Firm Growth and Resource Adjustment in a Control Theory Setting

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A review of literature related to the growth and resource adjustment of agricultural firms suggests two common yet perplexing features. First, the dynamic setting implied by firm growth is the most appropriate framework for generating useful explanations of managerial behavior, for prescribing desirable courses of action from the behavioral bases, and for predicting the outcomes of alternative courses of action. The dynamic setting helps to integrate the problem dimensions of time, uncertainty, and organizational detail in modeling the firm's decision making environment. Ideally a decision maker would follow an optimal growth path through time based on joint consideration of his objectives, behavioral features, levels and productivity of resources, and perceived values of state variables and identity of decision variables in all areas of the firm--production, marketing, and financial--over his life cycle.

In contrast, it is difficult to develop a pervasive theory of firm growth which is generally applicable to all types of agricultural firms. This difficulty stems from the heterogeneous agricultural environment including variations in the characteristics of resources, products and their markets, institutions, dependence on biology and climate, managerial behavior, and other such characteristics. Nevertheless, underlying these

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empirical features of agricultural firms are many common elements which jointly comprise a tractable theory of firm growth and resource adjustment over time.

Boussard [5] recently brought the notion of the turnpike theorem to bear on the derivation of optimal firm growth paths. The purpose of this paper is to explore further the conceptual features offered to firm growth analyses by the turnpike theorem and by control theory and to integrate the growth process with the more conventional terms of production economic theory. Special attention is also given to empirical features that are often encountered in the process of growth and resource allocation.[1]

Toward a Theory of Firm Growth and Resource Allocation

Capital and Control Theory

Dorfman [9] has argued that capital theory is formally identical with optimal control theory and that the main insights of control theory can be attained by strictly economic reasoning. Optimal control theory is a concept which has generally drawn on such mathematical devices as calculus of variations, dynamic programming, the maximum principle or mathematical programming to derive an optimal time path for certain variables. The path is often derived in the context of boundary conditions

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[1]An application of the theory is currently underway for some specific resource adjustment problems of farm producers in South Central Texas [4].

that stipulate the characteristics of an initial state and a desired terminal state for these variables.

The control theory idea can be applied in any context calling for an optimal path for variables over time. Its more popular and useful applications have been in the physical sciences where laws of motion hold far more exactly than in the social sciences. It has been invaluable in the space program for determining optimal missile trajectories. The planners of the Apollo Moon Program needed to derive a time path from the launch pad that would maximize terminal payload given a specific terminal position on the moon's surface and given a terminal velocity small enough that the men and equipment would survive the landing impact [16]. Control variables include the timing, magnitude and direction of various thrusts that can be exerted on the missile subject to such constraints as fuel, atmosphere and gravity. The thrusts can often be programmed in a "closed loop" system wherein deviations from the optimal path initiate actions to restore the missile to its optimal path. The laws of physics provide much of the information needed to develop summary equations of the entire process.

The analogy to firm growth is quite interesting. The launch pad for the firm is its initial capital structure, resource levels and states of technology, and organization of enterprises. Its propellants are the level and productivity of its resources and the savings to be fed back to the resource base over time. The firm is subject to various thrusts exerted upon it over time, only some of which can be controlled by the manager. Due to uncertainty the firm must provide for sequential feedback and processing of information to adapt or re-route its path in response to changing conditions. One of the firm's objectives may be to reach a desired terminal position with respect to level and structure of capital given a velocity of arrival sufficiently small that the manager will "survive the impact."

A typical example of desired terminal position occurs when an individual approaches the retirement stage of his life cycle. The relative importance of variables comprising his objective function may change considerably. The individual who goes from aggressive business behavior in one period to retirement, forced or voluntary, in the next period may have difficulty surviving the transition. The difficulty is increased if health, energy, and short-planning horizons are also effective constraints. A gradual and decelerating transition may likely be more tolerable. Changes such as these could well influence a firm's present decisions, and thus its optimal growth path, long before actual retirement occurs.

