Operations research
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POTENTIAL CONTRIBUTION OF OPERATIONS RESEARCH METHODS IN FARM MANAGEMENT*

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The methodology of operations research is not easily delineated and no attempt will be made here to do so. Nevertheless, it would seem that operations research is essentially application of the scientific method to operational problems, and the discipline as we observe it today has evolved from the many diverse applications which have been made since World War II in government, industry, and the military. These applications, and the research in mathematics stimulated by them, have given rise to a body of knowledge overlapping many disciplines, but still identifiable as a field of its own which we call operations research.

The bulk of this knowledge is of a mathematical nature, or at least highly refined logic, and is closely akin to economics as well as several other disciplines. Our concern is its relation to farm management research, and I am very optimistic about its potential application in this field. However, the full potential of operations research in farm management (and other fields of agricultural economics)

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will not be achieved unless it is realized that operations research provides more than a bag full of tricks or techniques. The techniques are only a means to facilitate application of the scientific method to operational problems.

No pretense is made that the group of techniques considered here is comprehensive. The ones chosen for discussion are the most promising in the opinion and limited knowledge of the speaker. The basic techniques of operations research are modern mathematics of optimization which are primarily linear, nonlinear, and dynamic programming. Many of the commonly cited categories of operations research such as inventory control, queuing models, and replacement rely heavily on dynamic programming applied to stochastic processes.

The term mathematical programming is used to encompass linear programming with which you are all familiar, and its straightforward extension to nonlinear objective and constraint functions. Such programming can include optimization over time, but the term dynamic programming is reserved for a special method quite different than traditional programming.

Several applications of mathematical and dynamic programming are outlined for problems in farm management research. It is anticipated that these applications will illustrate some of the leverage that operations research methodology has in coping with complex questions in farm management. Little attention is given to feasibility of estimating parameters in the models discussed, but hopefully, the suggested applications are not unrealistic in this respect. Only models for which efficient computational algorithms are available are considered, but efficiency is relative and some algorithms are more routine than others.

Mathematical Programming Applications

The ingenuity that has been shown in applications of linear programming to varied and complex farm management models is noteworthy. Use of linear programming as a substitute for budgeting of farm enterprises is almost a standard method of the profession, and few would dispute its usefulness. A natural extension was its application to allocation of resources over time in the framework commonly called dynamic linear programming.[1] It is surprising that there have been so few applications of linear programming to temporal resource allocation, but from my observations, there is a renewed interest in it.

Dynamic linear programming would seem to be an appropriate model for analyzing the important factors affecting growth of the firm. Few empirical answers have been provided to the question of what limits farm size. Is it increasing risk, capital rationing, management limitations, imperfections in the land market, or what? Imaginative implementation of dynamic linear programming could help answer this question.

For example, initial capital constraints and equity requirements on financing could be utilized in such a model to learn something of the effects of capital rationing. A land purchase activity made accessible only at widespread points in time would yield an imputed value to land which is available at discontinuous points in the firm's existence.

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[1] L. D. Loftsgard and E. O. Heady, "Application of Dynamic Programming Models for Optimal Farm and Home Plans," _J. Farm Econ._, Vol. 41, pp. 51-62 (1959).

This latter device might shed some light on the forces behind land prices currently being observed, particularly if parametric programming were applied to both land and capital constraints.

Some aspects of firm growth that might want to be examined are likely to lead to nonlinear relationships. If one were to postulate management as an important force in limiting firm size, a nonlinear objective function would result. The decreasing returns associated with management would probably hold for each activity taken separately, and with respect to groups of activities or all activities as an aggregate. One of the simplest nonlinear objective functions, but yet quite versatile, is a general quadratic relation in the activities. Several efficient quadratic programming algorithms are available, [2,3] but the number of time periods in the model would have to be kept fairly small. Empirical measures of the diminishing returns associated with management would surely be difficult to obtain, but so are many other empirical estimates.

It is surprising that the early work of Freund, which brought risk into farm management activity analysis, has not been used to more advantage.[4]

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[2]Philip Wolfe, "The Simplex Method of Quadratic Programming," Econometrica, Vol. 27, pp. 382-398 (1959).

[3]George B. Dantzig, Linear Programming and Extensions, Princeton Univ. Press, 1963, pp. 490-498.

