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Research Note

Max-min fuzzy programming approach for compromise farming: a case study

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Abstract Applying *max-min* fuzzy programming approach this paper shows how farmers can maximize income through re-allocation of land and other productive resources. In this approach, utilization of total cultivable land, achievement of the aspiration levels of production of seasonal crops and achievement of overall profit throughout the planning year, are fuzzily described. In model formulation and solution process the *max-min* fuzzy programming is used to qualify the 'compromise solutions' for which the highest degree of membership functions can be reached. A real case example is considered to illustrate the proposed method, and findings show considerable scope for land re-allocation to achieve the aspired levels of production of different crops. Achieved land re-allocation with existing resource constraints the farm profits almost get doubled.

Keywords Crop planning, Land utilization, Resource optimization, Fuzzy goal programming

JEL classification C02, C61

1 Introduction

In farm planning, the optimal production of crops depends on the efficient allocation of land and other productive resources. A number of mathematical programming approaches have been used to develop optimum crop plans since Heady (1954) demonstrated the use of linear programming for land allocation problems in agriculture (Nix 1979; Glen 1987).

Most planning problems involve multiple objectives. The conventional goal programming (GP) methods (Romero 2004) based on the satisficing philosophy are widely used to develop optimal crop plans (Wheeler & Russel 1977; Romero & Rehman 2003). However, a major weakness of this approach is that it ignores the aspiration levels. In real life, most decision-making problems are inexact in nature, and the conventional methods cannot provide solutions with inexact data. To overcome shortcomings of the conventional goal programming, the fuzzy programming (FP) approach (Bellman & Zadeh 1970) based on the concept of membership functions in fuzzy sets introduced by Zadeh (1965) is considered as an alternative to solve imprecisely defined multiobjective decision-making (MODM) problems. In the past few decades, this approach has been used extensively (Zimmermann 1978, 1987; Slowinski 1986; Mazoyer & Roudart 2006; Biswas & Pal 2005; Pal & Kumar 2014). However, the application of fuzzy programming approaches for multiobjective decision analysis of farm management problems in inexact environments is lacking.

This paper applies *max-min* FP (Zimmermann 1987) approach for modeling and solving agricultural planning problems to achieve aspiration levels of production of seasonal crops by allocating land properly and utilizing productive resources efficiently.

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2. Methodology

2.1 Problem formulation

In the model formulation, the fuzzily described objectives of the problem are characterized by their associated membership functions to measure optimality of crop production by utilizing the productive resources. In the solution process, the *max-min* FP is used to qualify the 'compromise solutions' for which the highest degree of membership functions can be reached.

Find $\boldsymbol{X}$ so as to satisfy

$$Z_k(\boldsymbol{X}) \gtrsim g_k, \quad k = 1, 2, \ldots k_1$$

$$Z_k(\boldsymbol{X}) \lesssim g_k, \quad k = k_1+1, k_1+2, \ldots, K$$

subject to

$$A(\mathbf{X})\begin{pmatrix} \geq \\ \leq \end{pmatrix} b, \quad X \geq 0 \qquad \ldots(1)$$

Where, g_k is the fuzzy aspiration level of achievement of the k-the objective function $Z_k(X)$. $A(X)$ is a function (linear or nonlinear) of a constrained coefficient set, b is a resource vector. The symbols $\lesssim$ and $\gtrsim$, respectively, denote the fuzzy version of $\leq$ and $\geq$, respectively.

Let t_{ek} and t_{uk} be the lower- and upper-tolerance limits, respectively, for achievement of the aspired level b_k of the *k-the* fuzzy goal. Then, the membership function, say $\mu_k(X)$, for the fuzzy goal $Z_k(X)$ can be characterized as follows (Zimmermann 1978):

For $\gtrsim$ type of restriction, $\mu_k(\boldsymbol{X})$ takes the form:

$$\mu_k(\mathbf{X}) = \begin{cases} 1, & \text{if } F_k(\mathbf{X}) \geq g_k \\ \dfrac{F_k(\mathbf{X}) - g_k + t_{\ell k}}{t_{\ell k}}, & \text{if } g_k - t_{\ell k} < F_k(\mathbf{X}) < g_k \\ 0, & \text{if } F_k(\mathbf{X}) \leq g_k - t_{\ell k} \end{cases}$$

$k = 1, 2, \ldots, k_1$ …(2)

