Swine Prices

1973

MULTI-FREQUENCY-COBWEB MODEL: DECOMPOSITION OF THE HOG CYCLE*

Hovav Talpaz

Since the formulation of the Cobweb Theorem by Ezekiel in 1938 [2] attempts have been made to explain the hog cycle in the cobweb framework [1, 5, 11]. Harlow modified the Cobweb Model to reconcile the emerging four-year cycle with the theoretical two-year cycle [5]. An important approach was later suggested by Larson who considered the hog cycle as true harmonic motion arising from feedback, and closely related to the theory of inventory cycles [7]. Larson concluded that "...the nature of the supply response differs fundamentally from that of the Cobweb Theorem where producers' decisions are assumed to refer to a short run supply curve. This is the feature of the model that leads to a four-year cycle instead of the two-year cycle that most naturally emerges from the Cobweb Theorem" [7, p. 375]. In criticizing the Cobweb Model Larson points out that "...even when the cycle is operating in its purest, most uninterrupted manner, producers' decisions are in the appropriate direction half of the time (as opposed to the Cobweb Theorem where they are always wrong)..." [7, p. 386]. Nerlove's distributed lag approach was another significant contribution for the understanding of the dynamic behavior of the supply response [9]. It thus appears clear that each of the above three models, standing alone, fails to satisfy the requirements of a realistic, flexible, explanatory, descriptive and accurate model.

Hovav Talpaz is a former graduate student of Agricultural Economics at Michigan State University and is now Asst. Professor ~~a Research Scientist~~ at the Department of Agricultural Economics and Rural Sociology, Texas A & M University.

Contributed paper, AAEA, Edmonton, Aug 1973

The primary objective of this paper is to develop and apply a theory of the hog cycle as a Multi-Frequency-Cobweb Model arising from feedback signals, short, intermediate and long run disequilibrium positions. As will be shown, none of the above mentioned approaches will be dismissed, since all of them are important in building the model, and/or explaining its rationale.

The Model

The basic structure of the Multi-Frequency-Cobweb Model is shown in Figure 1. On a Price-Quantity plane, linear long run demand-supply curves are drawn (D_1 and S_1). Following F. V. Waugh [13] the following interpretation is offered: The D_1 curve shows how current prices are related to current market output. The S_1 curve ("lagged output") shows how current output is related to past prices [13, p. 733]. This convention can be distinguished from the demand-supply relationships of the static theory formulated by Cournot and Marshall.[1] For present purposes it becomes necessary to extend the Waugh definitions. Anticipating cyclical behavior as suggested by the Cobweb Theorem, "current" refers to one-half the time span required to complete one cycle. For example, D_1 refers to a time span of two years (one-half of a four-year cycle). S_1 represents the lagged supply response for the corresponding two-year period.

Point A is the long run equilibrium position. Points B_1, B_2, B_3 and B_4 are the traditional Cobweb price-quantity coordinates as developed by Ezekiel and his followers. The rectangle B_1 B_2 B_3 B_4 represents the assumption of a continuous Cobweb locus (neither converging nor explosive) which simplifies the exposition. The corners of the rectangle can be interpreted as the "intended" price-quantity coordinates or the long run disequilibrium points if no other forces are imposed and Ezekiel's three conditions hold [2]. One of these conditions implies a price lagged at a single and fixed time interval. If this time interval is exactly two years then cycle 1 should be completed in exactly four years.

A second lagged price is then introduced as another stimulus. The two signals are allowed to operate separately with appropriate weights corresponding to their influence on the producers' decision process. It should be clear that D_2 and S_2 are the intermediate demand-supply curves intersecting each other at the long run disequilibrium point B_1. The arbitrary rectangle C_1 C_2 C_3 C_4 forms a two-year Cobweb cycle (cycle 2).[2] Another way to put it is to say that if the two-year price lag stimulus did not exist, then what is left is the traditional Cobweb Model with a locus about the equilibrium point B_1. Postponing the discussion of the simultaneous time lags influence, let it suffice to say at the moment that in the simultaneous case neither cycles 1 nor 2 will remain the same as is presented in Figure 1.

Proceeding in the same fashion, additional price stimuli lagged six and three months will generate cycle 3 and cycle 4 about points C_1 and D respectively. Notice that the time period for completion of a cycle is exactly one half the time of the preceding cycle. Hence, by the time cycle 1 is completed two are completed for cycle 2, four for cycle 3 and eight for cycle 4.

The number of cycles and their duration for a particular system are determined by the behavior of that system. The geometric properties of each rectangle is fully determined by the system behavior. The size and shape of the rectangle has a crucial influence on the final overall locus of the price-lagged quantity intersection points. There is a direct relationship between the rectangle's size and its contribution to the final locus. As in the Cobweb Theorem case, the direction of the motion is clockwise simply because the output is lagged after the price and not vice-versa.

The Cobweb Theorem is somewhat inflexible when it suggests that the time path between any two corners (B_1 and B_2 for example) should be a straight line. While this assumption might be true in situations involving some annual cash crops, it is completely inadequate in livestock production where the adjustment process is a continuous one generating a very interesting time path of too great an importance to be ignored (See Figures 4.1 and 4.2). In fact, under a continuous adjustment process the traditional Cobweb with its four rectangular corners (B_1, B_2, B_3, B_4) are replaced by a circle or ellipse. As proposed by Larson the adjustment may be described as a process of harmonic motion.[3]

To see this point let us formally consider the Cobweb formulation. In the simplified Cobweb case there are two basic functions for each cycle considered separately and independently.

(1a) Demand or Price: $P_{it} = a_i - b_i Q_{it}$

(1b) Supply or lagged output: $Q_{i(t + k_i)} = C_i + d_i P_{it}$

$$i = 1, 2, \ldots, n \text{ cycles}$$

b_i and d_i are the slopes of the demand and supply curves respectively for the cycle i where $d_i = -1/b_i$ for a continuously oscillating model, since for the linear case a necessary condition is given by $(b_1 d_1)^2 = 1$. The time period required to complete one half of cycle i is k_i. It is clear from (1a) and (1b) that if what is happening on time intervals between t and $t + k_i$ is of interest, the Cobweb formulation is incapable of providing this information.

Larson [7, p. 378] suggests using the trigonometric function cosine to express the price and quantity:

(2a) $P_t = \cos \frac{2\pi t}{48} + e_q$

(2b) $Q_t = \cos \frac{2\pi t}{48} + e_q$

where the 48 stands for a four-year cycle with months as units and e_p and e_q are phase angles depending on initial conditions. This set of equations gives a fixed amplitude four-year circular cycle. The contribution of this model is its intuitive explanation of power and the fact that the rate of change with respect to time, or the first derivative, is another similar trigonometric function which is actually the same function phased in time and different in amplitude - a very important property possessed by systems with feedback. However, for practical purposes Larson's model falls short as a sufficient and workable model[4] because of two basic shortcomings. First, it is implied that the frequency and amplitude of price and quantity are fixed and equal to each other. Second, the model dismisses the existence of other shorter cycles (higher frequency cycles). These two points can be seen to be incorrect by a quick look at Figures 2 and 3. There is not a pure single cosine curve in either the hog-corn price ratio or the number of sows farrowed variables.

The Econometric-Mathematical Model Representation

For representation and approximation of the price and quantity over time the Fourier Series was chosen because of its desirable and convenient properties some of which will be discussed here. The Fourier Series [12, p. 655] may take several different forms from which the following for the stochastic cases are selected:[5]

$$(3)\quad \phi_m(t) = \sum_{n=0}^{m} (a_n \text{ COS}(nw_o t) + e$$

where: $\phi_m(t)$ is the time variable to be approximated

m = integer, the maximum number of terms in the series.

w_o = $2\pi/T$ is the fundamental radian frequency related to the base period T.