[4]R. J. Freund, "The Introduction of Risk into a Programming Model," Econometrica, Vol. 24, pp. 253-263 (1956). For an alternative utility function specification which gives the same model see D. E. Farrar, The Investment Decision Under Uncertainty, Prentice-Hall, 1962, pp. 19-22.

When looking at our farm management research, we find arbitrary constraints imposed on high risk crops within a standard linear programming model, instead of a utility maximization model that incorporates risk. Some empirical research on the magnitude of this risk aversion factor would be very welcome; but without it programming models patterned after Freund's using risk aversion factors which produce reasonable results would seem superior to arbitrary constraints on various high expected return activities.

A similar conclusion could be reached in regard to estimating supply response with linear programming where arbitrary constraints are imposed on the adjustment process in order to get reasonable results. Introduction of risk into the programming analysis might alleviate the need for adjustment constraints.

The risk model of Freund does not appear readily adaptable to resource allocation over time, but it could be used in conjunction with dynamic programming by delineating efficient combinations of activities. The role of risk as a deterent to firm growth is discussed later under sequential decision models.

Let us now take a look at how income tax considerations might be incorporated into a multiperiod programming model. We know that income taxes have an important influence on investment decisions of the firm through the differential in capital gains compared to income taxes, particularly since the income tax is progressive and the tax on capital gains is a fixed proportion. Probably all of us have encountered situations where temporal allocation of resources are affected quite directly by these taxes, but I have not seen a study where their effects were analyzed in much detail.

A nonlinear analogue of dynamic linear programming can be used to analyze these tax aspects of farm investments and growth. Income producing activities are routed into the capital constraint, and in the objective function they enter nonlinearly in a tax payment relation. Special activities are created to transfer capital into income or from one time period to another and to make investments of longer life than one time period. These special activities enter the objective function nonlinearly. All the constraints can be kept linear except possibly one for each time period which would prevent deficit financing of tax payments on taxable portions of investments and provide a precise tax free amount for family living.

Computer hardware and applied mathematics have advanced to a state where nonlinear programming is operationally feasible, and it is primarily a question of whether the required resources for its application to a particular analysis are warranted. A promising algorithm for general application is the sequential method of Fiacco and McCormick.[5] Their method is general in the sense that nonlinearities can occur in both the objective function and the constraint set. Another nonlinear programming algorithm, which utilizes the simplex method of linear programming and is apparently performing satisfactorily computationally, is that of Hartley and Hocking.[6] In cases where all the restrictions

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[5] A. V. Fiacco and G. P. McCormick, "The Sequential Unconstrained Minimization Technique for Nonlinear Programming, A Primal-Dual Method," Management Science, Vol. 10, pp. 360-366, also pp. 601-617 (1964).

[6] H. O. Hartley and R. R. Hocking, "Convex Programming by Tangential Approximation," Mgt. Sc., Vol. 9, pp. 600-612 (1963).

are linear, the objective function can often be approximated as quadratic and a quadratic programming routine applied to obtain an approximate solution.

Another segment of mathematical programming that shows promise in farm management research is integer or discrete programming. Quite often an activity has a realistic interpretation only at integer values. For example, machinery purchase activities in farm management applications of linear programming should be restricted to integer levels because a fraction of a machine is an impossibility. Also, labor and land must be purchased in discrete quantities in commonly postulated conditions under which the farm firm operates. There are undoubtedly many additional situations where activities would ideally be restricted to integer levels. Economies of scale studies are particularly in need of implementation of integer restrictions on part of the activities since many scale economies are a direct result of these integer restrictions in the real world.

The most promising algorithms for integer, and mixed integer and continuous, programming models are those pioneered by Gomory. For these methods and references to the original papers, you are referred to Dantzig[7] or Hadley.[8] Computational results from these algorithms have not been as good as might be expected. Although convergence is insured

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[7]Dantzig, op. cit., ch. 26.

[8]G. Hadley, Nonlinear and Dynamic Programming, Addison-Wesley, 1964, ch. 8

for a finite number of iterations, the number has been very large in several applications of rather modest dimension.[9] Nevertheless, progress in this area has been very significant and we can expect improvements to be forthcoming.

In farm management applications, nonlinear programming is sometimes an alternative to restriction of activities to integers. A tractor purchase activity can be designed so that a choice of various size tractors is permitted with nonlinearities in the objective function. Where tractor services are used in the resource constraint vector, relaxation of the constraint by a tractor purchase activity would take place at decreasing costs per service unit up to the largest practical tractor size. This nonlinear cost relationship could be used with the activity constrained to less than or equal to unity, where the unit designates the largest tractor permitted. Thus a fractional level of the activity would approximate a tractor some size less than maximum. The same approach could be used on labor or land since presumably either labor or land could be purchased in arbitrarily small amounts _if the price paid were high enough_.