Again, for $\lesssim$ type of restriction, $\mu_k(\boldsymbol{X})$ becomes

$$\mu_k(\mathbf{X}) = \begin{cases} 1, & \text{if } \mathrm{F_k}(\mathbf{X}) \leq g_k \\ \dfrac{g_k + t_{uk} - \mathrm{F_k}(\mathbf{X})}{t_{uk}}, & \text{if } g_k < \mathrm{F_k}(\mathbf{X}) \leq g_k + t_{uk} \\ 0, & \text{if } \mathrm{F_k}(\mathbf{X}) > g_k + t_{uk} \end{cases}$$

$k = k_1+1, k_1+2, \ldots, K$ …(3)

Following the fuzzy decision of Bellman and Zadeh (1970) together with the defined membership functions in (2) and (3), the problem of finding the optimal decision is to determine the optimal decision $\boldsymbol{X}^0$ as:

$$\mu_D(X^0) = \max_{X \geq 0} \min_{i=1,\cdots,K} \{\mu_k(X)\} \qquad \ldots(4)$$

Then, introducing a new variable λ that corresponds to (4), the final decision problem appears as:

Find $\boldsymbol{X}$ so as to:

Maximize λ

subject to

$$\lambda \leq \frac{F_k(X) - g_k + t_{\ell k}}{t_{\ell k}}, \; k = 1, 2, \ldots, k_1$$

$$\lambda \leq \frac{g_k + t_{uk} - F_k(X)}{t_{uk}}, \; k = k_1+1, k_1+2, \ldots, K$$

$$0 \leq \lambda \leq 1,$$

$$A(\mathbf{X})\begin{pmatrix} \geq \\ \leq \end{pmatrix} b, \quad X \geq 0 \qquad \ldots(5)$$

2.2 Farm planning problem

The decision variable and different parameters associated with the problem in the farming context are introduced as follows:

Decision variables

L_{cs} = Allocation of cultivable land for crop c in season *s*, *c = 1,2, ..., C; s = 1,2,...,S.*

Fuzzy parameters

LA_s = Total cropped area in season *s*.

P_c = Annual production level (in metric tons) of crop *c* per ha of land cultivated in season *s*.

TP = Estimated total profit in the planning year.

Crisp parameters

MH_s = Estimated total machine- hours (in hrs) required during the season *s*.

F_{fs} = The estimated total amount of fertilizer *f* (*f = 1,2,..., F*) required during the season *s*.

W_s = Estimated total amount of water consumed (in million cubic meters (MCM)) during season s.

BA = Estimated budget allocation in the plan period.

Crisp coefficients

MH_{cs} = Man hours (in hrs) required per hectare (ha) of land for cultivating crop c during season s.

F_{fcs} = Amount of fertilizer f required (in kilogram (kg)) per ha of land for cultivating crop c during season s.

W_{cs} = Estimated amount of water consumed (in million cubic meter (MCM)) per ha of land for cultivation of crop c during season s.

P_{cs} = Estimated production of crop c per ha of land cultivated during season s.

A_{cs} = The average cost for the purchase of seeds and different farming assisting resources per ha of land cultivated for crop c during season s.

MP_{cs} = The market price at the time of harvest, of crop c cultivated during season s.

2.3 Description of fuzzy goals and constraints

Farm management fuzzy goals

The following fuzzy goals are associated with the proposed model.

(i) Land utilization goal

The fuzzy goal for the total land utilization appears as

$$\sum_{c=1}^{C} L_{cs} \lesssim LA_s, \quad s = 1, 2, \ldots, S \qquad \ldots(6)$$

(ii) Production goal

The fuzzy production goal for each crop cultivated in all seasons appear as

$$\sum_{c=1}^{C} L_{cs}.P_{cs} \gtrsim P_s, \quad s = 1, 2, \ldots, S \qquad \ldots(7)$$

(iii) Profit achievement goal

A certain level of profit is expected on the horizon of cultivating crops. The fuzzy description of profit goal then appears as

$$\sum_{c=1}^{C} (MP_{cs}.P_{cs} - A_{cs})L_{cs}. \gtrsim TP \qquad \ldots(8)$$

Farm management constraints

(i) Water supply constraints

Water is an essential component for crop-yield, and the minimum level of water required is the water requirement constraint:

$$\sum_{c=1}^{C} L_{cs}.W_{cs} \geq W_s, \quad s = 1, 2, \ldots, S \qquad \ldots(9)$$

(ii) Fertilizer requirement constraints

Fertilizers are essential for plant growth and sustainable production system. Sustainable agricultural production seeks to optimize management and use of fertilizers. The fertilizer requirement constraint takes the form

$$\sum_{c=1}^{C} L_{cs}.F_{fcs} \geq F_{fs}, \quad s = 1, 2, \ldots, S \qquad \ldots(10)$$

