CONSIDERATION OF TIME AND CARRYOVER EFFECTS IN MILK PRODUCTION FUNCTIONS*

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Livestock production functions must be interpreted with care, primarily because points on the production surface can be reached only by successively "feeding in" incremental feed inputs over time.[1/] Two questions related to "time" are immediately raised: (1) How does the "rate" of feeding through time affect output response? (2) How can carryover effects through time be measured and incorporated in the economic analysis? This paper concentrates on the second question. However, the first question is briefly discussed as prerequisite to understanding of the second.

The "Rate of Feeding" Problem

Figure 1 illustrates the "rate of feeding" problem. Line OA_1 is the production function if the cow is "full fed" on ration 1.[2/] Point A_1 is reached at the end of an entire lactation (say, 10 months). At a lower feeding rate (e.g., 80% full-fed) on ration 1, the production function is OB_1 with point B_1 reached at the end of the 10-month lactation. OB_1 lies below OA_1--at the lower feeding rate a larger percentage of the feed is used for body maintenance. Similar interpretations attach to lines OC_1 and OD_1 for lower rates of feeding,

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1/ In this regard, livestock functions have been contrasted with fertilizer functions where almost any feasible input level can be applied instantaneously. See Brown, William G. and George H. Arscott. "A Method for Dealing with Time in Determining Optimum Factor Inputs." Journal of Farm Economics, Vol. XL, No. 3, August, 1958, pp. 666-673.

2/ We ignore the sticky question of determining a "full feed." For a discussion of feeding problems in dairy cow experiments, see: Jacobson, N.L., "Problems in Designing Feeding Experiments From a Nutritional Standpoint," in Nutritional and Economic Aspects of Feed Utilization by Dairy Cows, Edited by Hoglund, C. R., Glenn L. Johnson, Charles A. Lassiter and Lon D. McGilliard, pp. 206-212.

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only summary will be published

with points C_1 and D_1 reached, respectively, at the end of the lactation period. Within a static framework, functions OD_1, OC_1 and OB_1 could be immediately eliminated as technologically inferior to function OA_1 (OA_1 provides greatest output for given feed input). In the static context, optimum input of ration 1 would be determined in the usual way by setting $\frac{d\text{ (Production function } OA_1)}{d R_1} = \frac{\text{Price } R_1}{\text{Price Milk}}$. However, this "optimum" input may lie at some point (e.g., c) corresponding to less than a full lactation--obviously an unrealistic solution. Heady argues, therefore, that a production function such as D_1A_1 is the relevant function for feeding decisions.[3/] The question here is: If the cow is fed for the entire 10-month lactation, which feeding rate is optimum? The relevant criterion here is: $\frac{d\text{ (production function } D_1A_1)}{d R_1} = \frac{\text{Price } R_1}{\text{Price milk}}$.

Limitations arise, however, in extending this argument to a production surface for the entire lactation (Fig. 2) where "choice of ration" as well as "feeding rate" is concerned. The isocline (Fig. 2) denotes the least-cost feeding path for a particular hay-concentrate price ratio. The isocline is not interpreted as the least-cost feeding path for a particular cow over its lactation; that is, the isocline does not mean, "feed ration R_4 for first, 4,000# milk, then switch to a lower concentrate ration (R_3) for the next 4,000#, then switch again (R_2) for the final 4,000# milk." Instead, the isocline shows that if 4,000# of milk is desired over the entire lactation, R_4 is the cheapest feeding method; if 12,000# of milk is desired R_3 is the cheapest

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3/ Heady, E. O., "Problems in Designing Dairy Feeding Experiments for Economic Analysis," in Hoglund, et. al., editors, op. cit., pp. 193-205.

feeding method. Thus, a surface derived for an entire lactation is limited to answering questions of "optimum ration" and "feeding level" only under a single ration feeding plan for the entire lactation. Heady, et. al.[4] handled this problem by including time (month of lactation) as a "shifter" in the production function, thereby allowing specification of a series of monthly production functions. Conventional profit maximizing criteria could then be applied to the monthly functions to solve for optimum monthly ration and rate of feeding.

