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ABSTRACT

MINIMIZING EXPECTED DISEQUILIBRIUM COSTS WITH UNCERTAIN PRODUCTION AND UTILIZATION

M. Dean Ethridge and Fred C. White

A theoretical framework is developed for optimizing planted acreage under yield and utilization uncertainties, the objective function being expected disequilibrium cost. Empirical application is demonstrated for peanut seed acreage in Georgia. Results indicate usefulness of the methodology for dealing with uncertainties in matching agricultural crop supplies with demands.

Key words: disequilibrium cost, optimum acreage, yield uncertainty, utilization uncertainty, peanut seed

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MINIMIZING EXPECTED DISEQUILIBRIUM COSTS WITH UNCERTAIN PRODUCTION AND UTILIZATION

by

M. Dean Ethridge and Fred C. White

The task of choosing appropriate acreage to plant in a given crop is of vital concern to agricultural producers and policy-makers. The difficulty of doing this is compounded by the fact that both per-acre yields and total utilization of an agricultural commodity are usually subject to significant random variation around their expected values.

Even if all production and marketing activities are performed in an economically efficient manner, added costs are incurred if actual production does not equal actual utilization.[1/] These costs may be termed "disequilibrium costs" or simply "additional costs." Thus, by definition, if actual production is equal to actual utilization, disequilibrium cost is zero. It is incurred only by failing to perfectly match production with utilization.[2/]

This paper presents a theoretical framework for choosing planted acreage in order to minimize the expected disequilibrium cost incurred by not matching total production with total utilization. Empirical application is demonstrated for peanut seed acreage in Georgia.

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THEORETICAL FRAMEWORK

For a given crop, define the following:

A = planted acreage

Y = yield per acre

$P = A \cdot Y$ = total production

U = total utilization

$X = P - U$ = excess production

$$C(X) = \begin{cases} \alpha X; & X > 0,\ \alpha > 0 \\ 0; & X = 0 \\ -\beta X; & X < 0,\ \beta > 0 \end{cases} = \text{additional cost of excess production}$$ [3/]

Assume that Y and U are independent random variables with normal distributions:[4/]

$$Y \sim N(\mu, \sigma^2), \qquad U \sim N(\nu, \delta^2), \qquad \mu, \sigma^2, \nu, \delta^2 > 0$$

Since Y and U are independent, it follows that X is also a normally distributed random variable:

$$X \sim N(A\mu - \nu, A^2\sigma^2 + \delta^2)$$

The relationships among acreage, excess production, and additional cost are illustrated by the 3-dimensional graph of Figure 1. Acreage and additional cost are non-negative while excess production may be positive (if production is greater than utilization) or negative (if production is less than utilization). For any given acreage there is an expected value of excess production and a distribution around the expected value. Figure 1 illustrates the fact that, as acreage increases, both the mean and variance of excess production increase. For $A = a_o$, the expected value of excess production is zero;

i.e., $E(X) = 0$. For $A = a_1$, $E(X) < 0$ and for $A = a_2$, $E(X) > 0$.

The expected disequilibrium cost $E(C)$ associated with any acreage is given as follows:

$$E(C) = \int_{-\infty}^{\infty} \frac{C(x)}{\sqrt{2\pi\ (A^2\sigma^2 + \delta^2)}} \exp\left[-\frac{(x - A\mu + \nu)^2}{2(A^2\sigma^2 + \delta^2)}\right] dx$$

Letting $a = A\mu - \nu$ and $b^2 = 2(A^2\sigma^2 + \delta^2)$,

$$E(C) = \int_{-\infty}^{\infty} \frac{C(x)}{b\sqrt{\pi}} \exp\left[-\frac{(x-a)^2}{b^2}\right] dx$$

$$= \int_{-\infty}^{0} \frac{-\beta x}{b\sqrt{\pi}} \exp\left[-\frac{(x-a)^2}{b^2}\right] dx + \int_{0}^{\infty} \frac{\alpha X}{b\sqrt{\pi}} \exp\left[-\frac{(x-a)^2}{b^2}\right] dx$$

$$= \frac{1}{b\sqrt{\pi}} \left\{\frac{\beta b^2}{2} e^{-(a^2/b^2)} - ab\beta \left[\frac{\sqrt{\pi}}{2} - \int_{0}^{a/b} e^{-v^2}\ dv\right]\right.$$

$$\left. + \frac{\alpha b^2}{2} e^{-(a^2/b^2)} + ab\alpha \left[\frac{\sqrt{\pi}}{2} + \int_{0}^{a/b} e^{-v^2}\ dv\right]\right\}$$

