, Sekovanov, 2013). The Mandelbrot set is closely related to the Julia set. Namely, if the value of the coefficient is inside the main cardioid of the Mandelbrot set, Julia set will represent the misshapen circle. This form of Julia set is typical for attractive fixed points (Kilyachkov A., Chaldaeva, & Kilyachkov N., 2017a; 2017b). When the value of a coefficient is moved to the edge of the main cardioid, Julia set will deform, keeping connectivity. If the coefficient values go beyond the main cardioid of the Mandelbrot set, the Julia set loses connectivity (Sekovanov, 2013). Due to the fact that the form of Julia set reflects the degree of sustainability of economic development, and the form of Julia set in its turn depends on the position of the parameter C inside the Mandelbrot set, knowing the position of the value of this parameter in relation to main cardioid of the Mandelbrot set, one can assess the ability of the economy to maintain stable state. Thus, the choice of the Mandelbrot set is determined by its close link with Julia set, which in the DDM is unique for each time interval (Kilyachkov A., Chaldaeva, & Kilyachkov N., 2017a; 2017b).

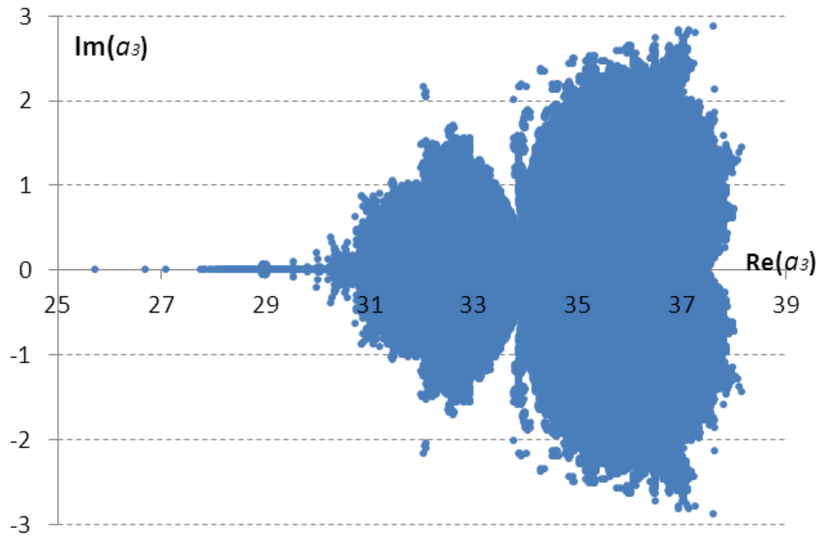
Our problem differs from its traditional formulation, as we used the polynomial (2) but not the binomial (3) as an iterated function. However, domain of convergence of equation (2) for the coefficients $\{a_k\}$ we will also call the Mandelbrot set.

We used the standard methodology of defining the Mandelbrot set for function (2). Values of three coefficients of a polynomial (2) were fixed and equal to the polynomial coefficients, which approximated the actual data of the world GDP annual change rate for corresponding time intervals mentioned above. The value of the fourth coefficient was changing within reasonable limits, allowing the creation of the Mandelbrot set. An expression (2) was iterated 25 times for each set of values of four coefficients $\{a_k\}$, i.e. X_{n+1}, calculated at the previous n-*th* step by formula (2), was used to calculate X_{n+2} for the next step. If the value of $| X_{n+1} |$ on 25 steps was ≤ 2, it was assumed that the iterated function converges. In other words, for each time period we took an approximating polynomial, fixed three coefficients out of four, and for the fourth one we built the Mandelbrot set.