Often, however, experiments are not designed ideally for economic analysis. In many cases, dairy experiments employ only "full fed" rations, thus providing only an estimate of the "stomach line" (Figure 2). The economic question here is: Which feed combination along the "stomach line" maximizes profit? As pointed out by Brown and Arscott,[5] this point may or may not lie on the conventional isocline.[6] In such experiments a series of monthly "stomach lines" could be estimated and the optimum "full-fed" rations determined month-by-month for the entire lactation. The necessity of a "complete" production surface appears to be sometimes overexaggerated. In most practical feeding situations cows are hand-fed a particular level of concentrate, then fed hay free-choice to reach the "stomach line." In such cases, an accurate estimate of the "stomach line" (a slice through the surface) may be more important than a complete surface with isoquants, isoclines, etc.

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4/ Heady, E. O., John A. Schnittker, N. L. Jacobson and Solomon Bloom. Milk Production Functions, Hay-Grain Substitution Rates and Economic Optimum Dairy Cow Rations. Iowa Agr. Exp. Sta. Bul. 444., October, 1956.

5/ Brown and Arscott, op. cit.

6/ In fact, if the economic "optimum" derived in conventional manner from the entire surface lies at some infeasible point "outside" the stomach line (e.g., point F), the "true" economic optimum may lie off the isocline.

The "Carryover" Problem

The above methods of obtaining monthly economic optima appear reasonably adequate, except where "carryover" effects exist. As pointed out by Frick and Mighell:[7/]

> "One of the problems associated with these monthly production surfaces is that they suggest that production in any month of lactation is independent of production in preceding stages....It would be interesting and valuable to determine just how much independence exists between feeding levels and sucessive production rates as the months of lactation progress."

<u>Possible experimental designs for estimating carryover effects.</u>

Dairy feeding experiments are either (1) continuous trials or (2) change-over trials.[8/] In a continuous trial each animal remains on the same experimental treatment (e.g., ration) throughout the trial; in a change-over trial each animal receives a sequence of two or more treatments (rations). Thus, change-over trials are often used because they permit comparison of more treatments per cow per unit of time than do continuous trials. However, change-over trials have been criticized because carry-over effects may preclude proper estimation of treatment differences and their errors. In practice, however, dairymen do normally change rations, first increasing and then reducing the percentage of concentrate as lactation progresses. Thus, carryover effects from changing rations, if present, need to be estimated and incorporated into the economic analysis. The change-over design allows a measure of cow performance when rations change, therefore may be useful in estimating carryover effects.

Figure 3 shows the treatment sequences for the first two 28-day periods of a change-over trial at California designed to test four hay:concentrate pelleted rations.[9/] The symbols R_{ij} $(i, j=1,...4)$ refer to the ration sequences fed

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7/ Frick, G. E. and Ronald L. Mighell. "Adequacy of Dairy Feeding Input-Output Research Data From the Viewpoint of the Economist." In Hoglund, et. al., editors, <u>op. cit.</u>, pp. 107-115.

8/ Lucas, H. L., Jr., "Experimental Designs and Analyses for Feeding Efficiency Trials with Dairy Cattle." In Hoglund, et. al., editors, <u>op. cit.</u>, pp. 177-191.