Within our own profession there is the recent contribution of Maruyama and Fuller to the area of mixed integer programming.[10] It

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[9] _Ibid._, p. 252.

[10] Y. Maruyama and E. I. Fuller, "Alternative Solution Procedures for Mixed Integer Programming Problems," _J. Farm Econ._, Vol. 46, pp. 1213-1218 (1964).

is too early to evaluate their method as a general algorithm, but it apparently has an advantage for situations where most of the activities are continuous and there are only a few activities restricted either to zero or unity.

Dynamic Programming Applications

Dynamic programming could be briefly described as backward mathematical induction applied to sequential decision problems. Rather than try to explain the method, the type of problem where it is feasibly applied is described, as well as the form of decision rules evolving under its use.[11] Although dynamic programming finds its largest use in decision processes through time, it can also be fruitfully applied to problems where the sequential nature of the process is artificially created to exploit the method.

Some terminology is introduced as it would be applied to a sequential decision process taking place in time. A stage of the process is associated with an interval of time, and although not necessary, the time interval is assumed constant from stage to stage. The state of the process is defined for each stage (time interval), and the state describes the decision process in some meaningful way at a particular stage.

An example will clarify these two terms. Suppose a dryland wheat farmer makes a decision each year on whether to leave a tract of land fallow or to plant it to wheat on the basis of soil moisture at wheat planting time. The stage of the process is denoted by a year within

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[11] A good reference is R. E. Bellman and S. E. Dreyfus, Applied Dynamic Programming, Princeton Univ. Press, 1962.

the farmer's planning horizon and the point in time within that year which is significant is wheat planting time. The state of the process at a given stage is level of soil moisture.

The state must describe the condition of the process in such a way that decisions are facilitated. In this example, the farmer seeks a conditional decision rule which says to plant wheat if moisture exceeds some point and fallow if it falls below that level; and he wants the rule optimal by some criterion such as long run average net returns or expected present value of net returns.

There are then three important factors in a sequential decision problem: (1) stages, (2) states, and (3) a conditional decision rule. In general, the decision rule is dependent on both the state and stage of the process.

It is convenient to introduce the notion of state and decision variables. State variables are those determining the state of the process such as soil moisture for wheat-fallow decisions. Decision variables are those subject to direct control at each stage of the process. In the wheat-fallow example, land utilization is the decision variable and it assumes only two values--one each for wheat planting and fallow.

A very desirable property of dynamic programming models is that stochastic elements in a problem are still manageable, and the conceptual framework is hardly changed when using an expected value criterion. Most stochastic dynamic programming models can be viewed as choosing a Markov process which maximizes expected present value of profits (or minimization of costs). Usually the stochastic part of the process arises from a decision in one stage specifying only the probability distribution of the state in the following stage. In the wheat-fallow example, soil moisture next year, for a given level this year, is a random variable under a choice of either fallow or wheat.

Crop Rotations

The example of wheat-fallow decisions was a very simple crop rotation model which has been explained in detail elsewhere.[12] It is easily generalized to choosing among several crops under dryland farming. The choice might be from among grain sorghum, corn, wheat and fallow. With both spring and fall planted crops, the year must comprise two stages. Even better decisions could be made if significant dependence exists between successive periods of rainfall. Such dependence would provide information in addition to soil moisture for choosing a crop.

Under intensive farming, crop rotations are sometimes required to effect disease or soil pest controls. Nematode infestation in soils producing sugar beets is an example. The nematode population density would be a likely choice of state variable in a dynamic programming model to determine whether to plant beets or an alternative crop which would help reduce the infestation in subsequent years. Presumably a critical level of infestation exists where sugar beets are optimal if the infestation is below that level and the alternate crop is optimal for greater infestation levels in spite of its lower immediate profits.

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[12] O. R. Burt and J. R. Allison, "Farm Management Decisions with Dynamic Programming," J. Farm Econ., Vol. 45, pp. 121-136 (1963).

Nematode population changes are undoubtedly subject to random variation which would require a stochastic model.