Hence,

$$E(C) = \frac{1}{\sqrt{\pi}} \left\{\frac{(\alpha + \beta)\sqrt{A^2\sigma^2 + \delta^2}}{\sqrt{2}} \exp\left[\frac{(A\mu - \nu)^2}{-2(A^2\sigma^2 + \delta^2)}\right]\right.$$

$$+ \frac{\sqrt{\pi}}{2} (\alpha - \beta)\ (A\mu - \nu)$$

$$\left. + (\alpha + \beta)\ (A\mu - \nu) \int_{0}^{\frac{A\mu - \nu}{\sqrt{2(A^2\sigma^2 + \delta^2)}}} e^{-v^2}\ dv\right\}$$

Figure 2 illustrates determination of the expected additional cost curve. The graph is reduced to two dimensions by omitting the A axis and simply corresponding each acreage with the appropriate distribution on the X axis. By integrating over the product of the probability of each X value with its corresponding additional cost, expected additional cost for that acreage may be determined, giving one point on the E(C) curve in Figure 2.

If the foregoing process is repeated for a sufficiently large number of A values, the entire E(C) curve may be traced. Due to nonzero variances of production and utilization, the expected additional cost curve will always lie above the additional cost lines. For a decision-maker, the more pertinent concern is the acreage at which E(C) is minimized. This will be examined now.

The conventional calculus method of locating the acreage which minimizes E(C) is to take the first derivitive of E(C) with respect to A, set it equal to zero, and solve for optimum A. In this case, the first derivitive of E(C) with respect to A will lead to at least a cubic polynomial in A with coefficients containing four arbitrary positive parameters, and only numerical methods are applicable to locate the optimum A value.[5/] However, use of electronic computers allows numerical approximation of E(C) to any desired degree of accuracy, so that behavior of cost-minimizing acreage may be observed with various values of the parameters.

Figure 3 illustrates the behavior of E(C) and optimum acreage as the cost parameters α and β vary, given that the sum of α and β is a constant magnitude and that the means (μ and ν) and variances (σ^2 and δ^2) are positive constants. For $E(C)_1$, $\alpha = 2\beta$, so that under-production by a given

