ENVIRONMENTAL VALUES AND THE OPTIMUM ROTATION PROBLEM IN FORESTRY ECONOMICS *

Dung Nguyen

Department of Economics
University of Maryland
College Park, Maryland 20742

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ABSTRACT

The model shows that, even for a forest that provides environmental services, it is always optimal to harvest, and the optimum harvesting age must be somewhere on the rising portion of the growth curve. Furthermore, an analysis based on the economist's criteria would not necessarily result in a much shorter cutting age compared to the forester's optimum.

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1. Introduction

To economists, the most interesting aspect of forest management seems to lie in the determination of the rotation length for a forest. Such a determination is of special concern to economists mainly because it can be formulated as a standard investment decision problem in capital theory. (See, for instance, Goundrey, and Hirshleifer). It is of course well-known that several criteria are available in forming an objective function for the optimum rotation problem. (For example, see Bentley and Teeguarden). Different criteria would likely lead to competing results on the rotation length for a forest. In particular, Samuelson shows that if a positive rate of discounting is used, the optimal rotation period of a forest determined by the economist's criteria is significantly shorter than that specified by the forester's maximum sustained (net) yield. Only if the interest rate is approaching zero, will the two criteria give the identical rotation age at which the net sustained yield is maximized. However, as in many conventional models, Samuelson's paper does not formally include in its analysis "environmental services" such as flood control, recreation, etc., which a standing forest may provide in addition to timber.

The growing public concern over the excessive exploitation of forests seems to give rise to the idea that one must take into account the intangible values that a forest offers in preserving environment. Recently, Hartman demonstrates that the harvesting of a forest may be delayed if environmental values are taken into consideration. He also states that (p.55) "...it may well be that the optimal age for harvesting a forest involves a negative rate of growth (of the forest)...", and "...(there are) situations in which the recreational values are sufficiently great that it is optimal never to harvest." (Italics added). The conclusion just cited is, in many respects, in contrast with that from the more conventional models where only the positively-sloped range of the growth curve is of relevance for an economic analysis.

The present paper attempts to reformulate the harvesting decision problem with certain modifications to be fully discussed later. Briefly, it may be stated that while all previous studies in the literature have assumed a simultaneous harvesting of all trees of a forest (i.e., a clear-cutting system is employed), an alternative technique, say, the selective-logging method, will be assumed in the present model. This modification in the assumption concerning cutting technique would lead to a rather simple way to introduce environmental considerations into the model: There exists a certain minimum level of the timber stock which must be maintained if environmental services were adequately provided.

The model incorporating the preceding features, as will be shown, would produce the standard result that the optimal harvesting age should be somewhere on the rising portion of the growth curve (i.e., at a positive growth rate), even when environmental values are included in the analysis. Moreover, in the same framework, the present discussion will suggest that an economic analysis based on present-value calculations would not necessarily yield a much shorter optimal harvesting age compared to that recommended by foresters.

2. Basic Assumptions

As already mentioned above, it has been assumed in the literature on this subject that all trees of a forest must be simultaneously harvested at the end of each harvesting period. Further, it has also been implicitly assumed that all trees belonging to a forest have a certain common age, or using a forestry economics term, an even-aged forest has been considered in previous studies.

While, on theoretical grounds, no objection is made here against these assumptions, we feel that clear-cutting is not a usual practice in forest management, especially for forests which, aside from timber-supplying function, provide valuable environmental services. Also, virgin forests, which are logically associated with environment-conserving functions, in their natural forms, do not seem to follow the even-aged distribution pattern. On the contrary, these forests appear to conform with the so-called uneven-aged distribution wherein trees growing in a forest belong to different age groups.