Optimal control theory [9, 11, 16] suggests a formal method of choosing a time path for values of state and decision variables connecting beginning and terminal points so as to maximize the value of the integral of a given function of the state variable, decisions taken that affect the state variable, and time. Dorfman adapts the control theory model to the decision problem of a firm that wishes to maximize its total profits over a given time interval (T) as

$$\text{Max } Z(k_o,\vec{x}) = \int_0^T u(k_t,x_t,t)e^{-\rho t}dt \qquad (1)$$

subject to the constraint

$$\dot{k} = \frac{dk}{dt} = f(k_t,x_t t) \qquad (2)$$

Time, t, is measured in continuous units and is defined over the planning horizon $(t_o \ldots T)$.

The function $u(k, x, t)$ is a given continuously differentiable function per unit of time (t), discounted continuously, reflecting the profits arising jointly from the level and composition of capital stock (state variable k) and the firm's decisions (decision variable x) in each time period. The firm that starts with initial capital stock k_o and follows the optimal decision policies $\vec{x}$ will maximize the present value (Z) of the integral or summation of the annual profits (u). The process is constrained by the rate of change of the capital stock k which in turn is a function of its current level (k_t), the time period t, and the decisions (x_t). Thus decisions taken at any time influence both the rate of profit at that time and the level of capital stock carried forward to the following time period.

Dynamic optimization (control) problems, with given, continuously differentiable functions are generally solved by application of rigorous calculus techniques: calculus of variations, dynamic programming, or the maximum principle. Dorfman and others [9, 16] illustrate the optimality conditions and application of these methods. These calculus methods can be considered the dynamic analogue of static (comparative) optimization based on discrete time periods and drawing on Lagrangian methods for optimization with equality constraints or mathematical programming for optimization with inequality constraints. Thus the numerous applications of linear programming growth models can quite logically be considered in the control theory framework [17]. However, as we shall argue, some of the important features which control theory can offer for explaining or prescribing economic growth appear to have been largely overlooked or fragmently treated.

Economic Growth of the Firm

In the firm growth setting the optimal decision policies implied by $(\vec{x})$ in equation (1) can represent a Von Neuman equilibrium path that is purported to exhibit a maximum and constant rate of growth [13]. It is characterized by constant proportions of resources and products, continued vitality and optimality--no other path is more desirable [13]. In this sense the equilibrium path represents a target toward which firms in disequilibrium should be expected to direct their resource adjustment decisions. As a result of this adjustment period, an individual firm's optimal growth path will differ from the equilibrium path. The properties of the firm's optimal growth path and their influence on its current decisions are most clearly understood in the context of the turnpike theorem [5, 13]. The turnpike theorem suggests that a sufficiently long planning horizon (T) leads a firm toward the Von Neuman equilibrium path, irrespective of the characteristics of the firm's beginning state and its desired final state. The turnpike is analogous to an automobile trip in which the driver may not follow a direct, local route from his point of departure to his final destination due to the existence of a longer appearing, but more efficiently designed turnpike. Thus, deviating from the local route to travel on the turnpike may actually result in a quicker, more comfortable, less costly journey.

However, the turnpike route might not be warranted for short journeys. In fact, the traveler must consider his preferences toward cost, time, risk, scenery, traffic volume, etc.; the distance of the turnpike from his point of departure and final destination; the length, condition, travel costs and risks of the turnpike; and similar information for alternative travel routes. Joint consideration of all these factors helps to determine the direction of the traveler's first move at departure.

