THE MEASUREMENT OF STRUCTURAL VARIATION OVER TIME: THE CASE FOR ADVERTISING AND FCOJ DEMAND

ABSTRACT

A distributed lagged advertising model with coefficients having random and systematic adjustments was estimated. The paths of parameter adjustments clearly identify structural changes in advertising effectiveness and price responsiveness. The model is used to show improvements over fixed models for forecasting.

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THE MEASUREMENT OF STRUCTURAL VARIATION OVER TIME: THE CASE FOR ADVERTISING AND FCOJ DEMAND

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Forecasting and/or explanatory models are normally based on a regression function where the $\hat{\beta}_j$ represent either OLS estimates of structural parameters; OLS estimates of reduced-form coefficients; or reduced-form estimates derived from statistically estimated structural relationships. Assuming that the $\hat{\beta}_j$'s were estimated using time series data for observation periods 1 through n, a common forecasting problem is that for some period, n + s, $\hat{Y}_{n+s}$ tends to deviate from the actual or observed value of Y during period n + s. The problems above may be caused by a variety of factors including model misspecification, errors in the projected values for the independent variables (X_j's), and parameter change between period n and the forecast period n + s. This paper focuses on the problem of developing and estimating the effectiveness of advertising frozen concentrated orange juice (FCOJ) when the demand parameters may change over time.

Varying Parameter Regression Model

The parameter variation pattern assumed in the advertising response model of this study parallels that assumed by Cooley and Prescott (1973a, b, c).

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Parameter variation may come from a wide variety of sources and any hypothesized pattern of parameter variation must be sufficiently general to accommodate several possible sources. The Cooley-Prescott model is fairly general in that it assumes that the parameters of the model $Y_t = x_t \beta_t$ are adaptive in nature and are subjected to both permanent and transitory changes, where x_t is a 1 x (K+1) vector. The transitory changes are temporary shocks whose effects do not presist over time. Permanent changes reflect changes in behavioral, technological, and institutional aspects of the economic phenomenon being studied and are more likely to persist over time and be generated in a systematic pattern. An important feature of the model is that it picks up structural "drifts" as opposed to uniformly constrained shifts. Assume that the parameter vector β_t is subjected to:

(a) transitory changes $\beta_t = \beta_t^p + \mu_t$

and (b) permanent changes $\beta_t^p = \beta_{t-1}^p + v_t$

where μ_t and v_t are identically and independently distributed multivariate normal vector variables with zero mean vectors and covariance matrices Σ_μ and Σ_v. A particular covariance structure assumed by Cooley and Prescott is $Cov(\mu_t) = (1 - \gamma)\sigma^2 \Sigma_\mu$ and $Cov(v_t) = \gamma \sigma^2 \Sigma_v$ where Σ_μ and Σ_v are assumed to be known up to scale factors which implies one element of both Σ_μ and Σ_v can be normalized to unity. It is convenient to assume that the first elements are one ($\sigma_{11\mu} = \sigma_{11v} = 1$) when the intercept is subjected to the above pattern of variation.

This pattern of parameter variation is sufficiently general to accommodate a wide variety of causes. The proportions γ and $(1 - \gamma)$ of the total parameter variation can respectively be attributed to permanent and transitory changes. The parameter γ represents the speed of parameter adaption to structural changes in the phenomenon being studied. The larger (smaller) value of γ implies that the sources of parameter variation are more (less) of a permanent nature. Changing elements of Σ_μ and Σ_v imply varying rates of changes for the various parameters and different degrees of permanency of changes. Parameters are estimated using maximum likelihood procedures and properties of the estimators are discussed in Cooley and Prescott (1973a, b, c) and Cooley (1971).

Advertising Model

In this section a model of advertising effectiveness will be developed for which the varying parameter model is appropriate. Ward's [1976] advertising model shows that current consumption of frozen concentrated orange juice is related to the price, seasonality, and a distributed lag specification of advertising expenditures. The final model was specified in first differences to compensate for serial correlation problems as illustrated below:

$$(1)\quad \dot{q}_t = \alpha_0 + \alpha_1 \dot{p}_t + \alpha_2 S + \sum_{j=0}^{N} \lambda_{j+1} \dot{a}_{t-j} + \varepsilon_t$$

where: q_t = per capita consumption of frozen concentrated orange juice (gallons of single strength equivalent),

p_t = average quarterly price of FCOJ (\$/gal. deflated by CPI: 1967 = 100),

a_t = advertising expenditures (mil. \$),

s = quarterly seasonal dummy,

t = a series of consecutive numbers beginning with t = 2 in 3rd quarter 1967 through t = 35 in 4th quarter 1975,

$\dot{q}_t = q_t - q_{t-1}$,

$\dot{p}_t = p_t - p_{t-1}$,

$\dot{a}_t = a_t - a_{t-1}$.