Thus, in the following discussion, it will be assumed that the forest under question is of an uneven-aged type and that the selective-logging system would be the relevant cutting method to be employed. Since this cutting method always leaves a certain portion of a forest unharvested, its application would give rise to the possibility that the environmental function of the forest under question can be assured if a pre-determined minimum level of timber stock is maintained throughout the planning horizon. Let G(t) be the growth function of a representative forest over time t, and $\underline{G}$ be that certain minimum level of G(t) for necessary environmental services. It is assumed that $0 \leqq \underline{G} < G_h$ where G_h denotes the highest level on the growth curve G(t). The possibility that $\underline{G} \geqq G_h$ is ruled out for reasons to be discussed in Section 3.

There are a number of comments about $\underline{G}$ we would like to make. Although the determination of $\underline{G}$ may involve several complicated considerations, it seems that a study which takes into account relevant biological, ecological, economic, and environmental aspects would roughly give an appropriate level for $\underline{G}$. Note also that the trade-off between timber harvested and environmental values is implied in our theoretical framework. The higher the level of minimum timber stock required, the lesser would be the number of trees left for possible exploitation.

3. The Model Reformulated

Let us first consider the case involving a one-harvest planning horizon. The maximization problem can be stated as:

(1) $\quad \text{Max.} X(t) = e^{-rt}[G(t)-\underline{G}]$

where X(t) is the present value of $[G(t)-\underline{G}]$ and r is the discount rate used. The first-order and second-order conditions give, respectively:

(2) $$\dot{G}(t) = r[G(t)-\underline{G}]$$

(3) $$\ddot{G}(t) < r\dot{G}(t)$$

where dotted variables denote time derivatives.

The economic interpretation of the condition expressed in (2) is rather straight-forward and it should not bother us here. However, one feature in (2) is worth noting. For a forest whose environmental value is negligible, or in terms of this model specification, for $\underline{G} = 0$, equation (2) reduces to:

(4) $$\dot{G}(t) = rG(t)$$

which is a maximization condition for conventional models. A close examination on condition (2) may reveal the fact that the presence of environmental services would cause a delay in harvesting a forest. Conditions (2) and (4) are graphically shown in Figure 1 where it can be seen that $t^* > t^{**}$ with t^* obtained from (2) and t^{**} from (4).

It can also be seen that the higher the level of $\underline{G}$, the longer will be the forest's optimum harvesting age. Thus, in terms of Figure 1, one can conceivably imagine a situation in which the level of $\underline{G}$, where $0 \leq \underline{G} < G_h$, is sufficiently high to yield any specific level of t^*, given that $t^{**} \leq t^* < t_h$. This would eventually imply that, by taking into account environmental values, it does <u>not</u> necessarily follow that the economist's optimal rotation age, t^*, is significantly shorter than the forester's optimal rotation age, t_h.

One may naturally raise the question of the effect on (2) when $[G(t)-\underline{G}]=0$. The answer appears to be that $\dot{G}(t)$ would be equal to zero and this in turn means that the highest growth level G_h has been reached. The corresponding forest age is t_h. In this situation, then, we have $G(t) = G_h = \underline{G}$ which violates one of the assumptions stating that $\underline{G}$ should be strictly smaller than G_h. Some justification for this assumption is now in order. Let us suppose that $\underline{G} = G_h = G(t)$, then it is obvious that there is none to be harvested at time t_h since the value of the harvest is $[G(t)-\underline{G}]$ which is zero. This is not the end of the story though, since the forest stock does not remain at the same level but declines as time goes by. So the forest in question will never be able to provide adequately the required environmental services since thereafter, G(t) can never reach $\underline{G}$ which is in fact G_h. The same is also true in the case where $\underline{G} > G_h$. What policy makers may do under these circumstances is to improve the growth performance of the forest by appropriate silvicultural means as well as by biological methods so that G_h must be shifted to a higher level than $\underline{G}$. Thereafter, the optimum rotation may be determined by the conditions expressed in (2) and (3).

Thus, in short, our assumption requires that $G(t) > \underline{G}$ and this in turn, through condition (2), implies that $\dot{G}(t^*) > 0$, or that the optimum rotation should be somewhere on the rising portion of the G(t) curve.