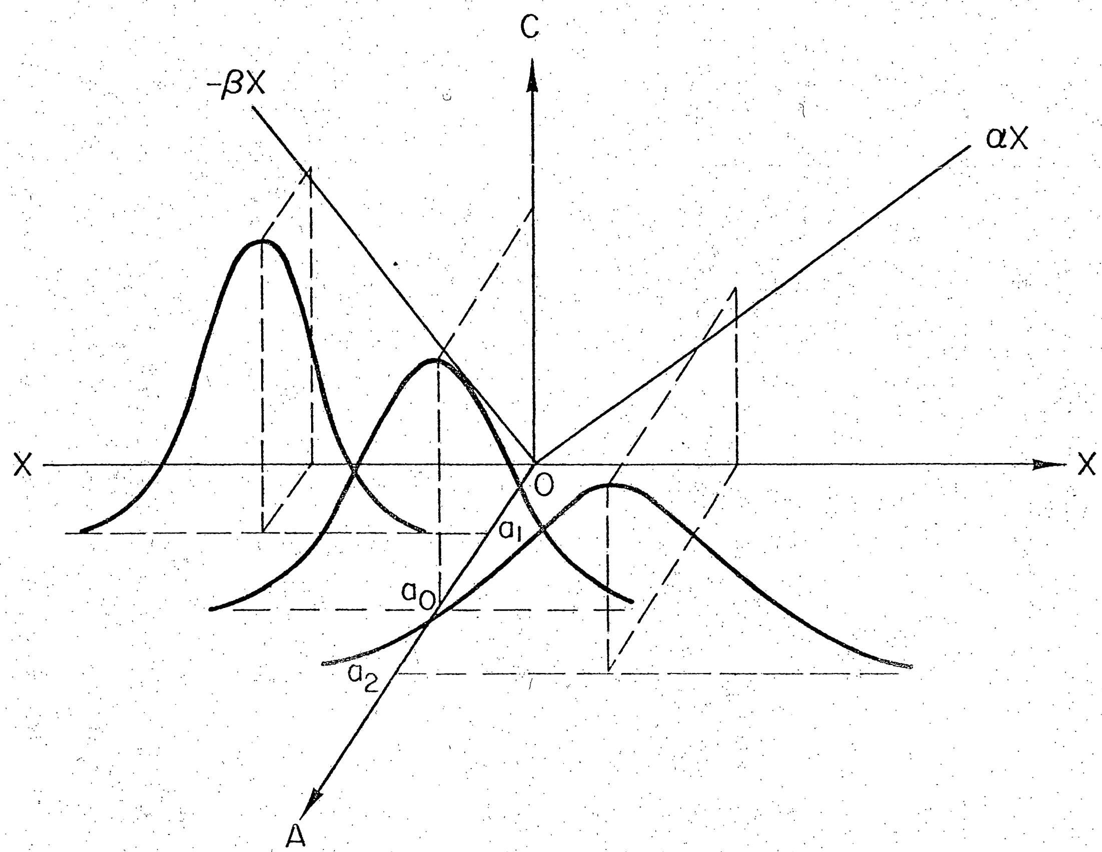

Figure I. Relationships Among Additional Cost(C), Excess Production (X) and Acreage (A).

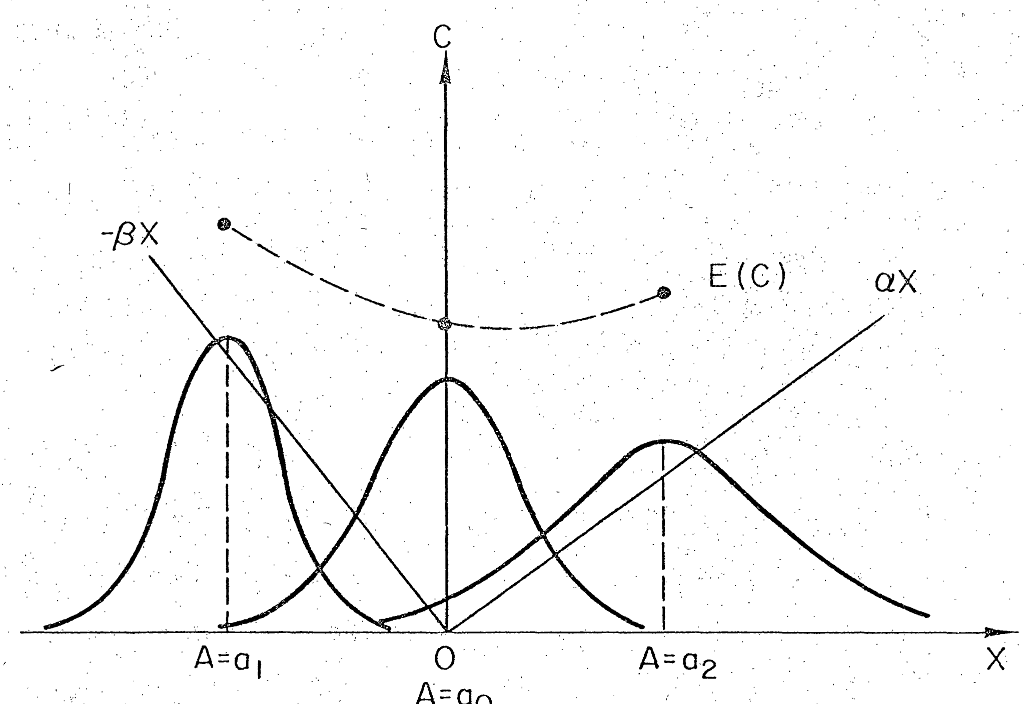

Figure 2. Determination of Expected Additional Cost E(C) of Excess Production, for Various Acreages

amount is only half as costly as over-production by the same amount. The opposite situation ($2\alpha = \beta$) is shown by $E(C)_3$. For $E(C)_2$, $\alpha = \beta$, so that under-production and over-production are equally costly. Intersection of the three expected additional cost curves locates the acreage (A^*) for which expected value of excess production is zero; i.e., $E(X) = 0$. This occurs because of the condition that α plus β is a constant. Furthermore, to the left of this acreage $E(X) < 0$, and $E(C)_1 < E(C)_2 < E(C)_3$ for any $A < A^*$. To the right of this acreage $E(X) > 0$, and $E(C)_3 < E(C)_2 < E(C)_1$ for any $A > A^*$. It follows that minimum $E(C)_1$ occurs at a smaller acreage than minimum $E(C)_2$, which in turn occurs at a smaller acreage than minimum $E(C)_3$ (i.e., $A_1 < A_2 < A_3$ in Figure 3).6/