This model obviously allows systematic adjustments in the constant, as is evident with α_2. The lagged effect was estimated using a polynomial approximation of different degrees [Ward, 1976]. The empirical results indicated that a first degree polynomial with up to four lags is an acceptable specification of the model.

The above model was initially estimated with advertising data up to 1973 and the specification restricted the parameters to remain fixed over the sample period. In contrast, advertising is designed to influence the consumers preference function and there is sufficient reason to suspect that the model specification is unduely restrictive, i.e., λ_j may change as additional exposure to advertising occurs. An alternative to the difference model giving a less restrictive specification would be to allow the parameters to vary with both the systematic and random components.

While the previous work by Ward consistently showed a first degree polynomial model to be satisfactory, a slight variation in the polynomial model is adopted in this paper. Consider the model below with the apparent lag structure λ_{j+1}, then the lagged structure is specified as $\lambda_{j+1} = \beta_0 + \beta_1 \sqrt[3]{j}$.

Using λ_{j+1} above, the parameters β_0, β_1 must be estimated. The parameters λ_{j+1} may have a systematic and random component and hence β_0 and β_1 may change systematically and with an added random component.

The revised model using the variation in the Almon lagged structure is:

$$(2) \quad q_t = \alpha_{0t} + \alpha_{1t}p_t + \beta_{0t}Z_{1t} + \beta_{1t}Z_{2t} + \varepsilon_t$$

and the Z_{it}'s follow from the Almon procedure

where:

$$Z_{1t} = a_t + a_{t-1} + a_{t-2} + a_{t-3} + a_{t-4}$$

$$Z_{2t} = a_{t-1} + 1.2570\ a_{t-2} + 1.4369\ a_{t-3} + 1.5801\ a_{t-4}.$$

An application of the variable coefficient model to (2) should show any path of parameter adjustments over the time period analyzed.

Random Coefficients Applied to the Advertising Model

Equation (2) represents the distributed lag structure where the parameters can be estimated without making transformations on the error terms. The parameters in (2) may also have systematic and random variation as suggested in the previous section. The results of a re-estimation of the fixed coefficient model developed by Ward but with the inclusion of twelve additional quarters of data suggest that changes in the advertising parameters may have occurred. Preliminary investigation of the results with the expanded data base raises serious questions as to the extended validity of the initial parameters. Also, the re-estimation using ordinary least squares does not provide a clear insight into the nature of any structural changes that may have occurred. Recognizing the evidence of change over time, use of random coefficients, which allows for systematic and non-systematic change, seems appropriate.

In the subsequent discussion we will consider one specific model for estimating equation (2) using variable coefficients. As indicated by Cooley and Prescott, all or some of the parameters may be allowed to vary systematically and/or randomly.

The model to be estimated is given below where σ^2 is the variance of the intercept and the diagonal elements are normalized on σ^2.[3]

$$\Sigma_\mu = \Sigma_\nu = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sigma_p^2 & 0 & 0 \\ 0 & 0 & \sigma_{Z1}^2 & 0 \\ 0 & 0 & 0 & \sigma_{Z2}^2 \end{bmatrix}$$

All
Variable
Parameters

OLS Base Model

The covariance Σ_μ is first estimated using an ordinary least square estimation of equation (2) (see equation 3). Further, the equation is slightly altered by including a time variable to permit the intercept to shift in fixed increments (i.e., time is a proxy for income and other demand shifters not explicitly included in the model).

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[3]The zeros on the off-diagonal elements of Σ_μ and Σ_ν imply independence among parameters. Alternative estimates using non-zero off-diagonal elements were not substantially different than those reported here.

$$(3)\quad \hat{q}_t = .5340 - .2346\ p_t + .0163\ Z_{1t} - .0153\ Z_{2t} + .0076\ t$$

$$\text{s.e.}\ (.1083)\quad (.0836)\quad (.0071)\quad (.0061)\quad (.0009)$$

$$R^2 = .9465 \qquad \text{D. W.} = 1.7023$$

From this equation the estimate of Σ_μ is:

$$\Sigma_\mu = \begin{bmatrix} 1.000 & 0 & 0 & 0 \\ 0 & .5950 & 0 & 0 \\ 0 & 0 & .0044 & 0 \\ 0 & 0 & 0 & .0035 \end{bmatrix}$$

Preliminary results suggest that the varying parameter estimates are robust with respect to slight deviations from the OLS estimates of Σ_μ.