T = the time period needed to complete the major cycle; in our case T = 48 months, also called the base period.

t = time count in month units.

e = the error term.

Using least squares method to estimate $\phi_m(t)$ we have

$$(4)\quad a_n = \frac{2}{T}\int_{t_o}^{t_o + T} \left(\phi\ (t)\ \text{COS}(nw_o t)\right) dt$$

$$b_n = \frac{2}{T}\int_{t}^{t_o + T} \left(\phi\ (t)\ \text{SIN}(nw_o t)\right) dt$$

This property also guarantees orthogonality which provides the following convenience. If the function $\phi_m(t)$ is approximated by the trigonometric polynomial $\phi_n(t)$ substituting (4) into (3) and then another approximation $\phi_k(t)$ using more terms (k>m) is taken, more terms may be added to (4) without changing any of the coefficients $a_o, \ldots, b_n$ used in the first approximation. For orthogonality to hold in a stochastic Fourier Series of the form given by (3) it was shown by Fishman [4] that the following conditions must hold:

(4a) $E(a_n a_k) = E(b_n b_k) = E(a_n b_k) = 0$ for n=k, and

(4b) $E(a_n^2) = E(b_n^2)$ for n = 1,..., n, [4, pp. 15-17].

The orthogonal property is of extreme importance because where n = o,..., m the result is what Fisher and Ando [3] called different "completely decomposable" sets of variables or absolutely ceteris paribus conditions. This is the property which makes it possible to decompose the hog cycle, to be elaborated later. In conclusion, something is gained by taking the Fourier Series model versus Larson's harmonic motion model <u>without losing</u> any of its representative and exploratory attributes. Making use of this characteristic the value of n is capable of representing a cycle with frequency equal to T/n. The motion of each cycle is totally independent of the motion in the other cycles. The coordinates of the price-quantity values is given by solving (4) for both price and quantity and substituting into (3). This implies that the ultimate location is determined by a summation of all the cycles and their corresponding time coordinates.

Methodology, Procedures and Data

To apply the model to the hog production industry, variables were selected to represent the "price" and "quantity" discussed above.

For "quantity" a policy variable was sought with an impact which is crucially decisive in determining the quantity supplied to the market, but yet reflecting short, intermediate and long run considerations and minimally affected by past decisions. A direct approach might lead to the selection of total quantity marketed (heads or pounds of hogs) but this variable is too much a result of past breeding rates decisions. The percent of sows bred could meet the qualification satisfactorily. It is this breeding rate that is the major operative (contrary to the strategic) decision made by the producers concerned with market price conditions. Unfortunately neither breeding rate nor accurate breeding herd population data are available on a national basis, but the number of sows farrowing (SF) is available[6] and may serve as a good proxy for the breeding rate decision made almost 4 months earlier.

For the "price" the Hog-Corn Price Ratio (HCPR) was selected to reflect both the product and input prices. Hog Price (HP) is the barrows and gilts average price/cwt. received by farmers in the 7 major markets. Corn Price (CP) is Corn No. 2 price received by farmers at Omaha [16, 17] (monthly time series). Figures 2 and 3 show the nationally reported HCPR and SF respectively. By Ordinary Least Squares (OLS) the coefficients a_n and b_n were estimated for both variables and using a Step-wise Delete Routine [10], applying the statistical-equivalent model to equation (3), all cosine or sine terms were deleted unless a 2% significance level was satisfied. The results are given in Table 1, covering the period 1964-1971 on a monthly basis.

Equations (5) and (6) below are the prediction equations for HCPR and SF as functions of time only. A special purpose computer simulation program was designed to map the price-quantity or more particularly HCPR-SF coordinates on the HCPR-SF plane. As mentioned above, equations (5) and (6) represent a linear summation of "completely decomposable" independent variable sets or cycles. Using this attribute a "filter" was imposed on (5) to yield

(6) capable of filtering through each individual cycle and by the same simulation program to trace out the time path of each cycle, combination of two or more cycles working simultaneously and finally, all of them together. Methodology and procedure will be discussed in the next section as the empirical results are interpreted.

Empirical Results and Interpretations

Estimated predictable equations

Applying equation (3) as a set up system for the Step-wise Delete Routine [10] n is allowed to take the values n = 0.5, 1, 2,..., 18 and the threshold significance level for including variables was 2%. Table 1 shows the estimated coefficients for the SF, the rest of the coefficients are not significantly different from zero.[7]

Table 1 and the other statistical measurements suggest that the SF variable is highly predictable and explainable by the independent variables set. The amplitude of each cycle is given by the absolute value of its regression coefficient if only one trigonometric function is involved at this frequency. For example the amplitude of the four-year cycle is 46.44. If both sine and cosine variables are involved at a particular frequency the amplitude is equal to a phase combination of the two coefficients.

Figure 3 shows the estimated SF using Table 1 coefficients. The success of the estimation is evident when it is compared with the reported SF.

Table 2 shows the estimated regression coefficients for the HCPR, including only those significantly different from zero.

Figure 2 shows the estimated HCPR using Table 2 coefficients. Five cycles were discovered for HCPR, namely: 48, 24, 16, 12, and 6 months cycles. The D. W. value shows a strong serial-correlation which tends to inflate the t-values, but Figure 2 still indicates a close approximation of the estimated and reported time paths.

Graphical decomposition of the estimated hog cycle

In a matrix formulation (3) can be written as follows:

$$(5)\quad \phi_m(t) = [a_o, \ldots, a_m, b_1, \ldots, b_m] \begin{bmatrix} 1 \\ \cos(1w_o t) \\ \vdots \\ \cos(mw_o t) \\ \sin(1w_o t) \\ \vdots \\ \sin(mw_o t) \end{bmatrix}$$

The coefficients row vector is a 1 x (2m + 1) matrix and the trigonometric column vector is a (2m + 1) x 1 matrix. If an identity matrix dimensioned (2m + 1) is inserted between the two vectors the value of $_m(t)$ will not be altered.[8]

$$(6)\quad \phi_m(t) = [a_o, \ldots, a_m, b_1, \ldots, b_m] \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \begin{bmatrix} 1 \\ \cos(1w_o t) \\ \vdots \\ \cos(mw_o t) \\ \sin(1w_o t) \\ \vdots \\ \sin(mw_o t) \end{bmatrix}$$

The identity matrix is easily converted into a "filter" matrix by (7):

$$(7)\quad \text{The "filter" matrix} \equiv \begin{bmatrix} 1 & & & & & & & 0 \\ & \alpha_1 & & & & & & \\ & & \ddots & & & & & \\ & & & \alpha_m & & & & \\ & & & & \beta_1 & & & \\ & & & & & \ddots & & \\ 0 & & & & & & & \beta_m \end{bmatrix}$$

where $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_m$ are binary zero-one variables. Using (7), any particular cycle, or combination of cycles, can be "filtered out". For example, if the four-year cycle is desired all α's and β's except $\alpha_1 = \beta_1 = 1.0$ would be set to 0. This idea was translated into a computer simulation program with the additional capability of locating the coordinates on the price-quantity plane. The results are shown in Figures 4.1 to 4.13. In each figure the starting point is indicated by a small arrow called t_o. The plane coordinates are mapped in alphabetical order by the computer; the connecting lines are manually drawn. The time path of the first year in most cases is marked by a continuous straight line.[9] For the second year broken lines were used and back to continuous lines for the third and so on. Where the cycle is shorter than one year or the sequential points are close to each other, only a continuous line is used and the time borders can be inferred by the alphabetical order.