Figure 4 illustrates, for $\beta > \alpha$ (i.e., under-production more expensive than over-production), how E(C) varies with different sources of uncertainty. Complete certainty ($\sigma^2 = \delta^2 = 0$) results in $E(C)_1 = C$. Introducing a utilization uncertainty ($\sigma^2 = 0$, $\delta^2 > 0$) results in $E(C)_2$. Adding a yield uncertainty ($\sigma^2 > 0$, $\delta^2 > 0$) gives $E(C)_3$. Inclusion of additional uncertainties cause the E(C) curves to shift successively upward and, since $\beta > \alpha$, optimum acreage to shift successively to right (from A_1 to A_2 to A_3).7/

APPLICATION TO PEANUT SEED ACREAGE

The methodology outlined in the first part of this paper was originally developed for the purpose of determining optimum acreage to plant for seeding purposes. Although subjected to limited analysis by economists, seed are a major farm input and of critical importance to commercial agriculture, which increasingly relies on an organized seed industry to supply high quality seed of known variety.

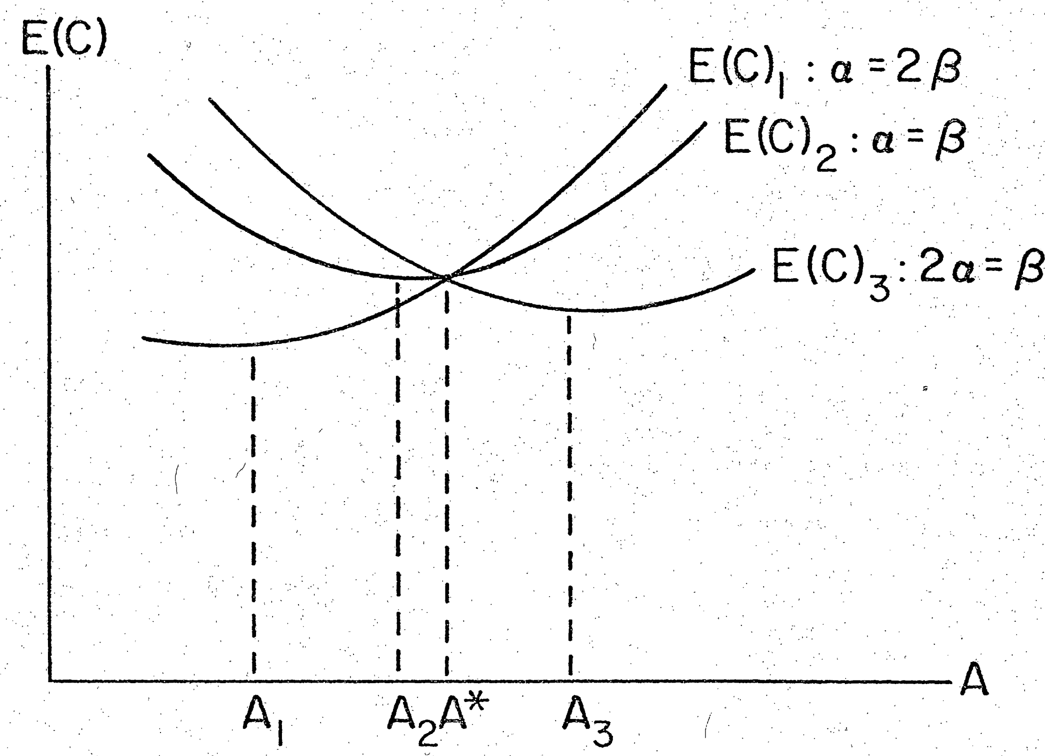

FIGURE 3. BEHAVIOR OF E(C) FOR VARIOUS VALUES OF α AND β, GIVEN THAT $\alpha + \beta$ IS A CONSTANT.