9/ Pelleting is ignored in the following discussion; hay and concentrate are assumed to be conventional separable inputs. For more detail on the particular experiment see: Magnar Ronning, "Effect of Varying Alfalfa Hay-Concentrate Ratios in a Pelleted Ration for Dairy Cows." <u>Journal of Dairy Science</u> Vol. 43, No. 6, 1960, p. 816.

pairs of cows in the first two periods; e.g., R_{13} refers to ration 1 in the first period followed by ration 3 in the second. Thus, the design measures performance by cows changing from each ration in period 1 to the other three rations in period 2.[10/]

A proposed method of determining economic optima with carryover present

The economic feeding problem is illustrated in Equation (1), where π = profit, P = price, M = milk, C = concentrate, H = hay, K = fixed costs (all

$$(1)\quad \text{Max}\,\pi = \sum_{i=1}^{n} P_{M_i} M_i - \sum_{i=1}^{n} P_{C_i} C_i - \sum_{i=1}^{n} P_{H_i} H_i - K$$

nonfeed inputs are assumed fixed for the time period) and i denotes the time period. Data from the California trial are used in demonstrating a possible method of determining economic rations in the first two periods with carryover present. However, the general method could be refined by additional variables and easily extended to three or more periods. Use of entirely pelleted feeds, with resultant low butterfat content from cows on high concentrate rations, suggests that the analysis here should be considered only illustrative.

The experiment was conducted only at "full-feeding" of each ration. Thus, instead of estimating an entire monthly "surface," the "stomach line" is estimated directly. The four equations estimated for the first two periods are:

$$(2)\quad M_1 = 10.1488 + .7398\,C_1 - .1354\,C_1^2$$

$$(3)\quad H_1 = 9.2450 - 1.2202\,C_1$$

$$(4)\quad M_2 = 8.9767 + .9157\,C_2 - .1324\,C_2^2 + .2235\,C_1 - .0639\,H_1$$

$$(5)\quad H_2 = 8.4884 - .4255\,C_2$$

Equation (2) measures the response of milk in period 1 to concentrate fed along the "stomach line." Equation (3) measures the amount of hay required to "fill

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10/ The design might be improved for measuring carryover effects by including additional cows fed on the same ration throughout the trial. In this trial, 8 additional cows would have been required in addition to the 24 used.

the stomach" for various levels of concentrate fed in period 1. Equation (4) measures milk response in period 2 along the "stomach line," incorporating the possibility of a one-period carryover effect; that is, milk in period 2 is a function of feed fed in period 1 as well as in period 2. With this data, coefficients of the "carryover" variables are not statistically significant. However, they are retained in the equation to complete the illustration. In view of the above-mentioned data limitations, estimation of more realistic, complex relationships (e.g., including "cow ability," nonlinear stomach lines and nonlinear carryover effects, etc.) did not appear warranted.

Profit for the two periods is defined in (6). By substituting Equations

$$(6)\ \pi = P_{M_1} M_1 + P_{M_2} M_2 - P_{C_1} C_1 - P_{C_2} C_2 - P_{H_1} H_1 - P_{H_2} H_2 - K$$

(2)-(5) in profit equation (6), this equation can be solved in terms of C_1, C_2 and the prices. Thus, assuming Price Set 1 (Table 1) the profit equation can be rewritten as (7). Taking the partial derivatives of (7) with respect

$$(7)\ \pi = 47.5387 + 2.9955\ C_1 - .5416\ C_1^2 + 1.3010\ C_2 - .5296\ C_2^2.$$

to C_1 and C_2, setting them equal to zero and solving simultaneously provides optimum values of C_1 = 276#, C_2 = 123#. Substitution of these values in (3) and (5) provide H_1 = 587# and H_2 = 797#. However, if independence between periods is assumed (i.e., Equation 4 replaced by $M_2 = 9.1548 + .8127\ C_2 - .1266\ C_2^2$), the optimum feed quantities at the same prices differ as follows: C_1 = 165#, C_2 = 88#, H_1 = 723# and H_2 = 811#.

Optimum rations for other sets of prices using the "carryover" model are illustrated in Table 1. As expected under changing feed price ratios, the optimum ration shifts toward the relatively cheaper ingredient (compare Price Sets 1, 2, 3, Table 1). Also optimum feed inputs in period 1 depend on prices in period 2 and vice versa. For example, if the milk price in period 2 is expected to increase, other prices constant, higher concentrate rations are optimum in both periods 1 and 2 (compare Price Sets 1 and 4, Table 1). This feature has obvious application where seasonal milk (or feed) price variation is expected.