Random Models

The variable coefficient (VC) model was estimated using quarterly observations for the 4th quarter of 1968 (t=7) through the 4th quarter of 1975 (t=35). Data for 1976 and 1977 were omitted from the estimation and used later for validation purposes. The parameters were allowed to change for each observation period and γ, showing the weighting of the permanent and transitory effects, indicated that 98 percent ($\gamma = .98$) of the parameter change was permanent. Note that the proxy trend variable initially included in the fixed model is now dropped since the permanent and transitory adjustments are reflected in the changing intercept estimates. Equation (4) represents the parameter estimates for period t=35.

(4) $\hat{q}_t = .7162 - .4138\ P_t + .0191\ Z_{1t} - .0057\ Z_{2t}$

s.e.(.1410) (.1693) (.0075) (.0064)

Our purpose in the remaining discussion is to illustrate the differences and dynamics of the VC estimates as they compare to the fixed OLS estimates.

Comparison of Intercepts

Figure 1 reports the pattern of intercept adjustments for the random and fixed models over the full time period. For the VC model the intercept α_{0t} exhibits an upward trend with some degree of seasonality while the intercept for the fixed model ($\alpha_0 + \alpha_4$ t) increases linearly. From a model development perspective, a more detailed analysis of Figure 1 may provide an indication of the type of fixed variables that could be included in the model in order to explicitly account for the change. However, it is evident that a simple time trend adjustment is unduly restrictive. In particular, the fixed model generated lower initial estimated values of the intercept when compared to the VC model. For the later periods the fixed month suggested much stronger growth patterns than what may have actually occurred.

Comparison of Price Coefficients

The VC model estimated only slight downward adjustments in the price coefficient over the time periods analyzed. Current statistical procedures are limited when attempting to test the significance of