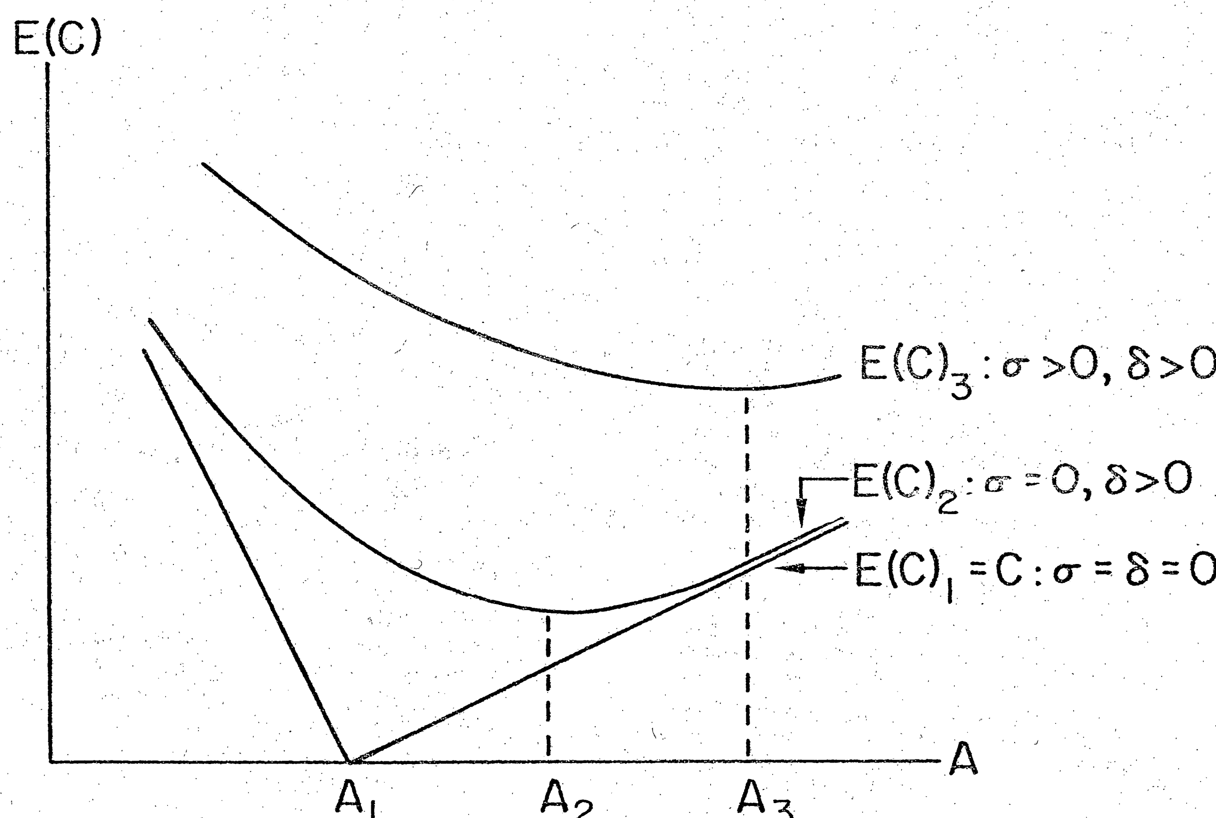

FIGURE 4. BEHAVIOR OF E(C) WITH VARIOUS DEGREES OF UNCERTAINTY, GIVEN THAT $\beta > \alpha$.

Seed is one of the major farm inputs that is produced as well as used by farmers. Supplies of these inputs - unlike agricultural chemicals, fertilizers, fuels and machinery - are subject to significant random fluctuations due to weather, disease, insects, etc. This is especially true for seed, which are living organisms and are therefore quite susceptible to destruction.

Just as seed supplies are subject to random yield fluctuations, seed demanded by farmers to plant a given acreage may also vary widely due to replantings occasioned by environmental conditions. Thus, matching quantities supplied with quantities demanded of a particular seed is an uncertain task at best. This uncertainty undoubtedly results in increased costs to the agricultural sector.

This section demonstrates the estimation of peanut seed acreage in Georgia that would minimize expected disequilibrium costs accruing to the state's agricultural sector from production of a given peanut crop acreage.[8/] Results have been obtained for each of the three major types (runner, Spanish and Virginia)[9/] of peanuts grown in Georgia; however, due to space limitations only results relating to total peanuts will be presented in this paper.