(iii) Manpower requirement constraints

Minimum manpower is required for cultivation, and the manpower constraint appear as

$$\sum_{c=1}^{C} L_{cs}.MH_{cs} \geq MH_s, \quad s = 1, 2, \ldots, S \qquad \ldots(11)$$

(iv) Budget allocation constraints

An estimated amount of money at a minimum level is to be allocated for the purcahse of seeds, fertilizers and other productive inputs. The constraint appears as

$$\sum_{s=1}^{S} \sum_{c=1}^{C} A_{cs}.L_{cs} \geq BA \qquad \ldots(12)$$

3 A case study of Jalpaiguri district, West Bengal

The crop planning problem is illustrated using data from Jalpaiuri district in the Indian state of West Bengal. The main field crops cultivated in the district in all the three seasons are as follows:

Table 1. Data description

Goals	Aspiration level	Lower tolerance limit	Upper tolerance limit
1. Total cropped area ('000 ha)	268.028		351.74
2. Production ('000 t)			
Rice	390.2	372.4	
Maize	27.5	25.4	
Brinjal	192.4	181.7	
Pineapple	56.2	54.1	
Jute	550.2	518.8	
Wheat	42.4	39.2	
Rape-mustard	6.50	6.0	
Cauliflower	111.6	105.7	
Potato	955.6	905.9	
Tomato	135.4	128.9	
Cabbage	142.5	130.6	
3. Profit (in Rs Lakhs)	8825.5	8455.2	

Source: West Bengal State Marketing Board, www.wbagrimarketingboard.gov.in/fruits/jalpaiguri.htmlý Government of West Bengal, India. Action Plan for the year 2014–2015 and 2015–2016.
Office of the Principal Agricultural Officer, Department of Agri-irrigation, Office of the Executive Engineer Department of Agri-irrigation, Jalpaiguri, West Bengal, India.

Pre-kharif (s=1): Aus paddy, Maize, Brinjal, Pineapple

Kharif (s=2): Aman paddy, Jute

Rabi (s=3): Boro paddy, Wheat, Rape and mustard, Cauliflower, Potato, Tomato, Cabbage.

The data associated with different fuzzy goals, say uses of land, production achievement and profit achievement are given in table 1.

The data on different crisp coefficients for annual production (t/ha), market price(Rs/t), mandays (days/ha), water consumption (inch/ha), different types of fertilizers (N, P, K) in Kg/ha, Expenditure (Rs/ha) are described in table 2.

The annual resource requirements are given in table 3.

Construction of model goals

Following the expressions in (6) – (12) and using the data described in tables 1-3 the model goals are constructed below.

Land utilization goals

The land utilization fuzzy goals are given by

$$L_{11} + L_{21} + L_{31} + L_{41} \lesssim 268028 \qquad \ldots(13)$$

$$L_{31} + L_{41} + L_{12} + L_{52} \lesssim 268028 \qquad \ldots(14)$$

Table 2. Data description for crisp coefficients

Crops	Production (mt/ha)	Average market price (Rs/t)	Mandays (days/ha)	Water consumption (inch/ha)	Fertilizer (kg/ha)			Expenditure (Rs/ha)
					Nitrogen	Phosphate	Potash	
Aus paddy	2.82	9350	80	25	50	25	25	20200
Maize	2.32	8000	48	12	25	30	20	20500
Brinjal	16.30	4500	180	60	100	80	80	27750
Pineapple	15.90	18200	150	30	272	120	180	26200
Aman paddy	3.32	8800	90	12.5	50	72	30	21250
Boro paddy	3.39	9100	110	48	110	65	48	27300
Jute	2.11	10500	124	60	45	22	20	18500
Wheat	2.55	12000	75	13	105	45	50	20400
Rape & Mustard	0.77	25000	45	6.25	75	40	30	15400
Cauliflower	18.30	5500	125	18	42	73	25	39300
Potato	27.86	4500	120	20.25	150	135	170	60250
Tomato	13.60	12750	110	15	48	24	12	43500
Cabbage	22.90	2670	55	10	80	50	25	37550

Source: National Committee on Plasticulture Applications in Horticulture, Government of India www.ncpahindia.com
Notes: Most crops are seasonal, but few crops like brinjal and pineapple are grown throughout the year and in all the seasons. For these crops, mandays, water consumption and fertilizer are assumed to be shared in the ratio of 1:1:1, 2:2:1 and 2:1:1, respectively.