Figures 4.1 and 4.2 show the reported time paths of two consecutive four-year HCPR-SF intersections. It can be seen that the year 1964 is similar to 1968 and that 1965 is similar to 1969. It can also be seen that 1966 resembles 1970 and 1967 resembles 1971 in the overall trend but differ in their detailed time path. This observation alone, should suggest that some particular multi-frequency cycles are present.

By using the coefficients of the regressions equations summarized in Tables 1 and 2 a time path of the relationship between estimates SF and estimated HCPR was derived and diagrammed in Figure 4.3. By comparing this diagram with those derived from reported data it can be seen that the estimated equations yield similar turning points on the cycle as well as similar slopes on the corresponding connecting lines in the diagrams. As would be expected with different pattenrs for the reported four-year periods, the estimated equations trace a time path somewhere between the two. It is appropriate now to use this estimated equation with the filter process described to observe the time paths traced by systematically decomposing the cycle.

Figure 4.4 depicts the four-year cycle as an ellipse with a clockwise motion. This time path clearly reminds one of a Cobweb continuous motion or of Larson's modified harmonic motion. Referring to the two-year cycle shown in Figure 4.5 it is noted that it looks very much like the four-year cycle with twice the velocity. Note, that the starting points are not in the same position, simply indicating that at t_o the two cycles are not in the same cycle phase. Again, there is a clockwise motion.

Some important points should be observed about the one-year cycle depicted in Figure 4.6. First, it appears that the ellipse's major axis has been rotated approximately 90 degrees. This fact leads to the following proposition: Where the four and two-year cycles are oscillating more widely on the price, the one-year cycle is oscillating widely about the quantity with a minor variation on the prices. This conclusion indicates the impact of the national business cycle and other outside forces operating in the longer run versus the internal industry supply-demand interaction operating in the shorter run. Secondly, the motion direction is counterclockwise, perhaps violating in this particular case both theories -- Cobweb and harmonic motion -- advocating an opposite direction. It is beyond the scope of this paper to establish a theory explaining this phenomenon. Nevertheless, a few alternative or complimentary explanations may be provocative enough to encourage more investigation. a) Pork is a storable item for periods of less than a year determined by taste and the fixed variable costs of inventory maintenance. Assuming a clockwise motion on the inventory control process with relation to prices, then a counterclockwise one-year cycle will adequately refill the storage b) Production cost differentials throughout the year coupled with consumer preference differentials throughout the year may be unrelated to each other and may bring about this motion.

c) After viewing the shorter cycles below, which again possess a clockwise motion, one may wonder if the one-year cycle is a kind of "overtone" caused by the industry and crucial to keeping balance among the other cycles. d) With regard to consumer preference for pork, such phenomena as holidays, religious customs and weather conditions (with annual periodicity) may contradict the long run elasticity expectations. Figure 4.6 shows an ellipse which is very clearly the one-year cycle but with a counterclockwise motion. The sharp variation on the quantity axis represents the two-peak, spring and fall farrowing with a relatively low farrowing in the summer and winter.

Figures 4.8 - 4.13 present the combined motions of two or more cycles. In the discussion to follow, perhaps it will help to keep in mind that as the total cycle is decomposed interest lies in analyzing the relative frequencies of its components. Figure 4.8 is a combination of the four and the two-year cycles. It shows two years of rapidly changing prices and two years of slowly changing prices along with corresponding quantities. With reference to the Cobweb and Harmonic Motion Theories which propose either a two-year or a four-year cycle respectively, this figure demonstrates that both frequencies are required for an adequate explanation of the true cyclical behavior. Figure 4.9 is another excellent example of the combination of high and low frequency cycles. As the four-year cycle moves slowly to complete its time path in 48 months, shown as a smooth ellipse in Figure 4.4, it is seriously disturbed by the aggressive one-year cycle which also determines the motions' direction. Figure 4.10 shows the interaction of the one and the two-year cycles which represents roughly one half the time behavior shown in Figure 4.9. An interesting reaction between the one year and 16 months cycles is revealed in Figure 4.11. The impact of the 16-months cycle can be interpreted as a vertical rotation of the axis of the one-year ellipse within the four-year period. Essentially it allows for the different slopes and modifications of each of the one-year cycles.

The high speed, short run cycles presented in Figure 4.12 emphasize the relatively oscillatory quantity behavior versus the moderate variation on prices in a one-year period. All of the long and intermediate run cycles are depicted in Figure 4.13. Here the same pattern as shown in Figure 4.3 is generally discovered. This expresses the estimated time path in the hog production industry, where it is found that two out of every four years low hog prices prevail with the upturn and downturn taking one year each tracing out two different paths. Particularly noteworthy is the observation that the downturn path is to the right of the supply curve in the case of expansion-contraction when substantial variable and fixed costs are involved in the decision making process.[10]

Distributed Lag Estimation

Having statistically tested and accepted the existence of the combined series of cycles operating simultaneously, it is appropriate to show a linkage with the Cobweb model. According to the Cobweb model the completion of the cycle by a price lag equal to one-half the cycle period is to be expected. Furthermore, these lagged price ratios are expected to be statistically significant in explaining the Sows Farrowing variable. To test this hypothesis a modified special case of Koyck distributed lag model [6] was chosen. Let the structural equation express the SF as a linear function[11] of lagged price ratio variables as follows:

(8) $SF_t = \beta_o + \beta_1 \, HCPR_{t-1} + \ldots + \beta_i \, HCPR_{t-1} + e_{SF}; \; i = 1, 2, \ldots, 50$

where: $\beta_o, \beta_1, \ldots \beta_i$ are constants to be estimated by least squares procedure.[12] i goes from 1 to 50 to include 48 months delay corresponding to an eight-year-cycle which has been statistically rejected by the frequency analysis, but is standing again for a test. e_{SF} is the disturbance term.

Applying a least-square-stepwise-add routine [10] the results summarized in Table 3 are reached. The variable selection procedure was to add a variable if it was the best candidate and was significantly different from zero at the five percent level, and to reject any variable previously included if it was no longer significant at the ten percent level.[13] The time lag in the price ratio was set at 4 time-units (months) earlier than the sows farrowing time lag to correspond to the approximate time of breeding. Since the time of breeding may be distributed throughout the month, it is safe to conclude that the actual farrowing could occur in a ± one month deviation from the time hypothesized. With this in mind Table 3 gives a great deal of support to the Cobweb theory behind each individual cycle. The first lagged price should be considered in agreement with the hypothesized lags simply because there are no fractions of lag units, so the 1.5 lag is unobtainable. The next two time lags are exactly in agreement, the fourth and sixth are within the range allowed for the time range in actual farrowing. The 32-months cycle "estimated" here is exactly twice the 16-month cycle in the frequency analysis above, for it may be simply because it is not compatible with a 48-month cycle. With regard to the 8-month cycle, although initially selected at earlier step, lost its level of significance in the stepwise-add routine by the dominance of other cycles.

Summary and Conclusion

A theory of the hog cycle as a Multi-Frequency Cobweb model, or as a linear combination of decomposable hog cycles has been presented. This model reflects an integrated multi-frequency decision process resulting from the feed-back of the production response to the price ratio signal through fixed multiple production lags. Decisions with long, intermediate and short run implications are continuously made and their impacts are projected to future decision and production process. The equilibrium and disequilibrium positions are under continuous attempts to adjust because of the existence of many simultaneous decision-response relation-ships. During periods of expansion, the hog producer builds or remodels facilities and invests in a larger breeding herd. These investments have different time spans in their consequences. During periods of contraction the producer still bears the consequences of the long term investment made during expansion. This may explain the four-year cycle.

