A PROGRAMMING MODEL FOR ANALYSIS OF NON-METROPOLITAN HOSPITAL SERVICES SYSTEMS*

Marvin R. Duncan**

INTRODUCTION

This paper describes a linear programming application of economic planning for hospital services delivery, developed as part of a project providing research support for a multi-county health planning council in north-central Iowa. The council's need was for an analytical tool to support its decision making process regarding formal proposals for service capability and capacity changes received from hospitals.

The linear programming technique has been applied to support decision making in several settings and other researchers have used it to examine aspects of health care systems. The technique was used to study health resource allocations in developing countries and for casemix planning (Feldstein), to eliminate bottlenecks and determine staffing patterns (Gurfield), and to balance tuberculosis control activities in developing countries (Revelle and Feldmann). But, it has not been applied to situations in which hospitals are already "in place" and where planners need to make decisions on marginal changes in capacity, which is precisely what health planning councils are asked to do.

PROBLEM SETTING

Health planning councils must make decisions within a planning framework. The objective function they wish to optimize is a complex one. The effect of changes in one

---

*The research for this paper was conducted while the author was a research associate in the Center for Agricultural and Rural Development at Iowa State University. Dr. Robert W. Crown contributed to the early conceptual phase of this research.
**Marvin R. Duncan is an agricultural economist with the Kansas City Federal Reserve Bank. Paper presented at AAEA meeting, Columbus, Aug. 10-13, 1975.

part of a hospital care system on other hospitals within the planning area and on accessibility to services must be weighed. Councils are not afforded the luxury of deciding where to approve new services as though none presently existed. Rather, decisions must be made within a framework in which substantial previous capital investment has taken place in health care facilities. Community trade patterns and social habits have adapted over the years to the existing facility capabilities. Capital recapture and physical, use, and locational obsolescence must occur before such facilities, even though severely underutilized, can be phased out of use.

A federal-state partnership in planning health care delivery was formed under the Comprehensive Health Planning and Public Health Services Act of 1966 and the Partnership for Health Amendments of 1967 and strengthened in 1972 and 1975 legislation (Comprehensive Health Planning Council). The federal legislation required that all capital expenditures planned by health care providers which 1) involved in excess of $100, 000, 2) changed the total bed capacity, or 3) resulted in substantial change in service in which federal reimbursements were anticipated for depreciation and interest undergo review procedures. (In Iowa, for example, a minimum of 40-45 percent of any acute care hospital's patients involve some federal reimbursement.) A negative recommendation could prevent the institution from receiving federal reimbursement for interest and depreciation. Implicit in the legislation is the need to investigate the cost effectiveness of proposed changes in health care delivery.

PROGRAMMING MODEL

The linear programming model of this study develops an optimal cost solution by allocating patient days of service demand to the hospitals in such a way that the summation of patient day service costs and transportation costs is minimized. Tradeoffs in patient allocation, resource use, and cost levels are explored. The model deals with marginal redistribution of service utilization among five major hospital services in a geographic planning area. These five services are: 1) medical-surgical, 2) obstetrics, 3) pediatrics, 4) intensive care, and 5) psychiatric. Not every hospital would necessarily have all five services.

To be of maximum usefulness to health planning councils, it was necessary to develop a technique that was parsimonious in its data requirements and which could use available data. Data requirements had to be limited to those available from hospital administrators and public sources, and overall operating costs of such a model needed to be modest.

DATA REQUIREMENTS

This model assumes the adequacy of cross section data of two major types. One type related to the utilization of hospital services and the origin of patients utilizing those services. The other related to the services capability and resource base of hospitals, and the cost for providing services. Input data into the programming model were collected by survey from each hospital in the planning region. Data requested were readily available in hospital records and financial reports.

Utilization data measured by patient days of services (classified by age category) extended in each of the five service categories for the fiscal year 1972 were collected. In the model these age categories were: 1) 0-14, 2) 15-44, 3) 45-64, and 4) over 64. Average lengths of stay in each service for each hospital and service capacity data were collected. Patient origin data were collected from each hospital indicating the town from which each patient had come and the number of patients originated from each town. These data were available from admittance records or community relations departments of hospitals. Human resources used in delivering hospital services and hospital bed capacity placed upper limits on the service capacity of hospitals. Data on full time equivalents of human resources available to each hospital were collected by the following categories: general practitioners, physician specialists, registered nurses, on LPN category, specialized medical personnel, and other personnel.