4. Results and discussions

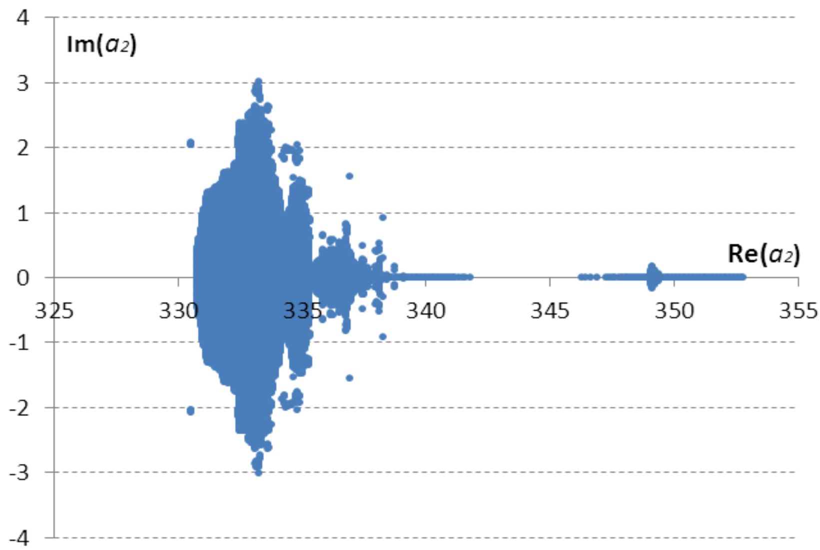
The described algorithm was used to build domains of convergence of the equation (2) regarding its coefficients $\{a_k\}$. The obtained Mandelbrot sets are presented in Figures 1-5.

FIGURE 1. MANDELBROT SET FOR COEFFICIENT *A3* IN THE INTERVAL (1998-2003)



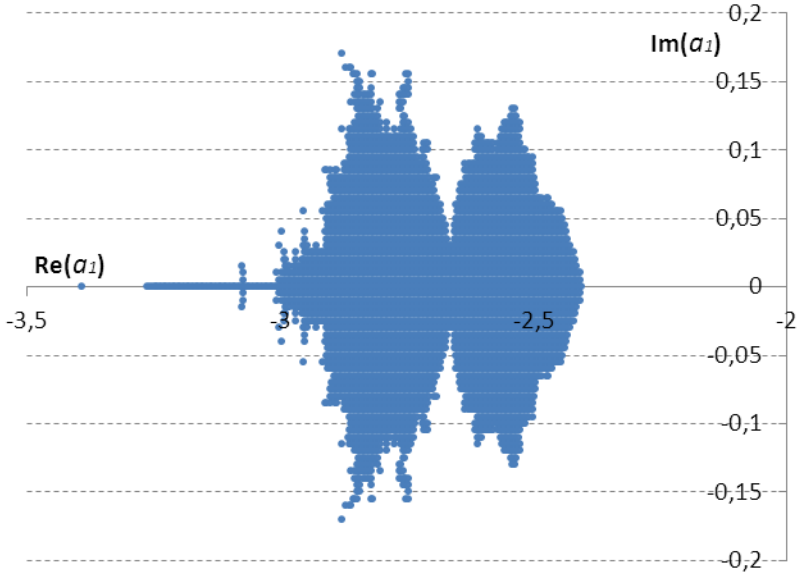
Source: Own elaboration.
Note: Approximating polynomial coefficient is *a3* = -2046.

FIGURE 2. MANDELBROT SET FOR COEFFICIENT A2 IN THE INTERVAL (1982 -1986)



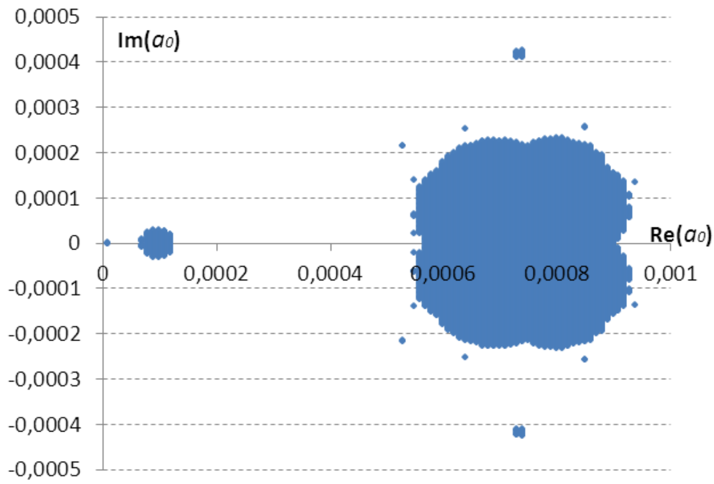
Source: Own elaboration.
Note: Approximating polynomial coefficient is a2 = -326.9.