A traditional Cobweb adjustment process may involve the two year cycle. The 16-month cycle may be the producers' evaluation of the profit prospects based on the relation between the corn supply of the current year and that of the previous year. The short run cycles of 6, 4, and 3 months may be explained by seasonal, weather and market signals coupled with capacity utilization of building and equipment subject to biological and technical constraints. Interactions with inventory control management are affecting the short cycles much more than they do to the long ones [7].

Theoretically the model incorporates three basic models: the Cobweb Theorem, the Harmonic Motion and the Distributed Lags Model. As was seen, each was unable to explain satisfactorily the supply response of the hog industry when used individually. The Cobweb and Harmonic motion models are essentially special cases of the present model.

Econometrically, the Fourier Series appears to be an excellent mathematical representation of a dynamic disequilibrium phenomenon. Inertia of adjustments is preserved as in cases where overshooting a target is gradually corrected. The characteristic of correction coming from previous motions preserves properties of macro-production systems with distributed delay behavior. The orthogonality property of the Fourier Series, although not completely achieved contributed to the equality of the estimation and permitted frequency analysis and decomposition of the cycle.

The Multi-Frequency Cobweb Model with some relaxation of assumptions may be appropriate in many other production cycles where integrated multiple frequencies decision making processes are simultaneously undertaken.

FOOTNOTES

* The author wishes to thank Professor Warren H. Vincent for suggestions, comments and inspiration in preparing this paper. Also appreciation for comments to Professors Lester Manderscheid, Glynn McBride, William Haley, and the two anonymous reviewers is expressed.

However, the author assumes full responsibility for any omissions and remaining errors.

The study is mainly supported by the U.S.D.A. through a grant to Michigan State University entitled: System Analysis of Economic Activity in Hog-Pork Subsector.

1. However this distinction is not critical for the present argument.
2. The corner B_1 was selected for convenience. Any other point on the rectangle could serve as well.
3. Not necessarily true harmonic motion. See [7, pp. 379-381].
4. It is possible that Larson did not intend to go beyond the basic theoretical features of his model.
5. Thomas [12, p. 655-657] and Manetsch and Park [8, p. 3-12 and 3-22] will be followed, for the mathematical properties of the Fourier Series.
6. Source: U.S.D.A. Hogs and Pigs 1968 to 1970, Pig Crop Report 1964 to 1967. Monthly farrowing for 1968-70 computed using 1955-67 average percent of quarterly total on reported quarterly data reported in units of 1000 head [14, 15].
7. All a_n, b_n coefficients will be set to zero unless accepted on the basis of the t-test with 2% critical area. Space consideration does not permit the reporting of correlation matrix but it is important to notice that the requirements given by relationships (4a) and (4b) were not absolutely met.

8. Obviously, the last two matrices are first multiplied then the product vector is multiplied by the first one.

9. It is assumed here that a straight line is as good as any other. Shorter intervals are of no interest here.

10. For an excellent treatment of the supply response under the condition of resource fixity see G. L. Johnson and C. L. Quance, Overproduction Trap in U. S. Agriculture, Johns Hopkins University Press, Baltimore, 1972.

11. Even though other mathematical transformations may be more suitable, linear transformation was used here for simplicity.

12. Applying ordinary-least-squares on (8) may result in violations of the assumptions underlying multiple regressions using least-squares techniques, but the interest here lies in the relative size, the sign and standard error of the coefficients. Therefore, heteroscedosticity, if it occurs, should not rule out least-squares procedure. The correlation coefficients among the independent variables, after the deletion process is completed, did not exceed 0.40.

13. Such a situation may occur when a single independent variable previously selected becomes insignificant where a combination of later variables better "explain" the dependent variable and is linearly correlated with the single independent variable.

REFERENCES

[1] Dean, Gerald W. and Earl O. Heady, "Changes in Supply Response and Elasticity for Hogs," Journal of Farm Economics, November 1958, p. 845-860.

[2] Ezekiel, Mordecai "The Cobweb Theorem," Quarterly Journal of Economics, Volume 53, February 1938.

[3] Fisher, F. M. and A. Ando, "Two Theorems on Ceteris Paribus in the Analysis of Dynamic Systems," American Political Science Review, Volume 56.

[4] Fishman, G.S., Spectral Methods in Econometrics, Harvard Press, 1969.

[5] Harlow, Arthur A., "The Hog Cycle and the Cobweb Theorem," Journal of Farm Economics, November 1960, p. 842-853.

[6] Koyck, L. M., Distributed Lags and Investment Analysis. Amsterdam: North-Holland, 1954.

[7] Larson, Arnold B., "The Hog Cycle as Harmonic Motion," Journal of Farm Economics, May 1964, p. 375-386.

[8] Manetsch, T. J. and G. L. Park, System Analysis and Simulation with Application to Economic and Social Systems, Preliminary edition, Michigan State University, September, 1972.

[9] Nerlove, Marc, The Dynamics of Supply: Estimation of Farmers Response to Price, Johns Hopkins Press, Baltimore, 1958.

[10] Ruble, W. L., Improving the Computation of Simultaneous Stochastic Linear Equations Estimates, Agricultural Economics Report No. 116 and Econometrics Special Report No. 1, Department of Agricultural Economics, Michigan State University, October, 1968.

[11] Shepherd, G. S., and G. A. Futrell, Marketing Farm Products, 5th Edition, Iowa State Press, 1969.

[12] Thomas, George B., Jr., Calculus and Analytic Geometry, 4th Edition, Eddison-Wesley Publishing Co., Reading, Massachusetts, 1968.

[13] Waugh, Fredrick V., "Cobweb Models," Journal of Farm Economics, November, 1964, p. 732-750.

[14] U.S.D.A., SRS, Hogs and Pigs, Crop Reporting Board, Washington, D.C., 1968-1970.

[15] U.S.D.A., SRS, Pig Crop Report, Crop Reporting Board, Washington, D.C., 1964-1967.

[16] U.S.D.A., ERS, SRS, Livestock and Meat Statistics, Supplement for 1970, Stat. Bulletin 333. Washington, D.C.

[17] U.S.D.A., Grain Market News, Consumer and Marketing Service, Independence, Missouri, 1964-1970.

Table 1. Regression Coefficients for the Sows Farrowing (SF) Estimation in Units of 1000 Head

Independent Variable	Cycle Period (Months)	Frequency in 4 Years	Regression Coefficient	Std. Error of Coefficient	t-value	Signif. Level
Constant	--	--	1029.88	10.31	99.86	<0.0005
$\cos(4w_o t)$	12	4	-207.91	14.36	-14.48	<0.0005
$\cos(8w_o t)$	6	8	-372.03	14.33	-25.96	<0.0005
$\cos(12w_o t)$	4	12	48.70	14.33	3.39	0.001
$\cos(16w_o t)$	3	16	99.09	14.33	6.91	<0.0005
$\sin(1w_o t)$	48	1	-46.44	14.51	-3.20	0.002
$\sin(2w_o t)$	24	2	55-15	14.62	3.77	<0.0005
$\sin(4w_o t)$	6	4	130.71	14.34	9.11	<0.0005
$\sin(12w_o t)$	4	12	57.72	14.33	4.03	<0.0005
$\sin(16w_o t)$	3	16	54.24	14.33	3.79	<0.0005

R^2 = o.9363; $\bar{R}^2$ = 0.9286; F-value = 120.9; Sig. level <0.0005;
Std. error of est. = 92.85; D.W. = 1.30*.

*D.W. is the Durbin-Watson statistic.