Analytical Framework

Peanut seed produced in the current season will be used to plant the crop for next season.[10/] The agricultural sector incurs additional costs whenever production of high quality seed in the current year is not equal to seed utilization in the following year. Costs associated with not having the desired quantity of seed may arise from three sources:

(1) Producing more seed than desired for storage.-- More seed may be produced than is expected to be required next year, due to a larger seed

yield per acre than anticipated. In this case the extra seed is sold in the edible peanut market and additional costs are incurred because production for seed is more expensive than production for edible purposes.

(2) Having too much seed at planting time.-- In this case, expected seed requirements have been overestimated and too much seed were processed and stored. If these extra seed are sold on the edible market at this time, additional costs have been incurred for production, processing and storage.

(3) Not having enough seed at planting time.-- In this case, expected seed requirements have been underestimated and too little seed were processed and stored. To make up for the shortage of high quality seed, the industry must either ship seed in from outside the state or resort to lower quality seed obtained from peanuts not produced specifically for seed purposes. Having to inship seed results in additional out-of pocket costs.[11/] Use of lower quality seed will cost the agricultural sector because of (a) reduced peanut yields, (b) increased planting rates in order to get an acceptable stand of plants in the field, and (c) additional cultivation, herbicides, or other treatments to reduce competition with the less vigorous plants.

The Model

Seed Production and Utilization

Quantity of seed produced in year t is given as follows:

$$P^t = A_t Y_t = A_t(\overline{Y}_t + e_1)$$

where:

P_t is seed production in year t,

A_t is acreage planted for seed in year t,

Y_t is seed yield per acre in year t,

$\overline{Y}_t$ is expected seed yield per acre in year t, and

e_1 is a normally distributed random variable associated with yields.

Thus, actual seed production is a function of planted acreage and the random variable per-acre yield.

If actual seed production is greater than quantity of seed needed to minimize expected cost, then excess seed production is sold in the edible market:

$$SE_t = \begin{cases} P_t - P^*_{t+1} & \text{if } P_t > P^*_{t+1} \\ 0 & \text{otherwise,} \end{cases}$$

where:

SE_t is quantity of seed sold in edible market in year t, and

P^*_{t+1} is quantity of seed that minimizes expected cost in year t+1.[12/]

The quantity of seed stored for next season's planting is equal to seed production unless some is sold in the edible market due to over-production:

$$SS_t = P_t - SE_t$$

where:

SS_t is quantity of seed stored in year t.

Total seed utilization is given by

$$U_{t+1} = u_{t+1}\, AP_{t+1} = (\overline{u}_{t+1} + e_2)\, AP_{t+1}$$

where:

U_{t+1} is total peanut seed used in year t+1,

u_{t+1} is seed utilization per acre planted in year t+1,

AP_{t+1} is planted acreage of peanuts in year t+1,

$\bar{u}_{t+1}$ is expected seed utilization per acre planted in year t+1, and

e_2 is a normally distributed random variable associated with number of times planted.

Thus, per acre seed utilization is assumed to be unaffected by seed price.13/ It is systematically related only to the planting rate.

Costs to the Agricultural Sector

If seed production is greater than the quantity which minimizes expected costs ($P_t > P^*_{t+1}$), the additional production cost for the excess seed is given by

$$C_1 = GR\ (P_t - P^*_{t+1})$$

where:

C_1 is additional cost of production for excess seed, and

GR is additional growing cost per pound of seed (i.e., additional cost of producing for seed rather than for food).

If quantity of seed stored is greater than seed utilization, added cost is given by

$$C_2 = (GR + PS)\ (SS_t - U_{t+1})$$

where:

C_2 is additional cost for seed kept over until next season, and

PS is processing and storage cost per pound of seed.

If quantity of seed stored is less than seed utilization, added cost is given by

$$C_3 = \bar{E}_{t+1} \cdot (\Delta \cdot \overline{YP}_{t+1}) \cdot (P_t + U_{t+1})/\bar{u}_{t+1} + CG\ [\Theta \cdot (P_t - U_{t+1})/\bar{u}_{t+1}]$$

$$+ CI[\Phi \cdot (P_t - U_{t+1})]$$

where:

C_3 is additional cost of not having enough high quality seed,

$\overline{E}_{t+1}$ is expected peanut price in edible market in year t+1,

Δ is proportionate decrease in peanut yield per acre due to low quality seed,

$\overline{YP}_{t+1}$ is expected peanut yield per acre in year t+1,

Θ is proportion of excess acres planted with low quality seed,

CG is additional growing cost per acre due to low quality seed,

CI is additional cost per pound of inshipping extra seed, and

Φ is proportion of deficit seed inshipped.