Table 1. Optimum feed combinations in periods 1 and 2 at various price relationships, as estimated from the "carryover" model.

Optimum feed quantities	Price of milk, concentrate and hay in dollars per cwt.			
	$P_{M_1}, P_{M_2} = \$4$ $P_{C_1}, P_{C_2} = \$3$ $P_{H_1}, P_{H_2} = \$1.50$ (Price Set 1)	$P_{M_1}, P_{M_2} = \$4$ $P_{C_1}, P_{C_2} = \$4$ $P_{H_1}, P_{H_2} = \$1$ (Price Set 2)	$P_{M_1}, P_{M_2} = \$4$ $P_{C_1}, P_{C_2} = \$2$ $P_{H_1}, P_{H_2} = \$2$ (Price Set 3)	$P_{M_1} = \$4, P_{M_2} = \$5$ $P_{C_1}, P_{C_2} = \$3$ $P_{H_1}, P_{H_2} = \$1.50$ (Price Set 4)
C_1	276#	128#	425#	304#
H_1	587#	768#	406#	552#
C_2	128#	8#	237#	167#
H_2	797#	845#	748#	778#

SUMMARY

The above example illustrates the necessity in incorporating carryover effects--where important--into economic analysis. More quantitative work is needed to determine the exact nature and importance of carryover in feeding dairy cows and other animals. The above analysis indicates that the change-over trial may provide one approach in "hooking together" monthly functions to determine economic optima for several periods simultaneously. Several interesting questions are raised by the analysis:

1. Can a "change-over" trial be used to estimate "carryover" where an entire monthly surface is estimated (from several levels of feeding as in Figure 2), rather than just a "stomach line"? For example, performance in period 2 probably depends on the level as well as combination of feeds in period 1. More "carryover" may result from switching levels of feeding the same ration than in switching rations.

2. Do carryover effects last more than a single period? If so, feed variables lagged more than one period would be required.

3. In more complex and realistic models, do the larger number of variables, coupled with relatively few monthly observations, cause estimation problems? For example, a measure of "cow ability," nonlinear lag relationships, etc. could lead to complex relationships.

These and other questions need to be examined. It is re-emphasized that particular optima derived above are only exploratory and illustrative. Another change-over trial at Davis this year, using hay and concentrate inputs in conventional form, should allow further exploration of carryover effects. Methods of analysis similar to those outlined above are anticipated.

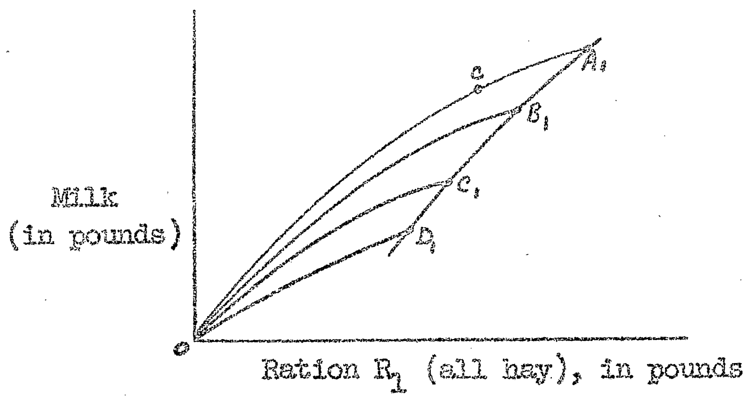

Figure 1. Hypothetical feed-milk relationships (single ration).