Table 2. Regression Coefficients for the HCPR Estimation (Unit: Bushels of Corn/cwt. Hogs)

Independent Variable	Cycle Period (Months)	Frequency in 4 Years	Regression Coefficient	Std. Error of Coefficient	t-value	Signif. Level
constant	--	--	16.63	0.18	94.95	0.0005
$\cos(1w_o t)$	48	1	-2.80	0.25	-11.13	0.0005
$\cos(2w_o t)$	24	2	2.27	0.25	9.01	0.0005
$\cos(3w_o t)$	16	3	-0.74	0.25	-2.93	0.0005
$\cos(4w_o t)$	12	4	-0.63	0.25	-2.57	0.012
$\sin(3w_o t)$	16	3	-0.60	0.25	-2.41	0.019
$\sin(4w_o t)$	12	4	-0.67	0.25	-2.66	0.009
$\sin(8w_o t)$	6	8	0.82	0.25	3.36	0.001

R^2 = 0.7831; $\bar{R}^2$ = 0.7631; F-value = 39.20; Sig. level <0.0005; Std. error of est. = 1.58; D.W. = 0.222.

Table 3. Sows Farrowing Estimation Using Lagged Hog-Corn Price Ratio, By LS, Stepwise Add Method

Independent	Time Lag About Breeding = i	Regress. Coeff.	Std. Error of Coeff.	t-value	Sig. Level	Cycle Period Selected Lag * 2	Cycle Period Hypothesized = 2 * i
constant	--	101.2	219.1	4.6	0.0005	--	--
HCPR(t-5)	1	-92.3	13.3	-6.9	0.0005	2	3
HCPR(t-7)	3	135.1	15.3	8.8	0.0005	6	6
HCPR(t-10)	6	-19.8	11.6	-1.7	0.091	12	12
HCPR(t-17)	13	-82.2	11.9	-6.9	0.0005	26	24
HCPR(t-20)	16	93.2	12.0	7.8	0.0005	32	--
HCPR(t-29)	25	-32.1	8.3	3.8	0.0005	50	48

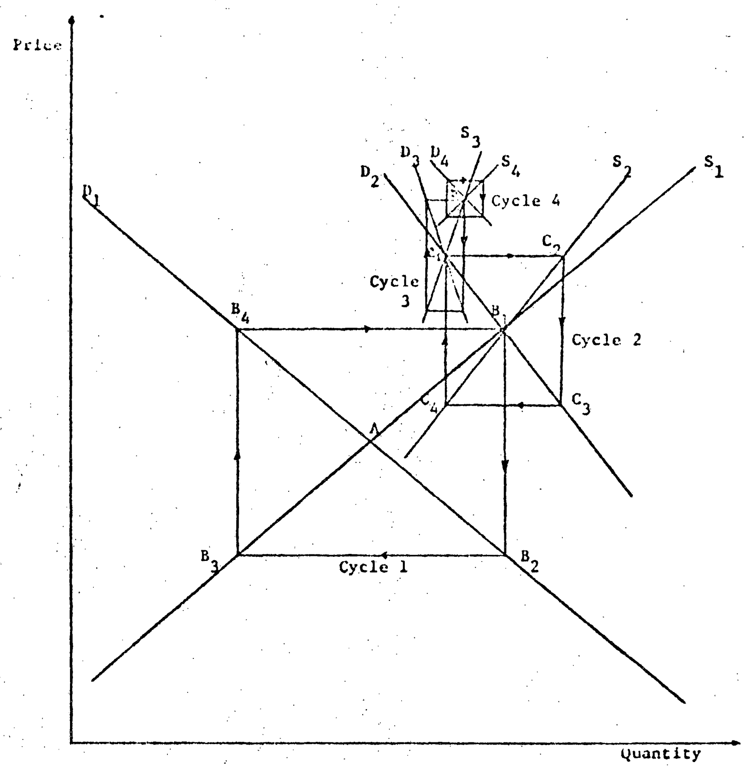


Figure 1. The Multi-Frequency-Cobweb Model with 4 frequencies or cycles.

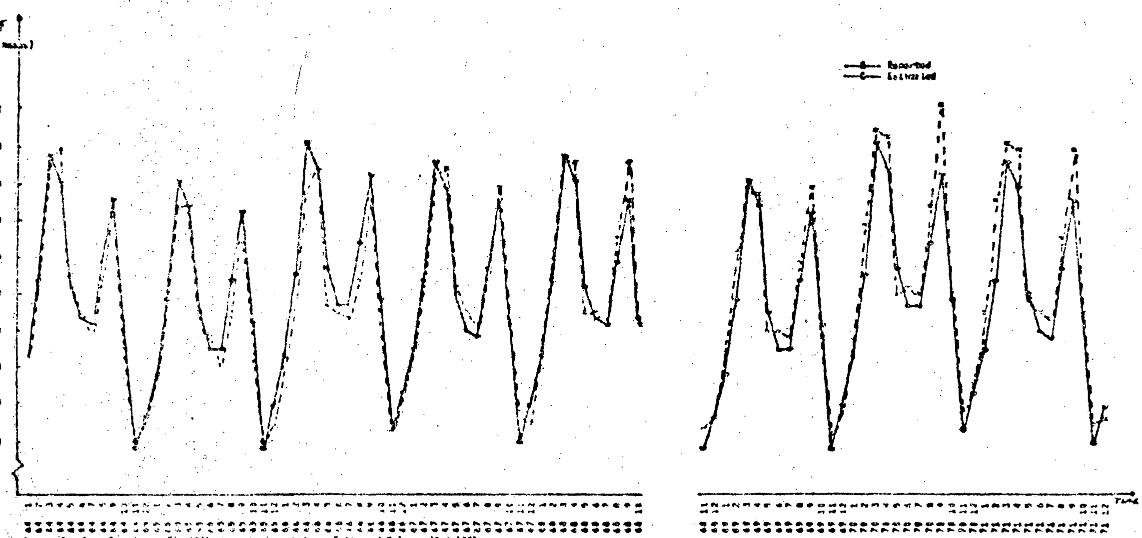


Figure 3: Sows Farrowing (SF) (000's Head), Reported vs. Estimated Values, 1964-1971.

Source: USDA, SRS, Livestock and Meat Statistics.

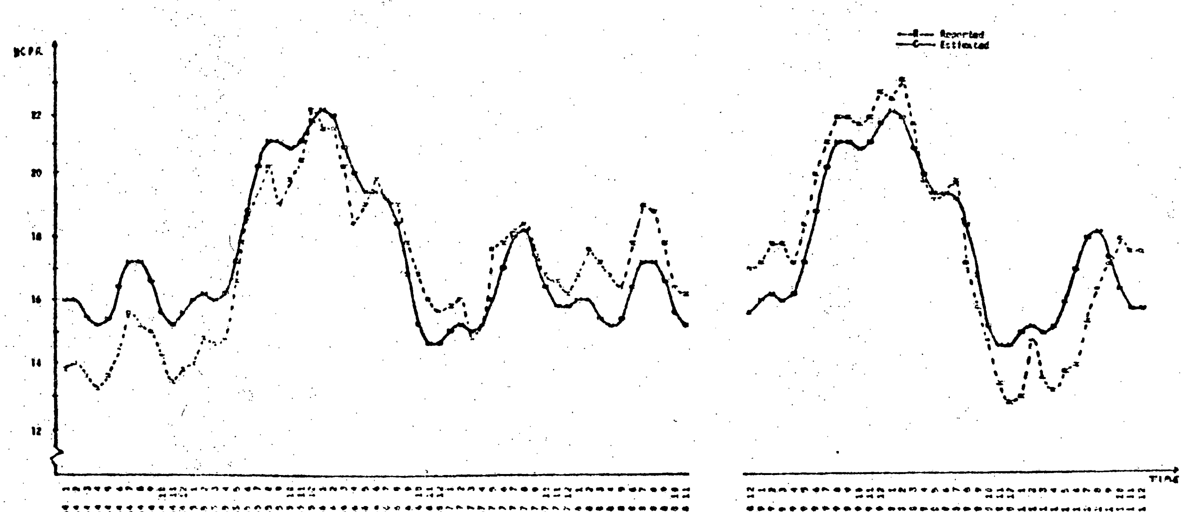


Figure 2: Hog-Corn Price Ratio (HCPR), (bu. corn/cwt. liveweight), Reported vs. Estimated Values, 1965-1971.