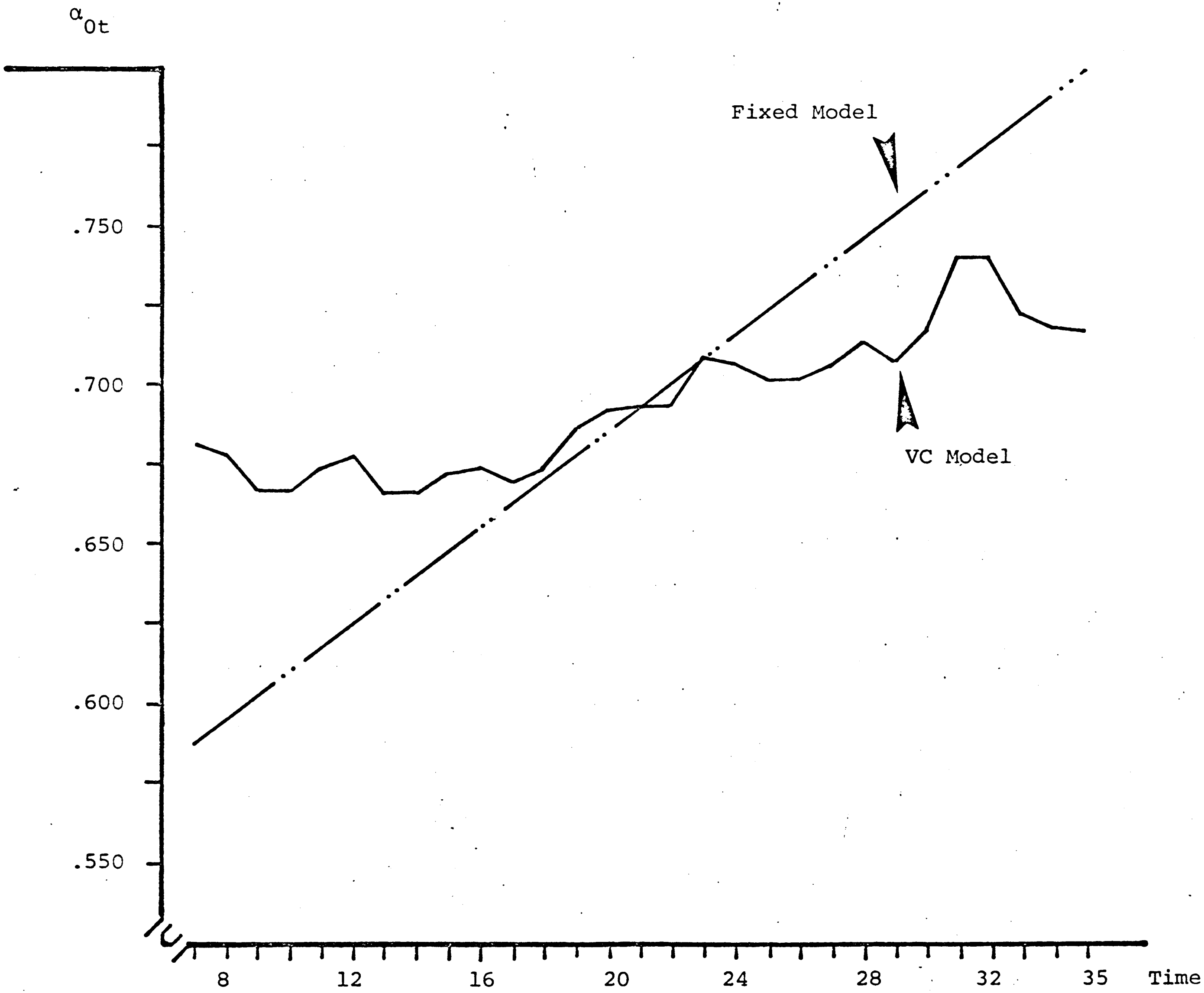

Figure 1. Intercept (α_{0t}) adjustment for fixed and VC models.

these parameter changes over time. However, a more important result from the VC estimates is found in the difference between the fixed and varying price coefficients. The fixed price coefficient was estimated to be -.2346 while the closest value of the random parameter was -.4044 (see Figure 2). In all fixed models where time was used as a proxy for income and growth trends, the price coefficient had small absolute values relative to alternative OLS models which included per capita income instead of time. Prices and time were negatively correlated over the period of the analysis and such correlation may have a direct effect on the estimated coefficient values. Deleting the time variable in the random model reduced the multicollinearity problem and gave a larger absolute price effect.[4]

The differences in OLS and VC model estimates can be further illustrated by comparing price elasticity estimates for the end of the sample period used for estimation. The price elasticity using the OLS model and price and quantity values for 4th quarter 1975 is -.276. The price elasticity estimate using the VC model is -.487.

Even though most discussions of VC models emphasize the forecasting merits of the procedure, the above results suggest that the procedure is equally useful of identifying specific problems with parameter values

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[4] A model similar to (3) with real per capita income rather than time was estimated. The statistics shown in eg. (3), however, suggested that its structure is preferable to the equation with income. Price was correlated with both time (t) and income (i) (i.e., $\rho(pt) = -.83$ and $\rho(pi) = +.80$) and the price parameter was estimated to be -.2346 with the time equation and -.5305 for the income equation. In contrast, the VC model with the multicollinearity problem removed gave price parameters in the mid-range of the fixed models.