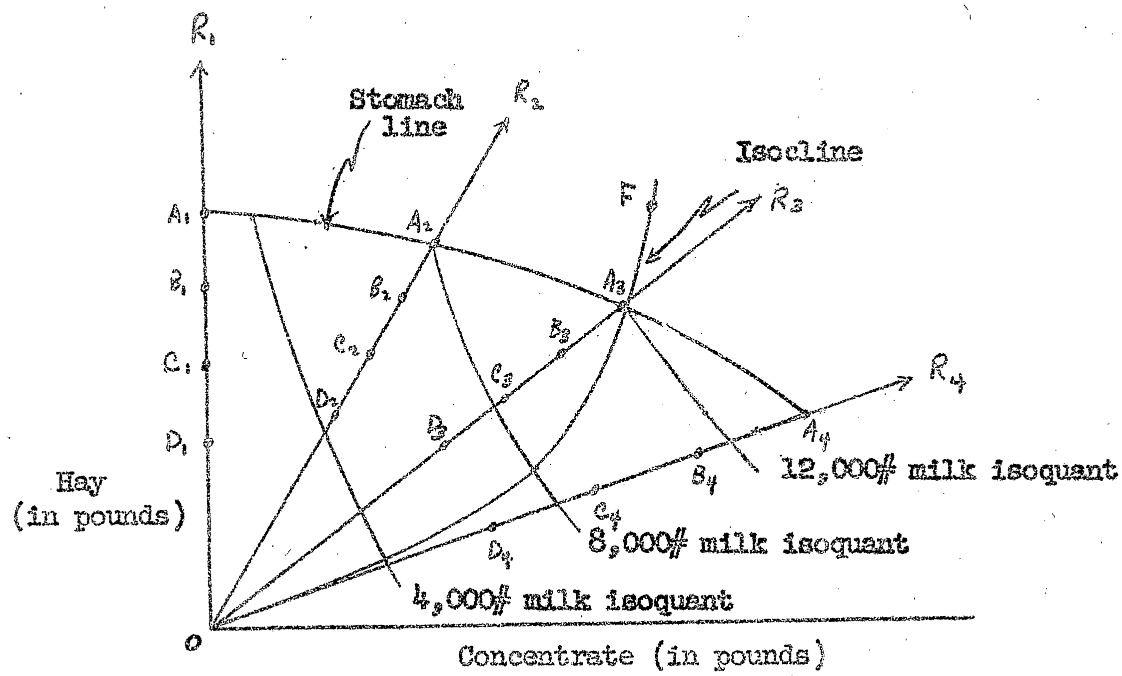

Figure 2. Hypothetical feed-milk relationship (production surface).

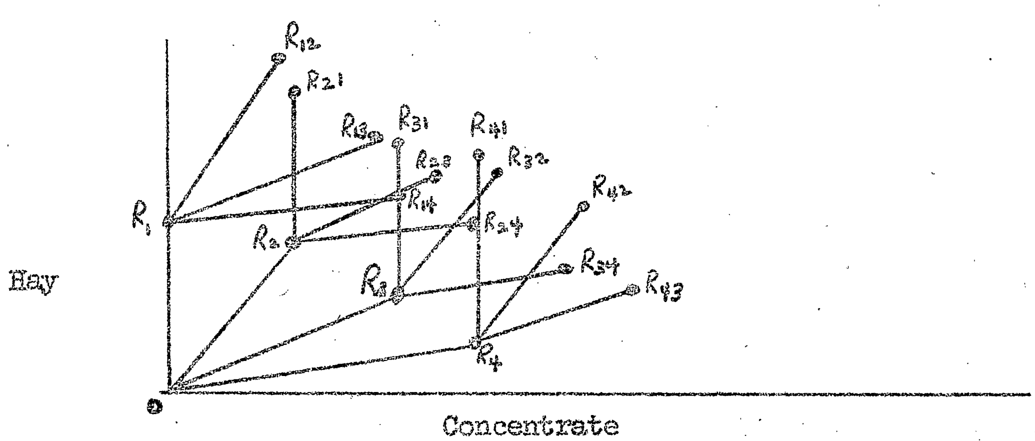

Figure 3. Diagram of treatment sequences for first two periods of change-over trial.