Source: USDA, SRS, Livestock and Meat Statistics; USDA, Grain Market News.

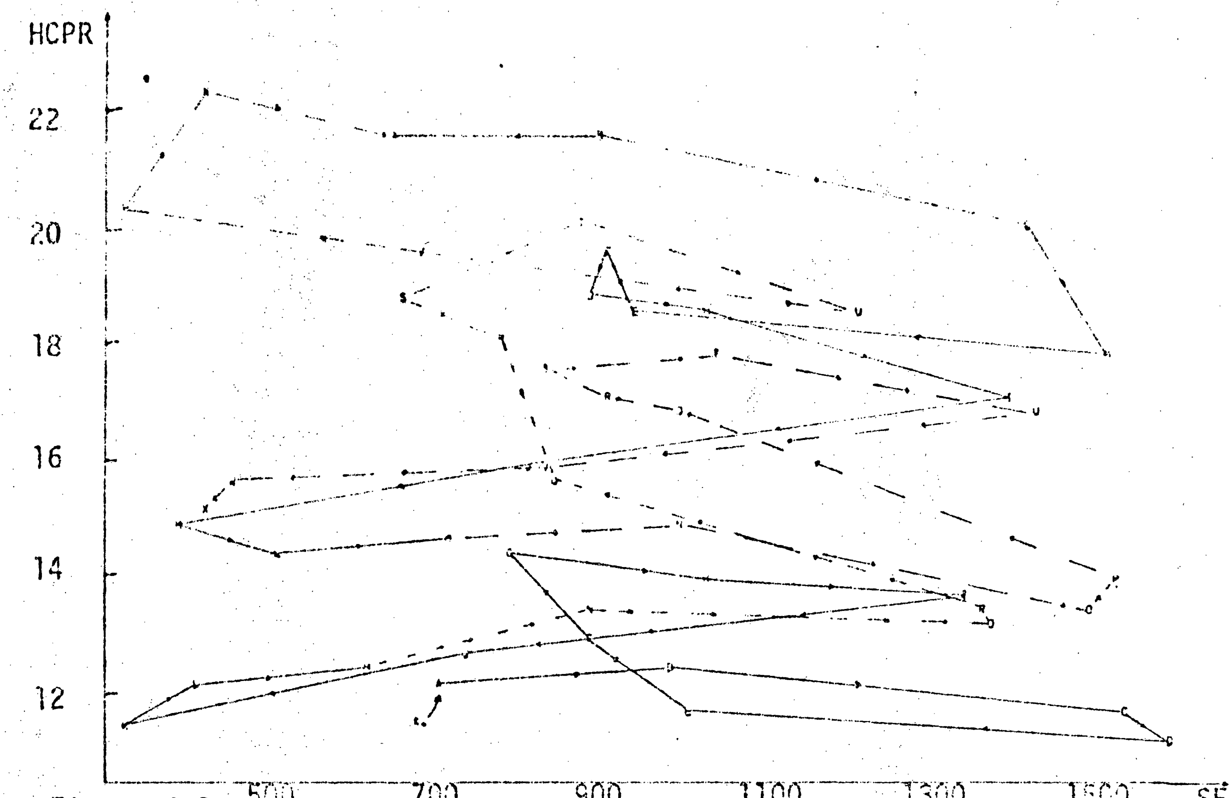


Figure 4.1 HCPR vrs. SF, Jan 1964 - Dec 1967, Reported Data