These same elements can be transposed to the growth and resource adjustment of the firm. In diagramatic terms of production economic theory, the equilibrium growth path is analogous to a firm's least cost expansion path in a factor-factor setting or to the profit-maximizing expansion path in a product-product setting. The latter is illustrated in a dynamic, deterministic setting in Figure 1. The horizontal and vertical axes represent levels of y_1 and y_2 respectively--two alternatives (in investment or production) that compete for the use of those resources of the firm that are allocable between y_1 and y_2. Curves P_t represent the frontiers of production for combinations of y_1 and y_2 that can be generated from a given bundle of resource services. The curves are continuously differentiable with increasing marginal rates of substitution between y_1 and y_2. Higher levels of P_t denote the expanded production possibilities <u>in future time periods</u> from expansion of the firm's resource base over time. Thus, movements in a northeasterly direction on the diagram represent the passage of time as well as added resource capacities. Only along expansion path AB, connecting tangency points between income lines (R_j) and successive production frontiers (P_t) do the required equilibrium conditions hold:

$$\frac{C_{1t}\,(e^{-\rho t})}{C_{2t}\,(e^{-\rho t})} = \frac{dy_{2t}}{dy_{1t}} \qquad (3)$$

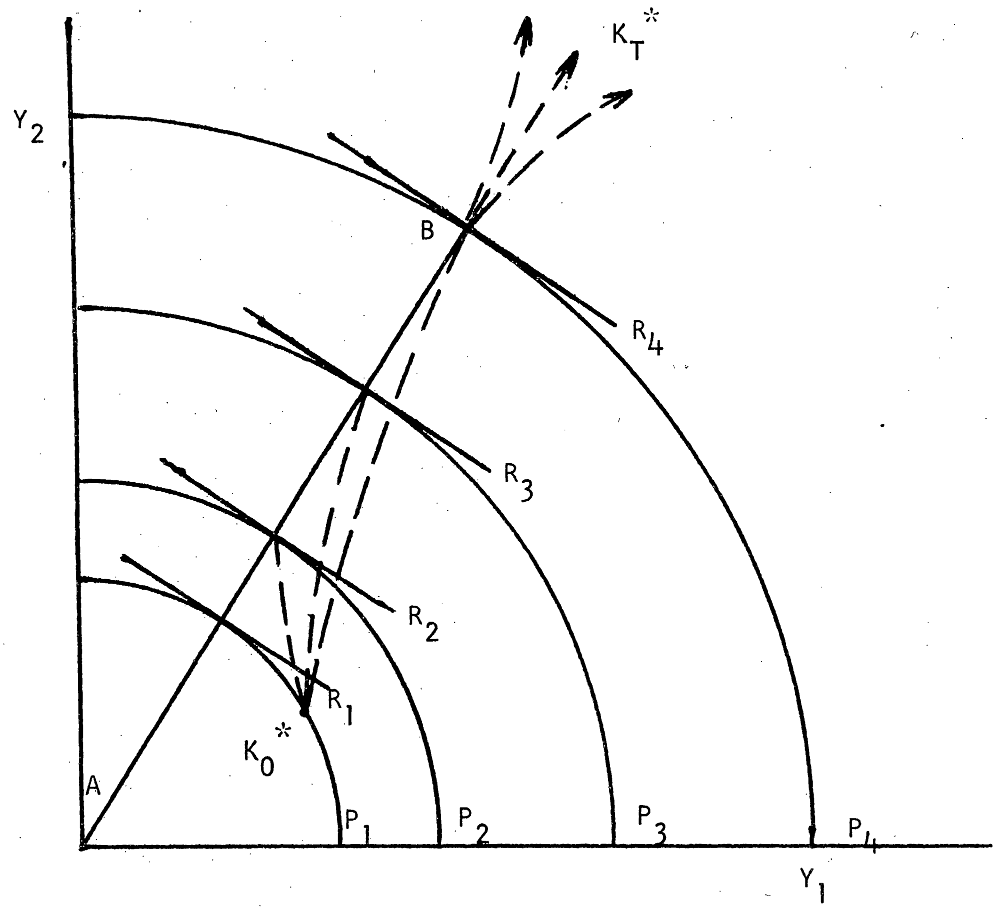

Figure 1. Derivation of equilibrium growth path and optimal growth paths in a dynamic, deterministic product-product setting.