A survey form was developed to collect hospital cost data in which data were categorized by service subcategories that could be assigned in whole or on a proportionate use basis to one of the five major service categories. Expense in each service subcategory was disaggregated by salaries, supplies, fees, and miscellaneous or other. Thirty-two service subcategories were identified.[1] The total cost assigned to each major service category provided by the hospital was then divided by the total patient days utilization of that service to identify a patient day cost for each service in each hospital.

Our experience in data gathering using the described format was that hospital administrators were quite readily able to provide data in the format requested and to indicate the service cost subcategories attributable to delivery of a given service. Thus, within the data set developed, reliable cross comparability of data among hospitals was achieved.

The linear programming model presented here could easily be adapted to more hospital services, more or fewer demand sectors, and a different sized transportation matrix. Also, a set of service activities to represent a composite of all out-of-region

---

1/These included such items as operating room, anesthesiology, laboratory-pathology, administrative, debt servicing, etc.

hospital services could be added or demand sectors could be created for out-of-region areas that generated patient demand for in-region hospital services.

MODEL SECTOR FORMULATION

The model incorporates an inter-hospital service comparative advantage production sector, a transportation network, and thirty-five service demand sectors subdivided by age grouping into 140 service demand activites. Model coefficients for production costs, transportation costs, and hospital services demand represent that experienced in the year 1972. The model minimizes the cost of satisfying hospital service demand and transporting that demand from a demand sector[2] to the hospital service at which the demand is satisfied (where the patient received care). This model has thirty-eight hospital service activities tied to service demand-generating activities[3] by five hundred fifty-one transportation activities. All demands generated in the model have to be satisfied by a hospital service.

The cost minimized is a summation of hospital services patient day costs and transportation costs (Agrawal and Heady). Algebraically the objective is to find a set of x's such that $f(C) = Cx$

is a minimum subject to these restraints:

$$A x \leq b$$

$$x \geq 0$$

2/The demand sectors are geographic units constructed from subcounty census reporting districts.

3/Service demand generating activities are column vectors that create patient days of service demand based on both the population of the age category in the activity and the coefficients that indicate patient days of each service demanded per person in the age category.

where:

C = the objective function value;

x = a column vector of hospital services, transportation, and demand generating activities;

A = a matrix of transformation coefficients; and

b = a column vector of resource restraints.

The model has 279 rows and 1,089 real variables; these real variables are hospital service provision, transportation, and demand-generating activities. The model, though of considerable size and complexity, solves quickly and inexpensively.

Each hospital service relates to from three to six demand sector activities. Each demand sector is linked via a transportation activity to every hospital service that data indicates it has related to or logically could be expected to relate to. Demand for hospital service is for a specific service rather than simply for services of a hospital. This approach insures that patients do not go to a hospital for services that hospital does not deliver.

Service demand sectors consist of multiples of subcounty census reporting districts. In Iowa these contain only one township, although, in many other states they are composed of from two to many townships. Demand sectors are built of subcounty census reporting districts that have these characteristics:

1) residents uniformly relate to one or more hospitals for satisfaction of hospital services demand as evidenced by hospital utilization data,

2) have access to common transportation network,

3) contains one central city, and

4) are constructed of contiguous subcounty census reporting districts.

Based on these criteria, the demand sectors can be of different geographic and population size.

Each of the thirty-five service demand sectors generate four service demand activities segmented by age.[4] Each demand activity has a fixed bound or limit at the level of the population of that age category in the demand sector. The volume of patient days of demand is determined by the size of patient demand-generating coefficients in the demand acitivity.

Demand coefficients are developed for each of the services demanded. The model uses coefficients defined by dividing the patient days of a service utilized for an age category by the total population of that age category within the planning region.

Algebraically $$d_{ij} = \frac{S_{ij}}{P_j}$$

where:

d_{ij} = demand coefficient for service i from age category j;
S_{ij} = patient days of service i utilized by age category j; and
P_j = planning region population of age category j.