Table 3. Data description for requirement of productive resources

	Season		
	Pre-kharif	Kharif	Rabi
Required mandays ('000 days)	296.5	148.7	482.1
Water consumption ('000 inch)	80.7	25.0	28.0
Fertilizer (t)			
Nitrogen	177.1	172.0	263.2
Phosphate	94.9	96.1	157.9
Potash	96.8	94.1	118.1
Total requirement of cash ('000 Rs)		748545500	

$L_{31} + L_{41} + L_{13} + L_{63} + L_{73} + L_{83} + L_{93} + L_{10,3} + L_{11,3} \lesssim 268028$ …(15)

Following the expression in (3) the respective membership functions for land utilization in three different seasons are successively given by

$\mu_1 = (1/83712)(351740–(L_{11} + L_{21} + L_{31} + L_{41}))$ …(16)

$\mu_2 = (1/83712)(351740–(L_{31} + L_{41} + L_{12} + L_{52}))$ …(17)

$\mu_3 = (1/83712)(351740–(L_{31} + L_{41} + L_{13} + L_{63} + L_{73} + L_{83} + L_{93} + L_{10,3} + L_{11,3}))$ …(18)

Membership function for production achievement

$\mu_4 = (1/17830)(2820L_{11} + 3320L_{12} + 3390L_{13} – 372400)$ (Rice) …(19)

$\mu_5 = (1/2100)(2320L_{21} – 15440)$ (Maize) …(20)

$\mu_6 = (1/10700)(16300L_{31} – 181700)$ (Brinjal) …(21)

$\mu_7 = (1/2120)(15900L_{41} – 54130)$ (Pineapple) …(22)

$\mu_8 = (1/31440)(2110L_{52} – 218800)$ (Jute) …(23)

$\mu_9 = (1/3200)(2550L_{63} – 49200)$ (Wheat) …(24)

$\mu_{10} = (1/500)(770L_{73} – 6500)$ (Rape- mustard) …(25)

$\mu_{11} = (1/5960)(18300L_{83} – 105690)$ (Cauliflower) …(26)

$\mu_{12} = (1/49720)(27864L_{93} – 905880)$ (Potato) …(27)

$\mu_{13} = (1/8550)(13600L_{10,3} – 126890)$ (Tomato) …(28)

$\mu_{14} = (1/12050)(22900L_{11,3} – 130500)$ (Cabbage) …(29)

Membership function for profit achievement

$\mu_{15} = (1/370300000)(6167L_{11} + 5940L_{21} + 14560L_{31} + 24058L_{41} + 7966L_{12} + 12549L_{52} + 5655L_{13} + 9200L_{63} + 6750L_{73} + 21350L_{83} + 35138L_{93} + 17593L_{10,3} + 21100L_{11,3} – 8455200000)$ …(30)

Resource constraints

(i) Manpower requirement constraints

As brinjal, pineapple are annual crop, the manpower requirement the three consecutive seasons is expressed as

$(80L_{11} + 48L_{11} + 60L_{31} + 50L_{41}) \geq 296552$ (Pre-kharif) …(31)

$(60L_{31} + 50L_{41} + 90L_{12} + 110L_{52}) \geq 148760$ (Kharif) …(32)

$(60L_{31} + 50L_{41} + 110L_{13} + 124L_{63} + 45L_{73} + 65L_{83} + 120L_{93} + 110L_{10,3} + 55L_{11,3}) \geq 482100$ (Rabi) …(33)

(ii) Water consumption constraints

$(25L_{11} + 12L_{11} + 20L_{31} + 10L_{41}) \geq 80730$ (Pre-kharif) …(34)

$(20L_{31} + 10L_{41} + 12.5L_{12} + 48L_{52}) \geq 25010$ (Kharif) …(35)

$(20L_{31} + 10L_{41} + 60L_{13} + 13L_{63} + 6.25L_{73} + 18L_{83} + 20.25L_{93} + 15L_{10,3} + 10L_{11,3}) \geq 28000$ (Rabi) …(36)

(iii) Fertilizer requirement constraints

- Nitrogen

$(50L_{11} + 25L_{11} + 50L_{31} + 136L_{41}) \geq 1771000$ (Pre-kharif) …(37)

$(25L_{31} + 68L_{41} + 50L_{12} + 110L_{52}) \geq 1720000$ (Kharif) …(38)

$(25L_{31} + 68L_{41} + 45L_{13} + 105L_{63} + 75L_{73} + 42L_{83} + 150L_{93} + 48L_{10,3} + 80L_{11,3}) \geq 2631500$ (Rabi) …(39)