unchanging price ratios or prices

where C_1 and C_2 can be considered as the respective prices of y_1 and y_2 in year t and ρ is the discount rate. Thus, line AB represents the Von Neuman path that is optimal with respect to the equilibrium conditions (3). Its slope and shape in this setting are determined by the decision maker's expectations on future production functions and prices.[2] Presumably firms that are currently in disequilibrium (e.g. K_o^*) will adjust their organization of resources and products so as to move toward AB.

The rate of expansion along the equilibrium path is the joint result of the firm's level of resource productivity, its rate of savings and the means of resource control, including financial leverage. Hence, the rate of firm growth can be expressed as [14]

$$g = rk \qquad (4)$$

where

g = the growth rate of equity capital

r = the net rate of return on total assets in the firm, as characterized by the equilibrium conditions in equation (3)

k = The rate of savings after consumption and payment of income taxes

If financial leverage is introduced, the model can be modified to

$$g = \left[\frac{rA - iD}{E}\right] k \qquad (5)$$

where A = the value of the firm's assets

D = the value of the firm's debts

E = the value of the firm's equity

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[2]This analysis assumes that technological change is known and that relative prices remain constant over time.

i = the interest rate paid on debt

Changes in rates of saving, return, and interest all influence the rate of growth. Moreover, if r exceeds i, and k remains constant, increasing the leverage ratio (D/E) will accelerate the rate of growth of equity thereby shortening the time span between specific levels of P_t in Figure 1.

The growth theory can be extended to a nondeterministic setting by introducing elements related to sources of risk, preferences toward risk and returns, and managerial responses to risk [3]. The result will likely be an equilibrium growth path, optimal with respect to the risk-returns elements of an individual's utility function, that will differ from equilibrium path AB derived in the deterministic setting of Figure 1.

It is common to consider the nondeterministic setting in the context of Markowitz's theory of portfolio selection. For the farm firm, the portfolio will consist of the various choices in production, marketing and investment (on or off farm) to which the firm's resources may be committed. Expectations on future events are no longer treated as single-valued events; rather decisions are based on ranges of possible outcomes expressed as probability distributions with attention given to the statistical characteristics of those ranges: mean values, variances, skewness, kurtosis. A typical approach is to derive expected returns (E) and their variance (V) with results summarized in an E-V setting so that a "portfolio decision" can be made.

While conceptually sound, this approach has encountered substantial problems in verification of decision behavior, empirical measurement, and method of analysis [7, 8]. Sources of risk must be identified and accurately measured for the firm's various resource uses. An efficient E-V frontier must be derived to obtain combinations of resource uses that yield minimum variance for alternative levels of income.[3] Then the decision maker's utility preferences toward E and V must be formulated and measured in order to obtain an equilibrium solution along the E-V frontier. Research efforts have only begun to make much progress toward

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[3]The derivation of efficiency frontiers is generally expressed in guadratic programming formulations [e.g. 15] although separable programming [20], minimization of total absolute deviations [12], simulation [10], and marginal risk constrained linear programming [6] have also been used.

In Figure 2, the product-product equilibrium is modified to incorporate the efficiency frontier derived on the basis of variance minimization for alternative levels of income [15].

A multiperiod quadratic programming format expresses the derivation of a complete E-V frontier:

$$\text{Max} = \lambda\sum_t[(c_{1t}\ y_{1t} + c_{2t}\ y_{2t}) - (y_{1t}^2\ \sigma^2 c_{1t} + y_{2t}^2\ \sigma^2 c_{2t} + y_{1t}y_{2t}\ \sigma_{tc1c2})]$$

$$\text{Subject to:}\quad \sum_{i}\sum_{t} A_{1it}\ y_1 + A_{21t}\ y_2 \lesssim B_{it} \qquad y_{1t}, y_{2t} \geq 0$$

where λ = a scalar to be parametrically increased from zero to unbounded.