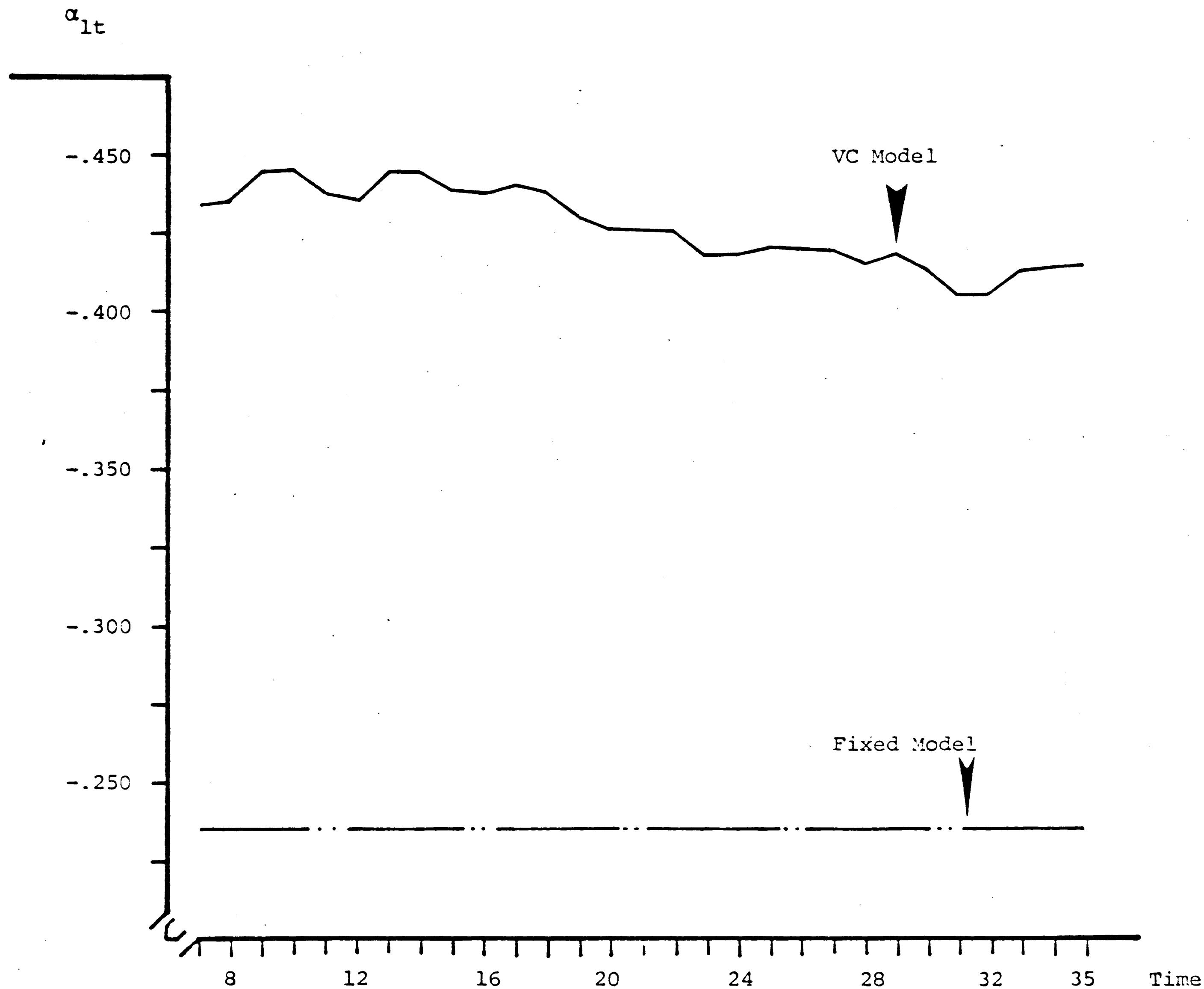

Figure 2. Adjustments in the price coefficient (α_{1t}) from the VC model as compared to the fixed model.

throughout the entire sample period. In this case, the price parameter is expected to have been underestimated because of multicollinearity. Whereas, if the trend variable is deleted from the VC model, the systematic adjustment in the intercept reflects the effects initially measured with the time variable in the fixed model.

Random Advertising Effects

The advertising components of the models were calculated where β_0 is the immediate effect and β_1 shows the decay. Figures 3 and 4 show a comparison of these parameters for both the fixed and VC models. The path of adjustment in β_0 clearly shows a positive trend, thus indicating that the immediate effect of advertising has increased in the latter periods. The systematic adjustments in β_0 suggest some seasonal variation in response to advertising in addition to the increased advertising effectiveness over time. Also, a comparison of the fixed and variable advertising parameters indicates a substantial numerical difference in the effects of advertising expenditures. This difference is obviously accentuated in the more recent periods.

The decay parameter β_1 also differs considerably from that of the fixed model as evident in Figure 4. This difference is important in that the fixed model suggests a very rapid advertising decay while the random model shows the effect of advertising to be extended over a number of quarters. In fact, calculating the lagged parameters from $\beta_0 + \beta_1 \sqrt[3]{j}$ shows that the rate of advertising decay has declined over the sample period. That is, not only has advertising become more