Selection of Farm Enterprises Under Risk

It was mentioned earlier that risk might want to be examined as a limitation to expansion of the firm; or at least, its role in influencing growth of the farm firm is of interest. A stochastic dynamic programming formulation is given for application to this type of question. Implicit in the model is the assumption of capital limitations, either direct or indirect through restrictions on the equity-debt ratio.

Let the stage of the process be a year and the state is primarily determined by the firm's liquidation value. It is assumed that the firm has considerable latitude in selecting a mixture of enterprises where the various mixtures yield profits which are random variables. The firm would like high expected profits with low variance, but the economics are such that these two criteria are inversely related for the various enterprise combinations.

High variability of profits causes difficulty because several bad years in succession could lead to bankruptcy. A zero or smaller liquidation value is defined as bankruptcy where the firm ceases to exist. Let our criterion be maximization of an expected utility function, with liquidation value at the end of a finite planning horizon its argument.

An optimal decision rule is sought which specifies the combination of enterprises as a function of firm liquidation value and the year within the planning horizon. In early years with few assets, one would expect considerable conservatism, while a large liquidation value would be expected to encourage taking high risks for the chance of large gains. This type of model would be particularly interesting to apply in regions where high risk enterprises such as truck crops or cattle feeding are a likely choice.

Replacement Decisions

There have been a few applications of dynamic programming to replacement of assets, but in our profession they have been stochastic only in a very limited manner.[13,14] Potential application is great in this area, particularly when integrated with broader problems. One such problem is culling of beef cow herds (essentially a replacement decision) in conjunction with optimal stocking of dryland pastures under climatic uncertainties. If we want to become even more ambitious, we could bring seasonal and cyclical cattle price variations into the picture. This overall cattle inventory and culling question could be posed in a dynamic programming framework, but a good deal of ingenuity would be required to obtain numerical results from an application.

Some other examples, not quite so ambitious, are farm machinery replacement, dairy cow culling, timing of pasture rejuvenation, and

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[13] A. N. Halter and W. C. White, "A Replacement Decision Process (An Application to a Caged Layer Enterprise)," Ky. Exp. Sta. Bul. 677 (March 1962).

[14] K. B. Jenkins and A. N. Halter, "A Multi-stage Stochastic Replacement Decision Model (Application to Replacement of Dairy Cows)," Oregon Exp. Sta. Tech. Bul. 67 (April 1963).

forest rotations. To give an idea of the potential sophistication achievable in replacement decisions, let us take a look at automobile replacement with which we are all familiar.

Some people will tell you that it is not the age but the mileage that counts, and the mechanically informed will tell you that it is the entire mechanical condition of the car which is important for replacement decisions. Assuming all of these observations have some merit, they must be reflected in an optimal decision rule. Thus, a conditional decision rule is sought which tells us whether to replace or not as a function of age, mileage, and mechanical condition. These are state variables in a dynamic programming context, and with some refinements, a model of this sort could be applied to automobile replacement. For a much simpler application to automobile replacement see Howard[15] which is also a good general reference for stochastic dynamic programming.

Other Operations Research Techniques

Simulation is gaining popularity in agricultural economics, but its use thus far is difficult to evaluate. Apparently it is looked upon more as a method for gaining general understanding of complex decision processes than as an optimization model. This current emphasis should not distract us from the possibilities simulation has as an optimization model. Simulation can be used to generate observations on complex phenomena much as a traditional experiment is used. Likewise, a criterion

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[15] Ronald Howard, Dynamic Programming and Markov Processes, Technology Press and Wiley, 1960, pp. 54-59.

function can be explored by the same methods used to explore statistical response surfaces. In some extremely complex problems, particularly those of a stochastic nature, simulation used in this manner is about the only recourse that we have for quantitative analysis.

As farming operations become more mechanized and larger scale, potential applications of operations research methods will grow. Specialties of operations research, which include sequencing, scheduling, and queuing, could become important in large scale agriculture. Problems such as comparison of farm work methods analyzed by Morris and Nygaard are becoming increasingly important.[16] It may be of interest that the optimizing path algorithm applied by them is identical to that arising from application of dynamic programming to the decision process.

Potential applications of operations research in farm management are probably far greater than we realize. If growth of linear programming uses over the past decade is any indication of what to expect in the future for other techniques, operations research is still in its infancy as far as farm management is concerned.

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[16] W. H. M. Morris and A. Nygaard, "Application of an Optimizing Path Algorithm in the Comparison of Farm Work Methods," J. Farm Econ., Vol. 46, pp. 410-417 (1964).