- Phosphate

$(25L_{11} + 30L_{11} + 40L_{31} + 60L_{41}) \geq 948850$ (Pre-kharif) …(40)

$(20L_{31} + 30L_{41} + 72L_{12} + 65L_{52}) \geq 960500$ (Kharif) …(41)

$(20L_{31} + 30L_{41} + 22L_{13} + 45L_{63} + 40L_{73} + 73L_{83} + 135L_{93} + 24L_{10,3} + 50L_{11,3}) \geq 1578600$ (Rabi) …(42)

- Potash requirement

$(25L_{11} + 20L_{11} + 40L_{31} + 90L_{41}) \geq 968500$ (Pre-kharif) …(43)

$(20L_{31} + 45L_{41} + 30L_{12} + 48L_{52}) \geq 940800$ (Kharif) …(44)

$(20L_{31} + 45L_{41} + 20L_{13} + 50L_{63} + 30L_{73} + 25L_{83} + 170L_{93} + 12L_{10,3} + 25L_{11,3}) \geq 11810000$ (Rabi) …(45)

(iv) Bubget allocation constraint

$20200L_{11} + 20500L_{21} + 27750L_{31} + 26200L_{41} + 21250L_{12} + 27300L_{52} + 18500L_{13} + 20400L_{63} + 15400L_{73} + 39300L_{83} + 60250L_{93} + 43500L_{10,3} + 37550L_{11,3} \geq 748545500000$ …(46)

Now, the general structure of the executable model can be presented as

Find

$X\,(L_{11} + L_{21} + L_{31} + L_{41} + L_{12} + L_{52} + L_{13} + L_{63} + L_{73} + L_{83} + L_{93} + L_{10,3} + L_{11,3})$

so as to:

Maximize λ

and satisfy

$\lambda \leq \mu_i$, i = 1, 2, ..., 15

$0 \leq \lambda \leq 1$,

subject to the system constraints in (31)-(46) and $\boldsymbol{X} \geq 0$

The software LINGO (ver. 12.0) is used to solve the problem. The resultant solution is given by

$L_{11} = 85.2$, $L_{21} = 10.2$, $L_{31} = 22.4$, $L_{41} = 14.6$, $L_{12} = 97.2$, $L_{52} = 71.9$, $L_{13} = 30.6$, $L_{63} = 18.2$, $L_{73} = 4.7$, $L_{83} = 35.2$, $L_{93} = 73.1$, $L_{10,3} = 24.0$, $L_{11,3} = 6.3$.

The resultant model solution together with the existing cropping plan (2014-2015) is displayed in the table 4.

Table 4. Land allocation and production achievement under the proposed model and existing cropping plan (2014-2015)

Crops	Proposed plan		Existing plan	
	Land ('000ha)	Production ('000 t)	Land ('000ha)	Production ('000 t)
Rice	213.0	662.4	123.9	390.2
Maize	10.2	23.7	11.8	27.5
Brinjal	22.4	365.1	11.8	192.4
Pineapple	14.2	225.8	3.5	56.2
Jute	71.9	151.7	267.7	550.2
Wheat	18.2	46.4	16.6	42.4
Rape- mustard	4.7	3.6	8.4	6.5
Cauliflower	35.2	644.2	6.1	111.7
Potato	73.1	2036.6	34.7	955.6
Tomato	24.0	326.4	9.9	135.4
Cabbage	6.3	144.3	6.2	142.6
Profit	Rs. 11726 Lakhs		Rs. 6924 Lakhs	

In the proposed approach, rice gains substantially in terms of both area as well as production. The other crops that stand to gain include brinjal, pineapple, cauliflower, potato and tomato. Jute loses its area significantly, and most of it goes to rice. Rapeseed and mustard also lose its area.

The estimated profit with the proposed land re-allocation (Rs. 11726 lakhs) is almost double of the profit under the existing cropping plan (Rs. 6924 Lakhs). These findings clearly show considerable scope for land re-allocation to achieve the aspired levels of the production goals.

6 Conclusions and implications

The *max-min fuzzy programming* method discussed in this paper for optimal crop plans with existing resources is a better approach to analyzing various farm management activities in an uncertain decision environment. The flexible nature of the proposed approach allows for incorporation of several parameters (precise or inexact) as well as region-specfic environmental constraints without much computational difficulty.

The approach can be extended to crop production planning problems with fuzzy description on the bounds of interval data of the model parameters for further relaxation on resource utilization. The method can be extended to land use planning for rain-fed agriculture to improving the environmental sustainability and water management.

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