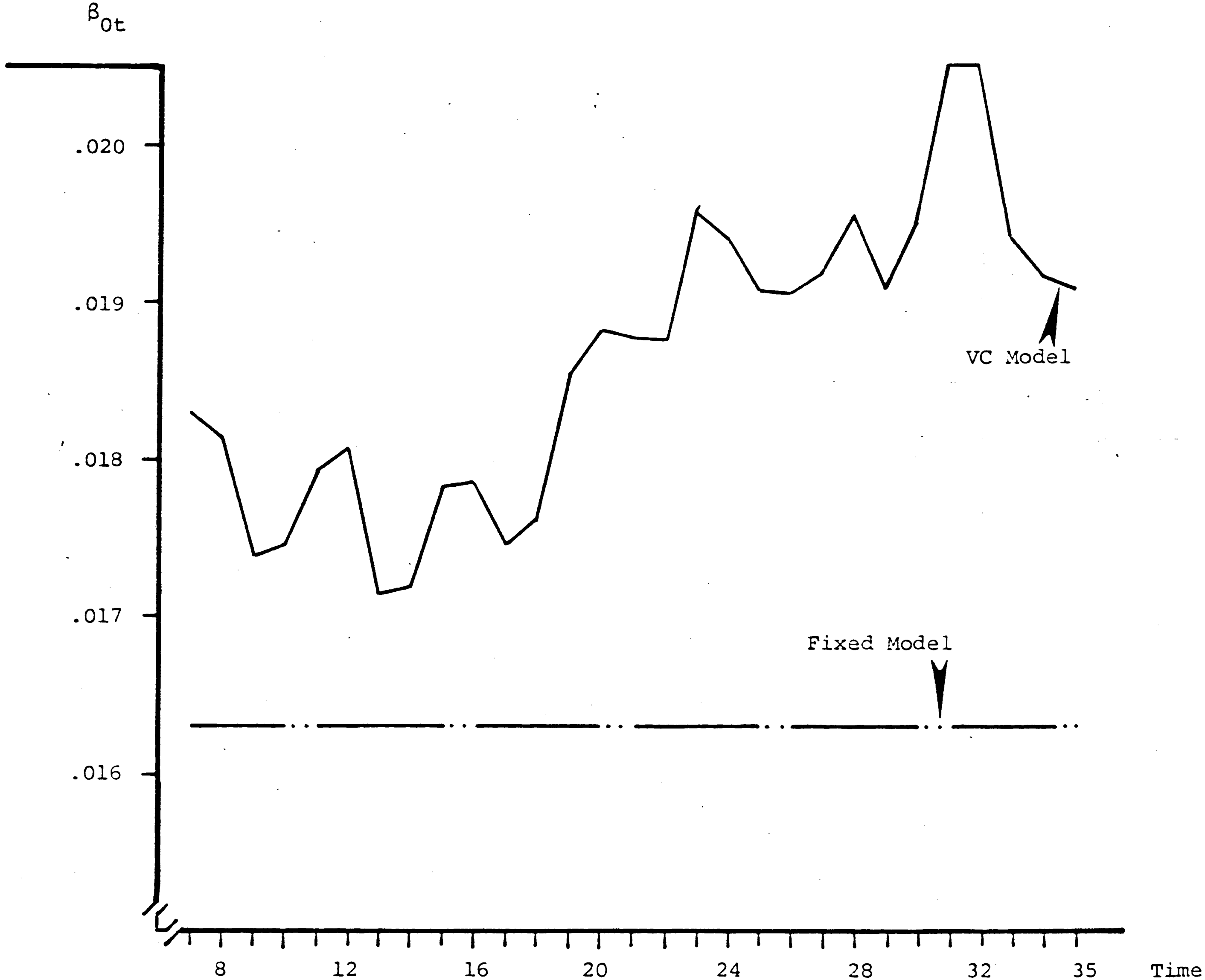

Figure 3. Adjustments in the advertising coefficient (β_{0t} from the VC model as compared to the fixed model.