σ_{tc1c2} = the covariance of returns between y_1 and y_2 in period t.

The nonlinear portion of the objective function generates an iso variance curve representing the locus of all combinations of y_1 and y_2 that produce a constant level of variance (V*), where $V^* = y^2_{1t}\ \sigma^2 c_{1t} + y_2{}^2\ \sigma^2 c_{2t} + y_{1t} y_{1t}\ \sigma_{c1c2}$. In general V* defines an ellipse with a center of zero variance at the origin of Figure 2. Changes in the level of V* result in a system of concentric ellipses or iso variance curves. The shape and direction of these curves depend on the covariance between y_1 and y_2 as reflected by the correlation coefficient ($c = \frac{\sigma_{12}}{\sigma_1 \sigma_2}$). When $c < 0$, the principal axis of the concentric ellipses extends from the southwest to the northeast portion of the diagram, thereby favoring diversification between y_1 and y_2. When $c > 0$, the principal axis extends from the southeast to the northwest portion of the diagram, thereby favoring a high degree of specialization in y_1 or y_2. When $c = \pm 1$, the ellipses collapse into their respective principal axes. When $c = 0$, the iso variance curves either become circles, when $\sigma_1{}^2 = \sigma_2{}^2$, or ellipses when $\sigma_1{}^2 \neq \sigma_2{}^2$ with the abscissa and ordinate serving as principal axes.

Minimum variance for specified income levels occurs at the point of tangency between the income line (R^1) and the iso variance curves. Line A^1B^1 represents a variance expansion path connecting tangency points on successively higher iso variance curves. Once A^1B^1 reaches the production frontier (P), income can be further increased only by moving to the profit maximizing solution P^1--identical to the linear programming solution. Point P^1 lies on iso variance curve V^1, a curve substantially above the minimum variance solution for the income level implied by R^1. Thus $A^1B^1P^1$ defines the E-V frontier for the risk programming problem. Any choice by a decision maker that lies below the profit maximizing point (P^1) will yield an equilibrium path, optimal with respect to his risk-return's utility function, that will differ from equilibrium path AB derived in the deterministic setting of Figure 1.