The Data[14/]

Time series data covering the period 1963-72 were used for the necessary seed production and utilization variables. Estimates of per acre seed yields (Y_t) and planting rates (R_t) were obtained from the Cooperative Extension Service of the University of Georgia. Data on seed utilization (U_{t+1}) came from secondary sources of the USDA [4], as did data on peanut yield per acre YP_{t+1} [3].

Additional costs of growing (GR), processing and storing (PS), and inshipping (CI) seed were estimated after consultation with key persons in the peanut seed industry, as were the percentage decline in per acre yields (Δ) and additional per-acre growing costs (CG) due to low quality seed. Trend of peanut prices, obtained from USDA data [2], was used to estimate expected peanut price ($\overline{E}_{t+1}$).

The proportion of a seed deficit made up by inshipments of high quality seed (Φ) may vary by areas and firms within the state, as well as according to how widespread the seed shortage happens to be.[15/] For simplicity, it is assumed that 50% of any deficit is filled by inshipments, with the remaining 50% of the acres planted with lower quality seed (Θ).

Estimation Procedure

The decision variable is acreage to plant for seed in the current year. The objective function to be minimized is expected additional (disequilibrium) cost, which is the summation of C_1, C_2 and C_3 as defined in the model section. For any designated level of seed acreage, all elements of these three costs are specified except seed yields and utilization rates. Random error associated with these two variables receive explicit consideration.

Per-acre seed yield was regressed on time and seed utilization rate was regressed on planting rate. The standard errors of these regressions were used to define the variances of random numbers generated from the normal distribution, thus generating 300 values of peanut seed yield and utilization rates.[16/] Using these data, expected disequilibrium cost associated with various peanut seed acreages could be calculated. To do this, an iterative computer procedure was used. Cost-minimizing seed acreage was then found by inspection.

Estimation Results

The unit of analysis used was 1,000 acres of peanuts. Since all cost estimates are assumed to be linearly related to acreage, optimum seed acreage for any number of peanut acres may be easily estimated by scaling these results.

The cost-minimizing solution in producing seed for 1,000 acres of peanuts is as follows:

Seed acreage planted---------------------------- 121.4 acres

Minimum expected additional cost---------------- $693.30

If utilization uncertainty is (erroneously) ignored, the seed acreage indicated is 114.1 acres - a reduction of 6%. This would result in an expected disequilibrium cost of $756.88 - an increase of over 8%. If both utilization and yield uncertainties are ignored, the seed acreage indicated is 106.4 - a decrease of over 12%. This would result in an expected disequilibrium cost of $1,019.90 - an increase of over 32%. Given the fact that indicated optimum acreage declines when these uncertainties are not allowed for, it follows that estimated costs to the entire agricultural sector are less for over-production than for under-production of peanut seed.

SUMMARY AND CONCLUSIONS

A theoretical framework has been developed for optimizing planted acreage under yield and utilization uncertainties, the objective function being expected disequilibrium costs. A model specifically for peanut seed acreage in Georgia has been formulated and used to estimate optimum seed acreage for planting a fixed peanut acreage. While the results are exploratory, they indicate the usefulness of the methodology for dealing with uncertainties in matching agricultural crop supplies with demands. Application of the methodology could be fairly general, depending on feasibility of estimating relevant costs and distributions of yield and utilization.

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FOOTNOTES

1/ Utilization may be specified to include more than just consumption in the market place; e.g., it may include desired levels of stocks, disaster relief grants, etc.

2/ The disequilibrium cost could conceivably be formulated for producers, marketing sectors, the entire agricultural sector, etc. - depending on a researcher's area of interest.

3/ Formulating additional cost as a linear function of excess production is somewhat arbitrary, but nonlinear relationships could be used if appropriate.

4/ The normal distribution seems the obvious one to use due to its wide applicability. Other distributions could be used if they are more appropriate.