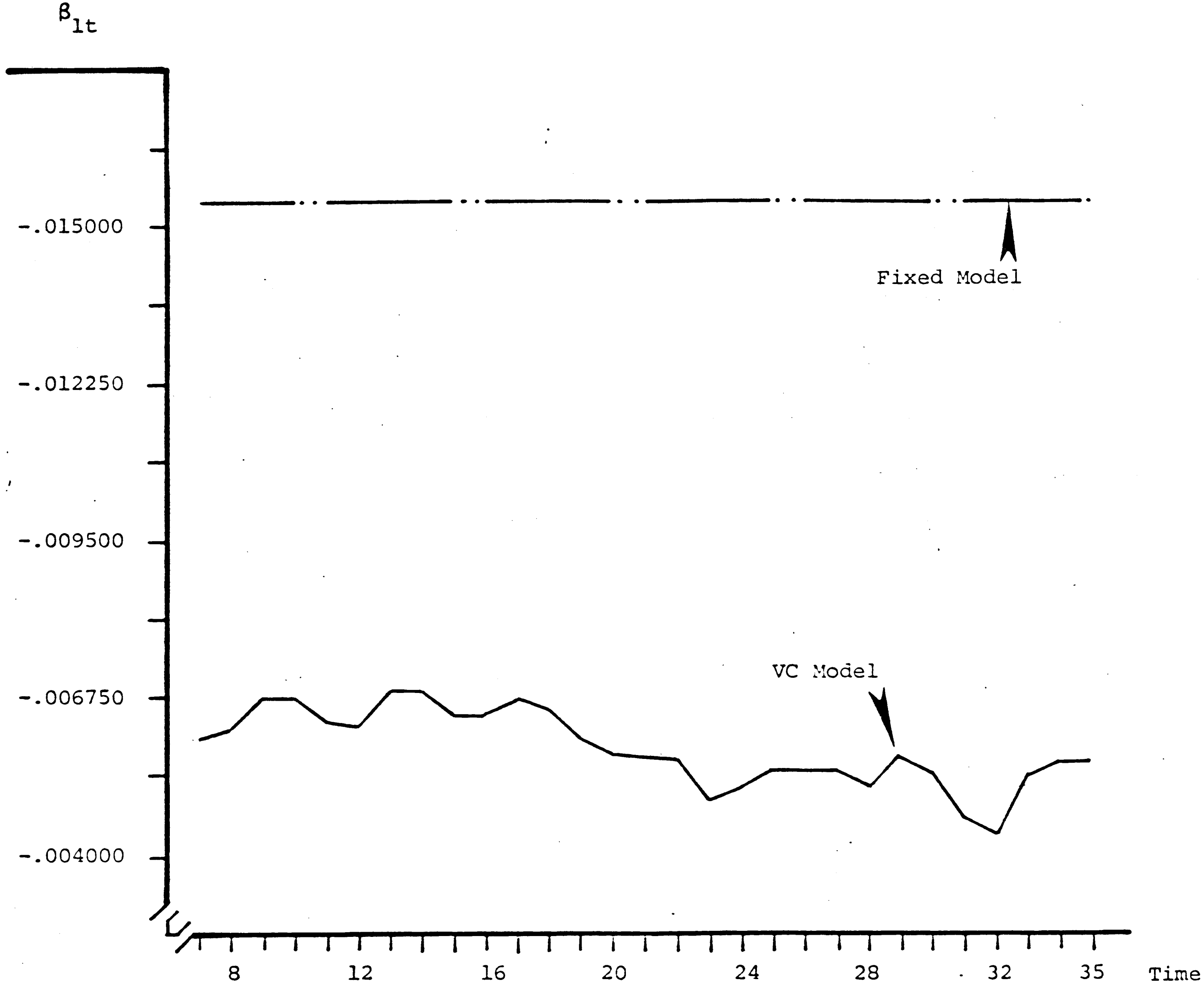

Figure 4. Adjustments in the advertising decay coefficient (β_{1t}) from the VC model as compared to the fixed model.

effective as an immediate stimulator to demand but has also become a more effective tool in that its effect is extended over a longer time period.

An idea of the magnitude of the policy implications is given by the estimated impact on concentrate sales per capita from a $1 million advertising expenditure during quarter t over a 5 quarter time horizon. The OLS estimates suggest increased per capita FCOJ sales of 0.017 gallons while the VC model suggests an increase of 0.065 gallons. Adjusted for a population of 220 million, the OLS estimates indicated added sales of 3.74 million single-strength gallons versus 14.3 million for the VC estimates. Clearly the advertising policy implications are obvious.

While current statistical procedure limit our abilities to statistically test these parameter differences, the numerical values suggest that considerable risk of policy error is associated with the fixed model relative to the VC model. In this regard, the VC coefficient procedure provides an extremely useful tool for modeling when structural change is suspected but the systematic component cannot be hypothesized a priori.

Forecasting Validation

Finally, we consider the forecasting accuracy of the OLS estimates versus that of the VC model. As indicated earlier, the models were estimated with data up through the 4th quarter of 1975 while complete data are available through 1977. These last 8 quarters have been reserved for evaluating the forecasting ability of the models.