FIGURE 3. MANDELBROT SET FOR COEFFICIENT A1 IN THE INTERVAL (1982 -1986)



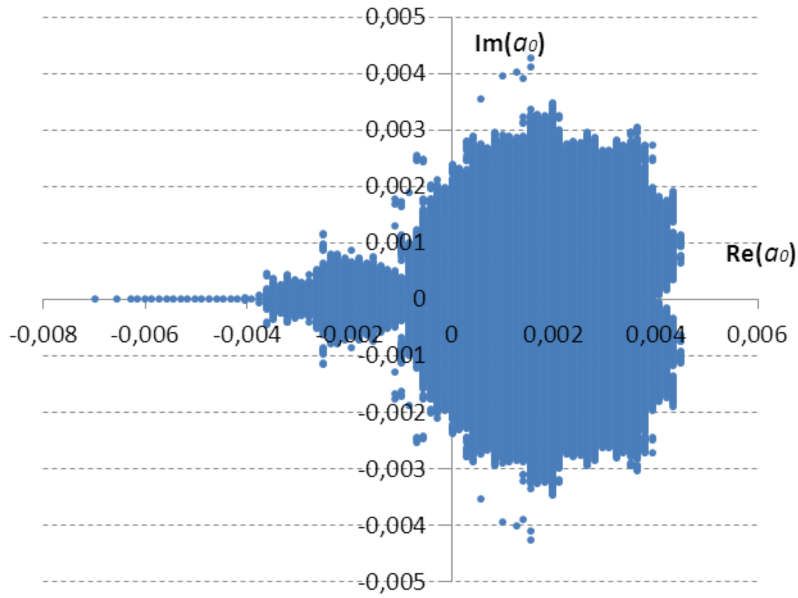
Source: Own elaboration.
Note: Approximating polynomial coefficient is *a1* = -7.86.

FIGURE 4. MANDELBROT SET FOR COEFFICIENT A0 IN THE INTERVAL (1982 -1986)



Source: Own elaboration.
Note: Approximating polynomial coefficient is *a0*= -0.0068.

FIGURE 5. MANDELBROT SET FOR COEFFICIENT A0 IN THE INTERVAL (2004 -2009)



Source: Own elaboration.
Note: Approximating polynomial coefficient is *a0*= -0.0564.

The results presented in Figures 1-5 allow the following conclusions. The Mandelbrot set cannot be built for all time intervals (blocks) mentioned above. The presence of the Mandelbrot set, containing the main cardioid is an exception rather than the rule. It was built only for (1982-1986), (1998-2003) and (2004 - 2009) time intervals. Under modification of the coefficients, the Mandelbrot set starts to "shrink" and break into more and more dwindling fragments. The values of the coefficients -- those that meet the best approximation of the world GDP annual change rate by a polynomial (2) at the corresponding time intervals -- lie outside the scope of the main cardioid of the Mandelbrot set. This can be seen from the comparison of the value of the corresponding coefficient with a value of $Re(a_i)$, changing within the scope of the main cardioid. The interval (1982-1986) years contains the Mandelbrot set for the greatest number of coefficients, namely (a_0, a_1, a_2). This gives grounds to assume that during this period of time, the world economy was characterised by the greatest ability to maintain a stable pace of economic development.

5. Conclusions

The obtained results show a weak ability of the world economy to maintain stable economic development, which indicates that it is in a fundamentally non-equilibrium state, and a slight change of economic parameters can lead to crisis escalation. That is why its parameters should be constantly adjusted in order to prevent economic crises. High rates of growth of the world's economy are not optimal in terms of its stable development. The ability of the economy to maintain stable economic development requires a moderate pace of its growth. Particularities of the interval (1982-1986), in which the Mandelbrot set exists for the greatest number of coefficients (a_0, a_1, a_2) reveals the need for a closer examination of the state of global economy in this period of time in order to identify signs of its capacity to maintain stable pace of economic development.

The Mandelbrot set is a graphical image for an infinite number of Julia sets (Sekovanov, 2013). In other words, Julia set is a "pictogram" of the state of the world's economy and the Mandelbrot set is a "pictogram" of the ability of the economy to maintain stable economic development.

Within the framework of the above-mentioned model, the following tasks arise, which, from our point of view, require solution. It is necessary to determine the domain of convergence of expression (2) with respect to the variable X_n and its change over time; and to compare these results with those obtained for the Mandelbrot set. Examining the state of the global economy on the interval 1982-1986 years to detect the signs of its ability to maintain stable economic development probably will give us key information to understand what economic indicators influence the coefficients$\{a_i\}$ and to find out how this influence manifests itself. These results will define the limits of permissible changes of the coefficients $\{a_i\}$ of the equation (2), in which DDM remains sustainable. Perhaps identifying the link between the ability of DDM to maintain a stable pace of development and the Mandelbrot sets we shall be able to formulate recommendations on stable economic development for different initial states.

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