5/ The first derivitive is given as follows:

$$\frac{dE(C)}{dA} = \frac{\alpha + \beta}{\sqrt{2(A^2\sigma^2 + \delta^2)}} \cdot \exp\left[- \frac{(A\mu - \nu)^2}{2(A^2\sigma^2 + \delta^2)}\right] \cdot$$

$$\left\{A^3\sigma^4 + \mu\nu A^2\sigma^2 + A(\nu^2\sigma^2 - \mu^2\delta^2 + \sigma^2\delta^2) + \mu\nu\delta^2\right\}$$

$$+ \frac{\sqrt{\pi}}{2} (\alpha - \beta)\mu + \frac{\alpha + \beta}{(A^2\sigma^2 + \delta^2)^2} \left\{u(A^2\sigma^2 + \delta^2) \cdot\right.$$

$$\int_o^{\frac{A\mu - \nu}{\sqrt{2(A^2\sigma^2 + \delta^2)}}} e^{-v^2} \, dv + \left[-A^3\delta^2\mu^2\nu\right.$$

$$\left. + A^2 (\mu\nu^2\sigma^2 - \delta^2\mu^3 + \delta^2\mu\nu^2) + A(2\delta^2\mu^2\nu - \nu^3\sigma^2) - \delta^2\mu\nu^2\right] \cdot$$

$$\exp \left[- \frac{(A\mu - \nu)^2}{2(A^2\sigma^2 + \delta^2)} \right] \Bigg\}$$

6/ Figure 3 also illustrates a fact that may not be realized at first: Cost minimizing acreage where $\alpha = \beta$ does not simply occur at the acreage for which $E(X) = 0$ (i.e., where $A = A^* = \frac{\mu}{\nu}$). Instead it will occur at a smaller acreage than this due to the fact that by reducing acreage, variance of production ($= A^2\sigma^2$) is reduced, which will initially result in a lowering of expected additional cost.

That A^* in Figure 3 is not an optimum when $\alpha = \beta$ may be rigorously proved by the method of contradiction. If $\alpha = \beta$ and $A = \frac{\mu}{\nu}$, then

$$\frac{dE(C)}{dA} = \frac{\alpha \sqrt{2}}{\mu^3(\sigma^2A^2 + \delta^2)^{\frac{3}{2}}} \quad \sigma^2\nu \ (\sigma^2\nu^2 + \mu^2\delta^2)$$

Since α, μ, σ and δ are all positive constants, this first derivitive cannot be zero. But this contradicts the assertion that $A = \frac{\mu}{\nu}$ is a critical value. Therefore, A^* does not locate the optimum acreage.

7/ With $\beta < \alpha$ (i.e., under-production less expensive than over-production), optimum acreage would shift successively to the left.

8/ The problem of estimating total peanut acreage planted for food purposes need not be dealt with here, since government programs have made planted peanut acres a predetermined variable. If desired, planted acreage could be estimated separately and results applied to this analysis in a stepwise fashion.

9/ In 1972, the total volume of peanut seed was approximately divided among these three types as follows: runners - 70%, Spanish - 25%, and Virginia - 5% [1, p. 9].

10/ Rarely are peanut seed carried over for two seasons because of prohibitive storage costs and/or deterioration of seed quality.

11/ Of course some peanut seed are brought into, as well as shipped out of, the state in a normal year. To the extent that this is planned by the industry, no additional costs are assumed to occur. When unplanned, however, the agricultural sector must bear at the very least some additional shipping costs.

12/ This quantity of seed minimizes expected disequilibrium cost in the presence of utilization uncertainty. Since total quantity of seed (rather than total seed acreage) is considered at this point, there is no per-acre yield uncertainty.

13/ This assumption was upheld by trial regressions of per acre peanut seed utilization on peanut seed price. Since peanuts are grown on a limited acreage and are a high valued crop, it is reasonable to conclude that the derived demand for peanut seed is almost perfectly inelastic with respect to its own price.

14/ It is assumed that production practices, yields and costs for certified peanut seed are representative for all "high quality" peanut seed. This seems a safe assumption, since in 1972 about 95% of all peanut seed handled by commercial seed processing plants in Georgia were certified [1, p.9].

15/ In the event of a widespread seed shortage, seed price could become quite inflated. But ramifications of this are not explored here.

16/ Statistics generated by use of these random numbers are constrained to have means and variances that are consistent with the regression error terms; therefore, the simulated data are necessarily consistent with the observed time series data.