The analysis is now extended to the case involving an indefinite-sequence-of-harvest planning horizon where it will be assumed that a partial harvest of the forest would not change the future growth performance. This assumption is demonstrated in Figure 2 where $\underline{t}$ is the forest age corresponding to $\underline{G}$, and t_i^*; i = 1,2,3,... denotes the optimum <u>year</u> to carry out the <u>i</u>th harvest. Due to this assumption, it should be clear that $t_1^*-\underline{t} = t_2^*-t_1^* = t_3^*-t_2^* = \ldots$, and that the value of $[G(t^*)-\underline{G}]$ is the same for all t_i^*'s.

The preceding notation requires some clarifications. In order to find a convenient expression for future growths, it may be thought that in year t_1^*, the age of the "next" forest drops to $\underline{t}$ after the first selective-cutting harvest is carried out, and thereafter the new growth level is a function of its "new age". Specifically, in <u>year</u> t_2^*, the <u>age</u> of the second growth is t_1^*. By a similar argument, it can be seen that all growths of future generations have the common harvesting age, namely, t_1^*. The value of t_1^* is determined by maximizing the following objective function:

(5) $$\text{Max. } Y(t_1) = [G(t_1)-\underline{G}][e^{-rt_1} + e^{-rt_2} + e^{-rt_3} + \ldots]$$

Noting that $t_2=2t_1-\underline{t}$, $t_3=3t_1-2\underline{t},\ldots$, and to simplify notation, let us denote the common harvesting age as t instead of t_1, then equation (5) becomes:

(6) $$\text{Max. } Y(t) = [G(t)-\underline{G}]\, e^{-rt}[1 + e^{-r(t-\underline{t})} + e^{-2r(t-\underline{t})} + \ldots]$$
$$= e^{-rt}[G(t)-\underline{G}] \,/\, [1-e^{-r(t-\underline{t})}]$$

The first-order and second-order conditions are, respectively:

(7) $$\dot{G}(t) = r[G(t)-\underline{G}] \,/\, [1-e^{-r(t-\underline{t})}]$$

(8) $$[1-e^{-r(t-\underline{t})}][\ddot{G}(t)-r\dot{G}(t)] < 0$$

Much of the earlier discussion concerning the assumption that $G(t) > \underline{G}$ is relevant here. Besides, this assumption implies that $(t-\underline{t}) > 0$. Thus, for $(t-\underline{t}) > 0$, we have: $0 < [1-e^{-r(t-\underline{t})}] \leqq 1$, and hence, the relation (8) becomes:

(3a) $$\ddot{G}(t) < r\dot{G}(t)\ .$$

This is obviously identical with the condition expressed in eq.(3) above.

It can be seen that the model shown in equation (6) is a generalization of the rotation problem, and the conditions in (7) and (8) would give a generalization of the well-known Faustmann solution. The first-order maximization condition for the conventional models can be derived from equation (7) by letting $\underline{G}$ (and in effect, $\underline{t}$) be zero, or:

(9) $$\dot{G}(t) = rG(t) \,/\, (1-e^{-rt})$$

The comparison between equations (7) and (2) shows unambiguously that if the planning horizon runs through an indefinite sequence of rotations, the length of the optimum rotation would likely be reduced compared to that from the one-rotation model. Back in Figure 1, one can see that the curve representing the R.H.S. of equation (7) must lie somewhere above the $r[G(t)-\underline{G}]$ curve since $r/[1-e^{-r(t-\underline{t})}]$ must be greater than or equal to r. Thus, the former would cut the curve $\dot{G}(t)$ somewhere at the left of t*. In other words, the rotation length is cut short.