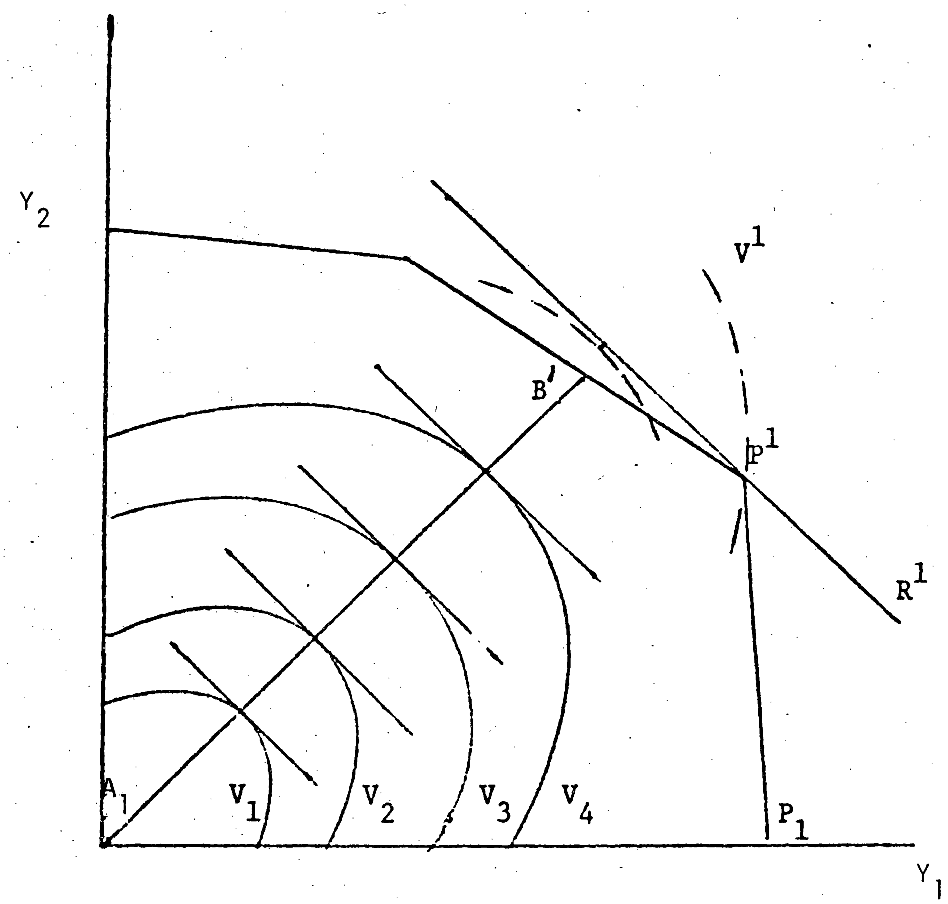

Figure 2. Derivation of E-V expansion path in a product-product setting

formulating and measuring such utility functions [eg. 18]. The task involves an exploration of a manager's strength of conviction about uncertain events, his ordering and preferences among outcomes of those events, and alternatives other than diversification for managing risks. Even then, little evidence is available to support the general stability of utility functions over time or for decision situations that vary with respect to size of potential gains or losses, frequency of occurrence, urgency of pending action, or subsequent flexibility.

Nevertheless, real world decision making does occur in a nondeterministic, dynamic world characterized by numerous problem situations. Proper understanding of managerial behavior helps contribute to decision making that leads to optimal growth--optimal with respect to the decision maker's utility function.

Problems in Resource Adjustment

A firm that is optimally organized with respect to equilibrium conditions (3) must be concerned with remaining on its equilibrium path (AB), maintaining a desirable rate of growth, and making proper resource adjustments as a result of changing objectives, technology and prices of resources and products. This may be no simple task. Such a firm can encounter resource adjustment problems that are identical to those faced by a firm that begins in a position of disequilibrium.

The firm that is currently in disequilibrium with respect to its equilibrium path must consider those factors affecting the feasibility, rate, and direction of movement toward the equilibrium path. A general listing of these resource adjustment factors is suggested below. Earlier they were outlined for the turnpike traveler, now they are cast in a farm business setting.

- The length of planning horizon (T).
- The discount rate and the objective function: reflecting the relative importance of (1) level, timing, and variation of annual consumption; (2) capital accumulation including level, growth, and variation of equity, structure of assets and liabilities, and desired terminal position; (3) risk preferences; (4) changes in these items in response to a life cycle of the business.
- Properties of the equilibrium path: its capital requirements, organization of products and resources, and level and timing of expected returns.
- Degree of disequilibrium: the structure, quality and endowment of the firm's current resources and distance of the beginning state from the equilibrium path.
- Resource fixity: the degree of divergence between acquisition costs and salvage values of resources, and their effects on resource allocation.
- Financing constraints as expressed by limits on rates of savings, cash and credit.
- Indivisibilities in the availability of land, labor, machinery, equipment and other resources.
- Economies (diseconomies) of size.
- Risk responses: attempts to reduce risks (product and resource diversification, market contracts) or provide for risk acceptance (organizational flexibility and demand for financial and production reserves).
- Lags in adjustment of management to new technology or to unfamiliar enterprises.

-Time patterns in returns due to biological, climatological or other characteristics of products.