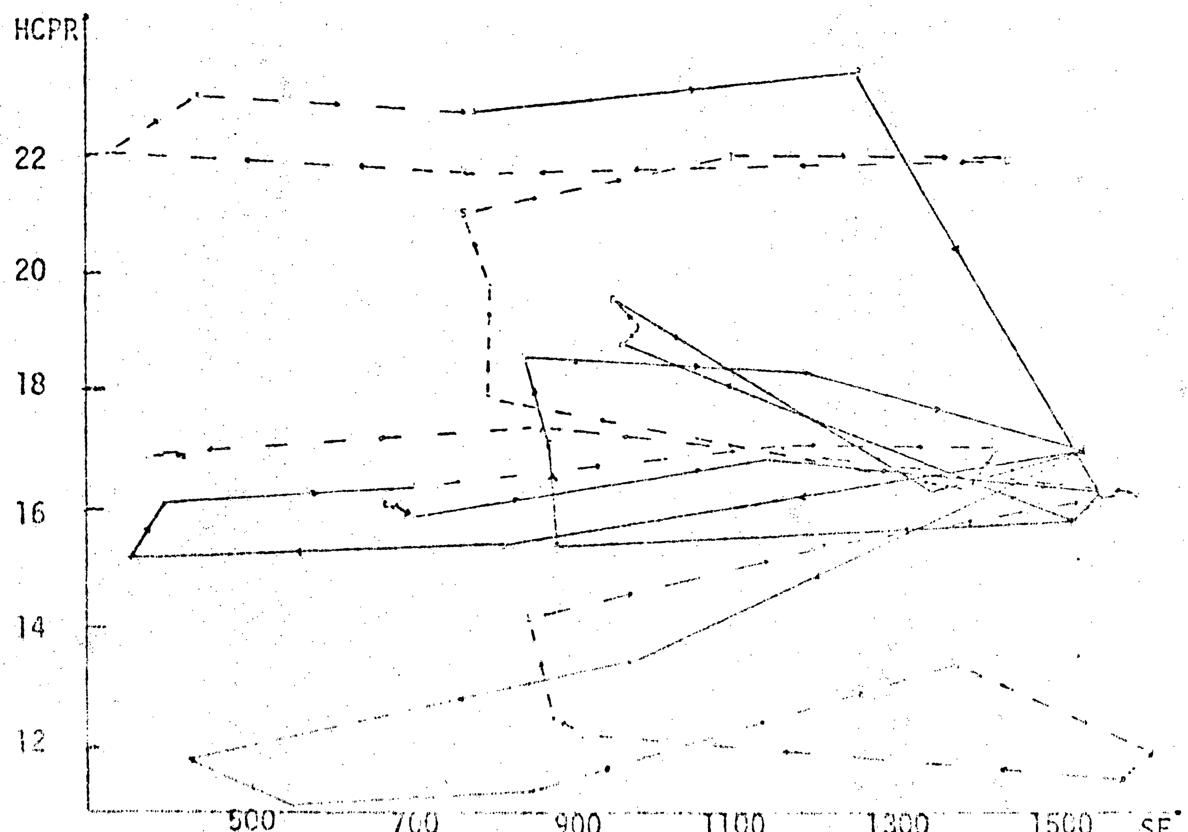


Figure 4.2 HCPR vrs. SF, Jan 1968 - Dec 1971, Reported Data

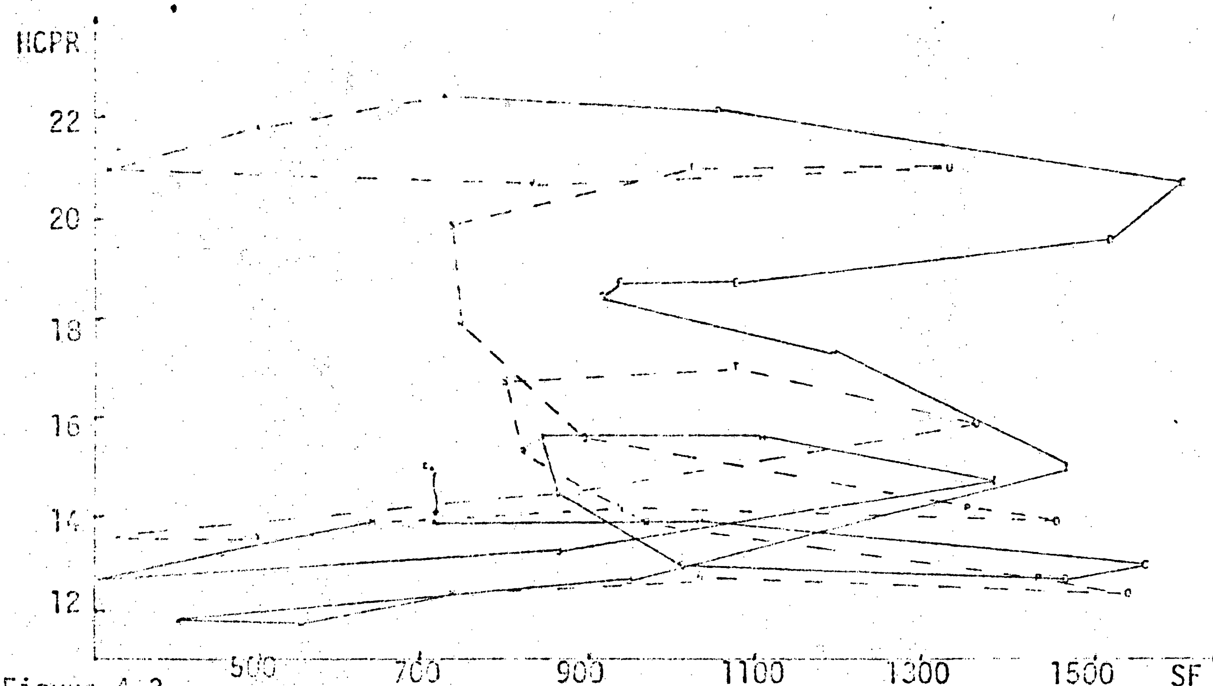
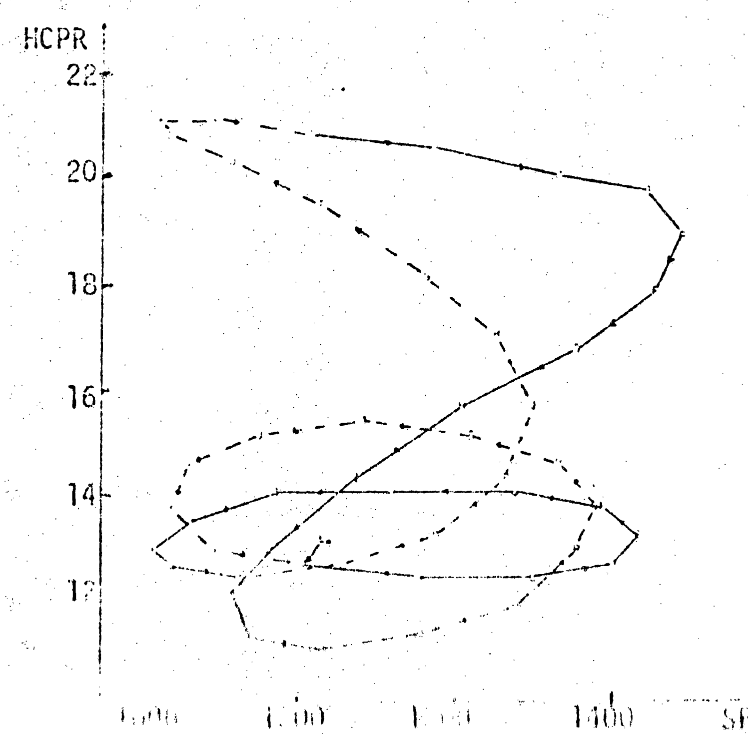