On the other hand, the comparison between (7) and (9) does not generally give a clear-cut answer as to whether the impact of variable $\underline{G}$ is to prolong or to shorten the rotation period that the more traditional models would indicate. However, it can be shown that if the level of $\underline{G}$ is sufficiently high, then the optimal rotation age obtained from condition (7) would be higher than that from (9). Thus, again, in this situation, economic analysis would not necessarily result in a shortening of the optimal harvesting age compared to the forester's rotation age.

At this stage, it is necessary to make an important distinction which has been intentionally neglected so far. It was suggested above that the common optimum harvesting <u>age</u> for <u>all</u> growths is t_1^* which is determined from the maximization problem (5). However, the optimal cutting length, i.e., the period from one (selective-logging) harvest of a forest to the next harvest of that forest, is represented by $t_1^*-\underline{t} = t_2^*-t_1^* = \ldots$ in Figure 2. Only when $\underline{t} = 0$, does the harvesting age coincide with the cutting period. This distinction was not made earlier since, in our notation, the assumption that $\underline{t} = 0$ is implied in previous studies on the subject, Hence, the terms "harvesting age" and "cutting period, or length" can be interchangeably used.

Such a distinction may be very crucial in judging the controversy over the optimal rotation problem, specifically between economists on the one side and foresters on the other. Foresters may have been proposing too long an optimal cutting age of a forest, but they may not have had any objections to a much shorter cutting period suggested by economists. For example, both economists and foresters may be satisfied with an optimal solution such as that with $t_1^* = 80$ years, and $t_1^* - \underline{t} = 20$ years.

Before concluding this section, it should be re-emphasized that, through condition (7) and the assumption $G(t) > \underline{G}$, we have $\dot{G}(t) > 0$, i.e., the optimal harvesting age should lie on the positively-sloped portion of the growth curve.

4. A Variant of the Model

The model discussed in the preceding section assumes that the growth performance is unchanged with respect to a partial harvest of the forest. This assumption may not generally hold in case a selective-logging system is used. It is a well-known fact that the forest considerably improves its growth performance after a selective-cutting program is undertaken for the first harvest. In this section, therefore, it will be assumed that after the first harvest, the growth function is shifted upward, and thereafter it remains the same. With a constant price for timber, this new assumption would give a scenario such as the one hypothetically shown in Figure 3.

Since there are now two time decision variables in the problem, some modifications in the notation will be helpful. Let the harvesting age of the first growth be denoted as t and its growth function be $G^1(t)$. Furthermore, let the common harvesting age of the second and all subsequent growths be denoted as x and their identical growth function be $G^2(x)$. It should be noted that, as a result of the assumption concerning the growth pattern as depicted in Figure 3, we have: $x = t_2 - t + \underline{t}$

The objective function is:

$$(10) \quad \text{Max. } Z(t,x) = [G^1(t)-\underline{G}]e^{-rt} + [G^2(x)-\underline{G}](e^{-rt_2} + e^{-rt_3} + \ldots)$$

$$= [G^1(t)-\underline{G}]e^{-rt} + [G^2(x)-\underline{G}]e^{-rt_2}\,[1+e^{-r(t_2-t)}+e^{-2r(t_2-t)}+\ldots]$$

since $t_2 - t = t_3 - t_2 = t_4 - t_3 = \ldots$, or:

(11) $$\text{Max. } Z(t,x) = [G^1(t)-\underline{G}]e^{-rt} + [e^{-rt_2}(G^2(x)-\underline{G})]/\ [1-e^{-r(t_2-t)}]$$

The expression in (11) is a familiar unconstrained maximization problem with two choice variables. The first-order conditions give:

(12) $$G^1_t(t) = r\ \{\ [G^1(t)-\underline{G}] + e^{-r(t_2-t)} \cdot \frac{[G^2(x)-\underline{G}]}{1-e^{-r(t_2-t)}}\ \} \quad ,$$

$$G^2_x(x) = r[G^2(x)-\underline{G}]\ /\ [1-e^{-r(t_2-t)}]$$

where subscripts denote the usual partial derivatives. The values of t and x can be obtained by solving these two equations simultaneously.