Of the above factors, as well as others unidentified and less general, those that are relevant to specific managerial situations need to be considered in deriving the firm's optimal growth or adjustment path from its initial point K_o^* (see dotted lines in Figure 1) to the equilibrium path AB--and thence to a desired terminal position K_T^* if one can be defined. One can then study the effects of variations in each relevant factor on resource adjustment and firm growth, thereby identifying the more limiting factors in the growth process.

For example, one could ask what length of planning horizon is appropriate for evaluating specifically defined objectives of the firm. An economically relevant planning horizon is thought of as the planning time needed in order to make the best decision for the first period [5]. If T is large enough, then the discounted present values associated with K_T^* would not significantly affect the first move at K_o^*. In the same fashion, higher discount rates (p) render lower present values for given values of T. If the planning horizon is sufficiently short, the discount rate sufficiently high, and/or the beginning and preferred terminal positions sufficiently off the turnpike, then any present movement toward equilibrium path AB could be substantially abated. An optimal growth path for these conditions may differ substantially from the turnpike or Von Neuman path.

On the other hand, if values of T and p are such that K_T^* can take on any value without influencing present decisions, then one can expect all present decisions to be directed toward a rapid movement to the turnpike. The optimal growth paths lie along a continuum depending on the values of T, p, K_o^* and K_T^*. Thus, a whole series of optimal growth paths could be derived for likely combinations of values of these variables (again, see dotted lines in Figure 1).

Even then, however, the shape and direction of the optimal growth path would also be dictated by other empirical factors cited above for the particular decision maker, planning environment, and characteristics of resources and products. Resource fixity, size economies, capital constraints, and resource indivisibilities, all influence a firm's optimal growth path and its rate of growth along that path. The greater the degree of resource fixity, the slower the adjustment to AB as it takes longer for resources to depreciate to the point where replacement by new, technologically superior resources is warranted. Market and/or technological forces generating economies of size may stimulate growth thereby alleviating resource fixity. Even along the equilibrium path, optimal patterns of resource replacement must be derived for maximizing the present value of a stream of benefits to the firm [19].

Similarly, constraints on rates of saving, capital capacities, cash, and credit may slow the adjustment process. Constraints on cash and credit often interact with resource indivisibilities so that large blocks of funds are needed to finance investments involving large size capital items. The greater the degree of indivisibility, the longer the period of capital accumulation, and the slower the speed of adjustment.

Credit evaluation and financing terms reflecting lender behavior may also influence resource allocation and investment choices as well as rates of firm growth [1, 14]. The manager's demand for liquid financial reserves or other excess capital capacities as a strategy for risk acceptance is closely related to his financing decisions. Large blocks of cash and/or credit reserves, for example, may provide valuable sources of liquidity that can help in controlling and adapting the optimal growth path to changes in the firm's decision making environment [3].

Finally, it is common to observe lags in adjustment of management to new technology or to new or expanded enterprises that affect the shape and direction of the optimal growth path. These lags in management may interact with the inherent characteristics of resources and products that cause differences in timing of returns.[4]

In summary, all these factors combine to shape the firm's optimal growth path and affect its likelihood of approach to the Von Neuman equilibrium path. The derivation of optimal growth paths requires a complete modeling of the firm's growth environment including proper treatment of all those relevant factors discussed above. Only on this basis can a decision maker begin to make confident and reliable decisions and constantly re-evaluate these decisions as time passes.

While expected changes in technology, prices and other environmental conditions may give an elusive property to the equilibrium path, the notions of control help the decision maker to at least aim for it. Once he has (1) selected measurable criteria to monitor his objectives, (2) determined the acceptable norm for each criterion, (3) established an information feedback system, (4) specified tolerance limits on deviations from norms, and (5) specified corrective action for situations when tolerance limits are exceeded, then he has a control system which should always keep his firm's performance directed toward its optimal growth path.

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[4]Investments to establish orchards, pasture improvement programs, and expand beef-cow herds generally have long payoff periods. On the other hand, investments in livestock feeding facilities generally pay off more quickly.

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