HCPR
22
20
18
16
14
12
500
700
900
1100
1300
1500
SF

Figure 4.3


HCPR
22
20
18
16
14
1400
SF

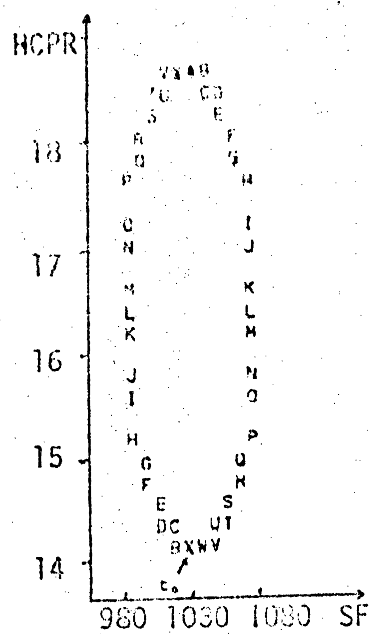


Figure 4.4 Four-Year Cycle

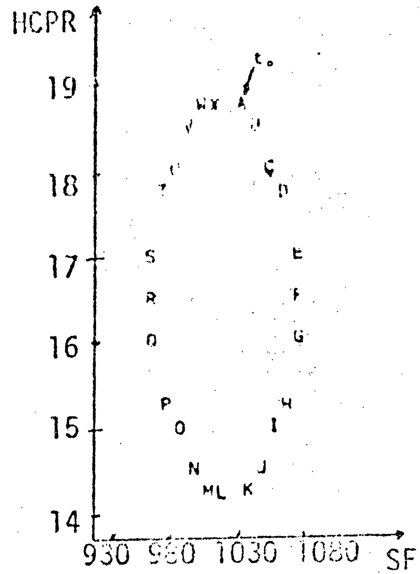


Figure 4.5 Two-Year Cycle

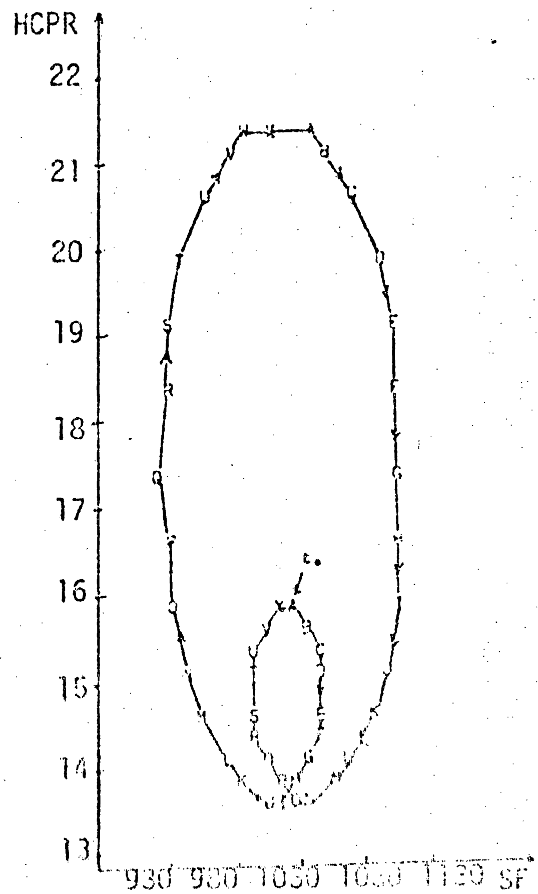


Figure 4.8 Four and Two Years Cycles

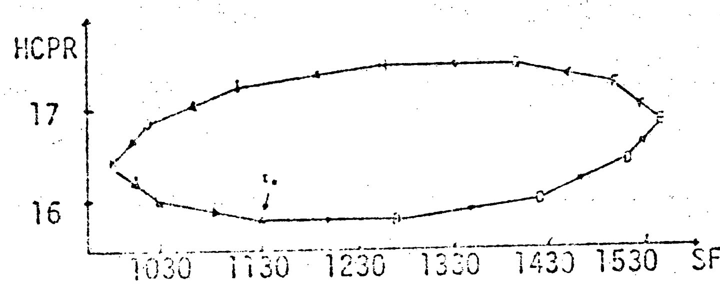


Figure 4.6 One-Year Cycle

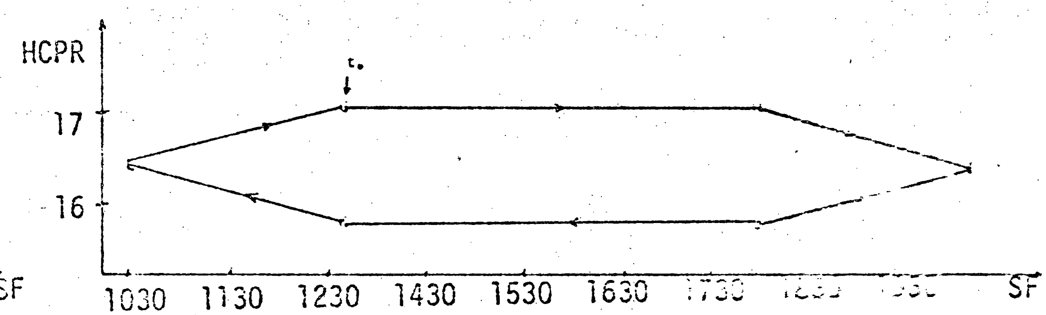


Figure 4.7 Six-Months Cycle

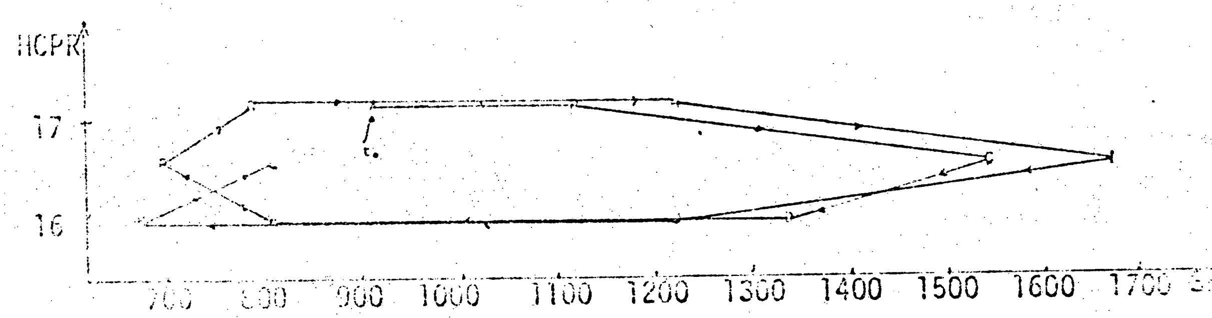


Figure 4.12 Six, Four and Three Months Cycles

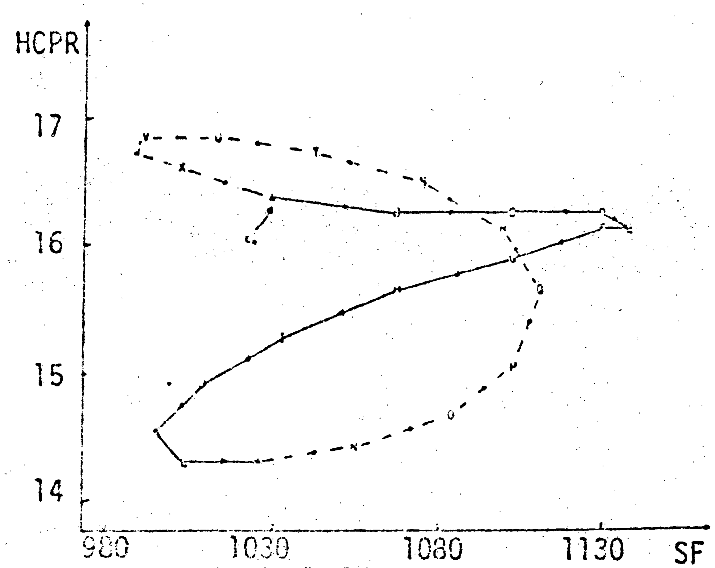


Figure 4.10 Two and One Year Cycles

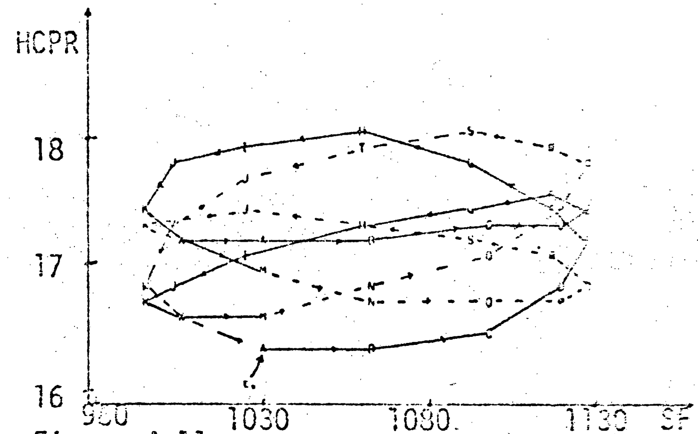


Figure 4.11 One-Year and 16-Months Cycles

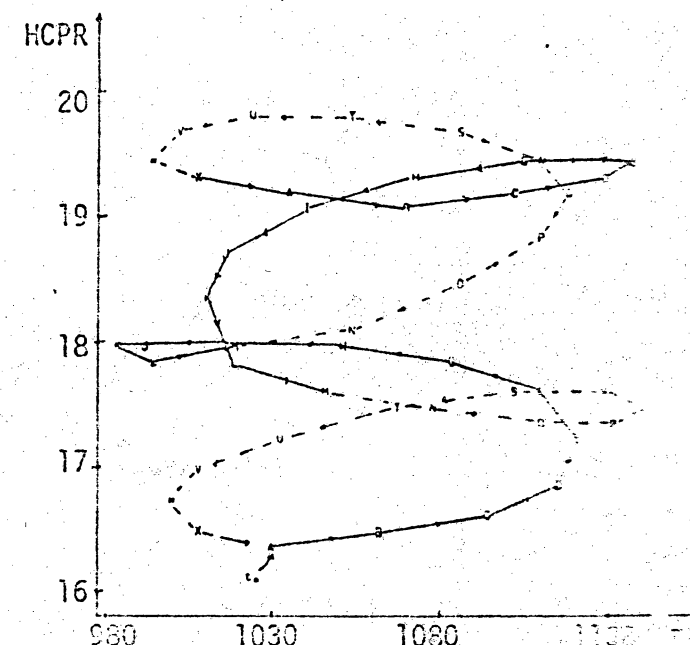


Figure 4.9 Four and One-Year Cycles