The second-order conditions may be expressed as:

(13) $$Z_{tt} = e^{-rt}[G^1_{tt}(t) - rG^1_t(t)] < 0 \quad ,$$

$$Z_{xx} = [1-e^{-r(t_2-t)}]\ [G^2_{xx}(x) - rG^2_x(x)] < 0 \quad , \text{ and}$$

(14) $$Z_{tx} = Z_{xt} = 0 < Z_{tt} \cdot Z_{xx} \quad .$$

Since both (e^{-rt}) and $[1-e^{-r(t_2-t)}]$ are positive, conditions in (13) are simply:

(15) $$G^1_{tt}(t) < rG^1_t(t) \quad ; \quad \text{and} \quad G^2_{xx}(x) < rG^2_x(x)$$

which are similar with those in (3) and (3a).

There is little to be said of these conditions as they are already treated in Section 3. It may be of interest to note, however, that if $G^1(t)$ and $G^2(x)$ are identical, i.e., if there is no improvement in the growth performance of the forest after the first selective-logging harvest, (hence x=t, and $t_2-t = t-\underline{t}$), then conditions in (12) would reduce to that in (7).

5. Concluding Remarks

Among the conclusions emerging from the preceding analysis are the following:

(i) For the one-rotation planning horizon case, the presence of environmental services would cause a delay in the harvesting of a forest. In the indefinite-rotation planning horizon case, the impact of these services appears to be ambiguous. However, if environmental values are sufficiently high, the reformulated model would give a longer optimum harvesting age than that derived from the more conventional models.

(ii) Perhaps with the exception of a negligible number of instances, it is always optimal to harvest a forest, and more importantly, the optimum harvesting age must lie somewhere on the rising portion of the growth curve.

Conclusion (ii) re-establishes the result of conventional analyses that only the positively-sloped range of the growth curve is of relevance for an economic study.

On the other hand, conclusion (i) implies that, in the presence of environmental services, an economic analysis would not necessarily result in a much shorter harvesting age compared to the forester's optimum. In this connection, it seems that the distinction between the cutting age and the harvesting period for a forest may be a crucial factor which must be considered in the context of the optimal rotation debate. More specifically, it is suggested that a longer cutting age accompanied by a shorter harvesting period may well satisfy both the forester's and the economist's criteria.

REFERENCES

Bentley, W. and D. Teeguarden, 1965, "Financial Maturity: A Theoretical Review," Forest Science, 11:76-87.

Goundrey, G.K., 1960, "Forest Management and the Theory of Capital," Canadian Journal of Economics, 26:439-451.

Hartman, R. , 1976, "The Harvesting Decision when a Standing Forest Has Value," Economic Inquiry, 14:52-58.

Hirshleifer, J., 1970, Investment, Interest, and Capital, Prentice-Hall, Inc., Englewood Cliffs.

Samuelson, P.A., 1976, "Economics of Forestry in an Evolving Society," Economic Inquiry, 14:466-492.