Figure 5 is used to compare the performance of each model. Generally, the parameter variation model gave predicted values nearer to actual levels of per capita consumption and were better at predicting turning points (i.e., the Theil μ statistics were μ_{ols} = .197 and μ_{ran} = .132). As the forecast is extended further beyond period 35, the non-stochastic model consistently generated larger forecasting errors relative to the random model.

Conclusion

Parameter variation models greatly expand the capability to better specify models, improve estimating efficiency, and reduce forecasting error. An FCOJ advertising model has illustrated how these random coefficient procedures can be adopted to distributed lag specifications.

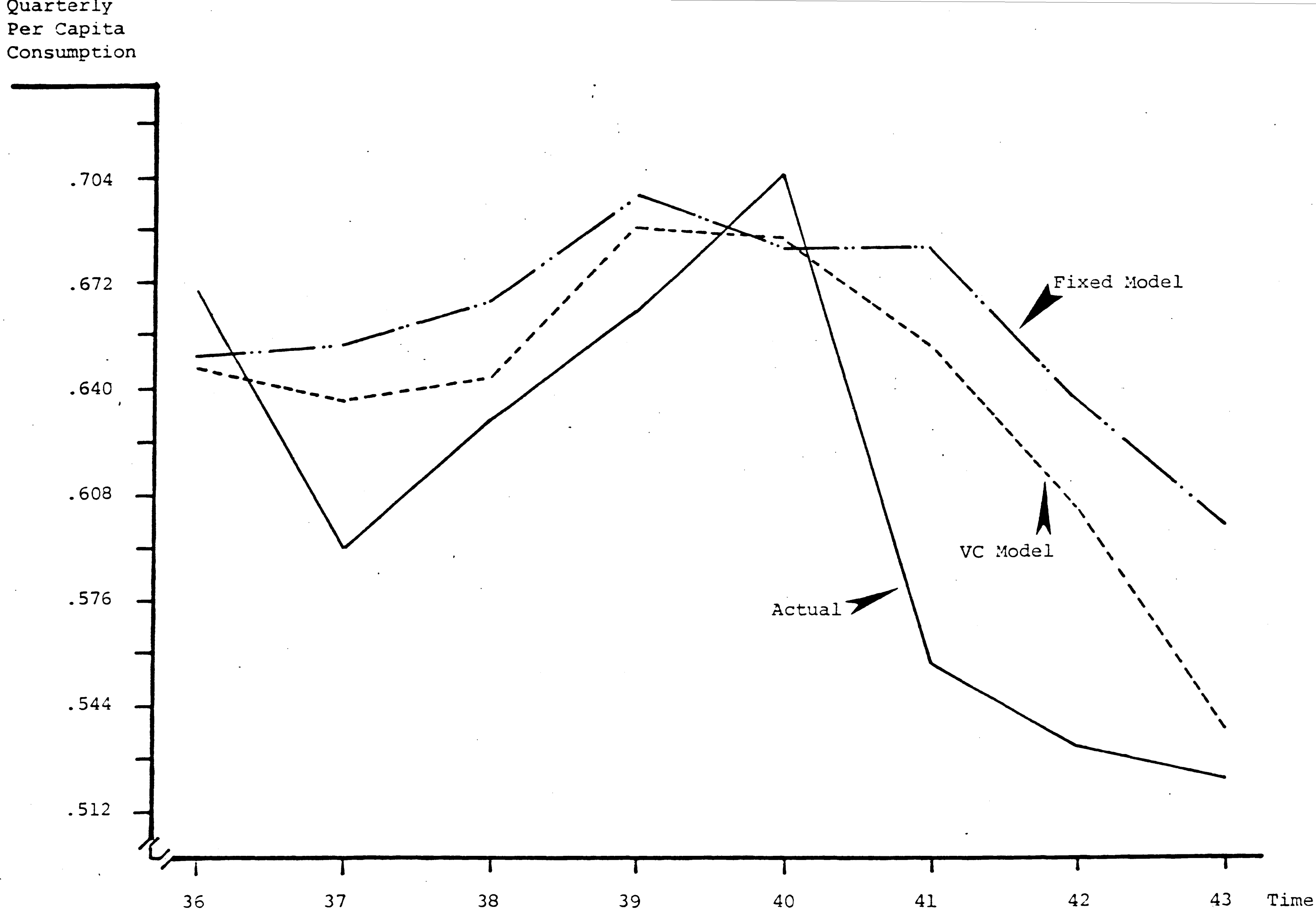

Figure 5. Comparison of forecasting with VC and fixed models.

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