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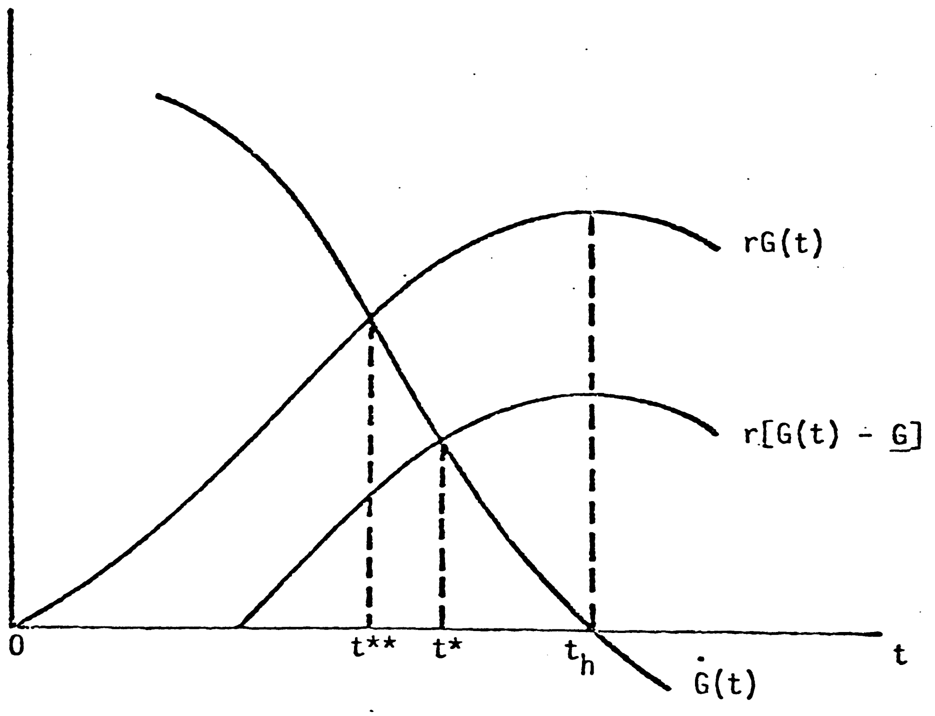

Figure 1

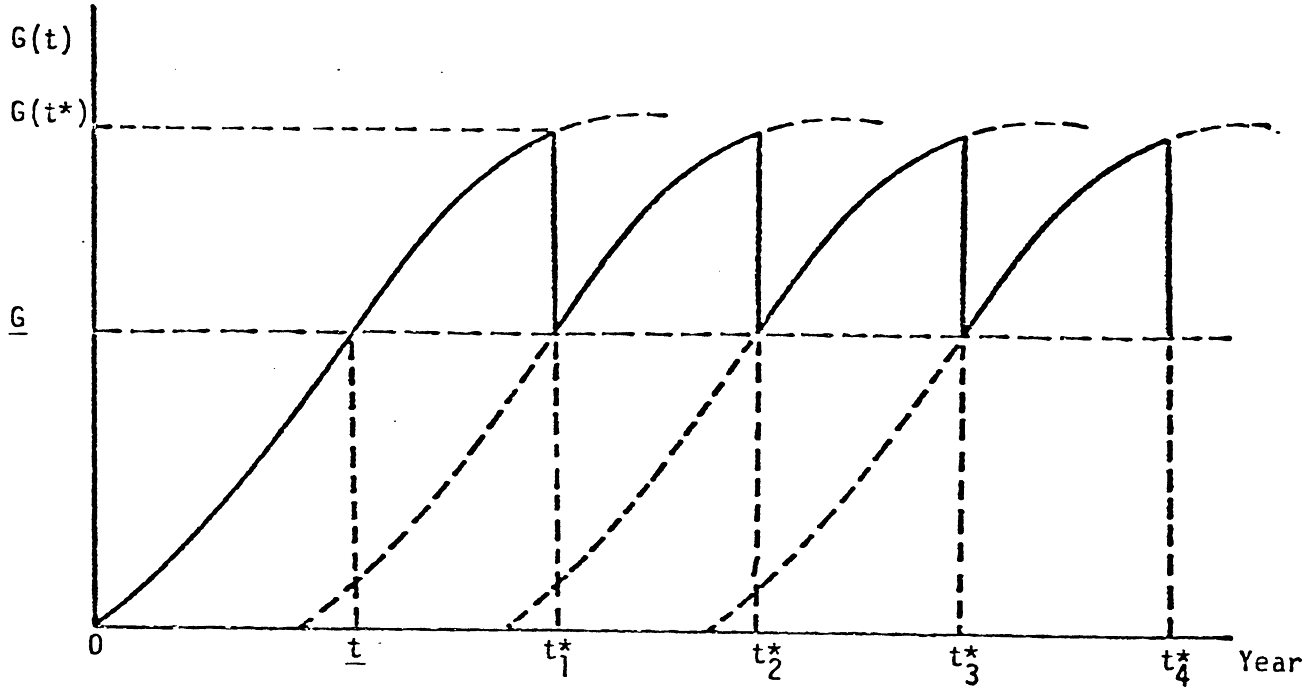

Figure 2

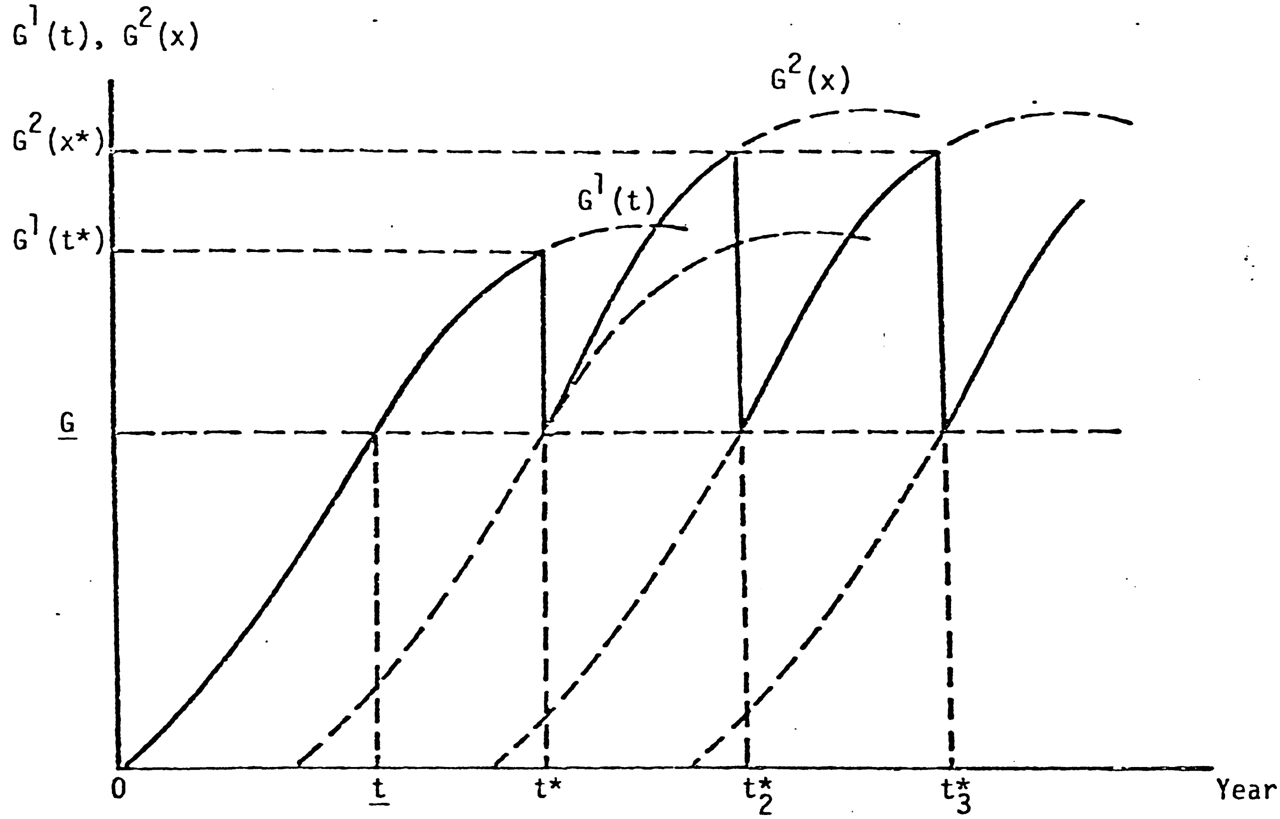

Figure 3