Not all services are demanded by each age grouping. For example, pediatrics is demanded only by activity 1 (age category 0-14).

Use of demand coefficients generated with time series data and regression techniques could be readily incorporated into the model. Though coefficients developed

4/It is important to remember that the hospital service activities provide services to patients, and service demand activities generate patient days of demand that utilize those hospital services provided.

for an entire state or multistate region from time series data or cross section data might be preferable to those developed from one planning area, data gaps difficult to resolve are confronted when attempting to secure such data.[5]

Transportation activities are defined in the row section of the programming model as equality rows. Thus, every patient day of demand that enters a transportation activity must be transported to a proper hospital services activity.

The transformation coefficients for a hospital are determined on the basis of human resources used in that hospital. Since competition is among services of different hospitals and not within a hospital, each service competing for a unit of patient demand has a different set of transformation coefficients. Here one might insert engineering coefficients in the event a new facility is contemplated and its impact on existing facilities is to be determined.

The objective function value for a hospital service is the patient day cost of delivering that service at the level of utilization required during the relevant data period. Patient day cost is a summation of the professional salaries, supplies, fees, miscellaneous and other, and administrative and fixed expense used in a particular hospital to deliver the historical level of patient days of the service under consideration. Since the model is primarily concerned with marginal changes in utilization, the assmuption of constant patient day costs over a limited range of utilization is not unreasonable.

Transportation activities contribute to the objective function whenever the level of movement in any given activity is greater than zero. Transportation cost is determined to be a function of time and distance for the patient demanding hospital services

5/Third party payers of hospital costs have in-house data though they are reluctant to release it to researchers. A few data processing firms can develop costly approximations of uncertain reliability.

and for those persons who visit the patient while confined to a hospital. The functional form of the transportation cost equation is:

$$TC = F_1 (T_1) + F_2 (D_1) + F_3 (T_2) + F_4 (D_2)^6$$

where:

TC = transportation cost;

T_1 = time expended by hospital patient in round trip to hospital of choice:

D_1 = distance traveled by hospital patient in round trip to hospital of choice;

T_2 = time expended by visitors in traveling round trip to visit hospital patient;

D_2 = distance traveled by those visiting hospital patient.

MODEL ASSUMPTIONS

It is recognized that certain other constraints inhibit the movement of patient demand to the service offering least cost satisfaction. Such constraints include the hospital service preference of the admitting physician (based on his preference function and very important in determining utilization patterns), the patients subjective evaluation of service quality in a hospital relative to quality in other hospital services in the planning area, and trade patterns for other goods and services. Recognizing an inability to accurately specify such constraints the hospital service activities are restrained to fall within 70 to 130 percent of historical utilization patterns. Historical utilization patterns are assumed to reflect such constraints in addition to patient day cost of the

---

6/ $F_1 (T_1)$ is the round trip distance to the hospital service used divided by an average speed of travel times a time charge (federal minimum wage) and divided by average length of stay in the hospital service.

$F_3 (T_2)$ is the round trip distance to the hospital being visited divided by an average speed of travel times the number of visits per day times the number of visitors per visit times a time charge (federal minimum wage).

$F_2 (D_1)$ is the round trip distance to the hospital service used times a mileage charge and divided by the average length of stay in the hospital service.

$F_4 (D_2)$ is round trip distance to the hospital being visited times the number of visits per inpatient day times a mileage charge.

service and transportation costs.

The planning area is assumed to be essentially a closed system with as many persons leaving the area for service as coming into the area for hospital service. Therefore, excess capacity in a hospital in the model can only be filled by patient demand presently being serviced at another area hospital. Other model assumptions are those normally encountered in a regional modeling problem (Isard).

MODEL USES

The model can be useful in answering a number of questions. In the North Iowa Health Planning Area it was used to determine the impact on the area's hospital utilization patterns of consecutive decreases in manpower availability in area hospitals. The impact on utilization patterns resulting from deletion of a service capability in a particular hospital or deletion of an entire hospital was tested. Population projections were incorporated into the model to determine probable utilization patterns at a future point in time. Changes in patient day cost resulting from new construction and transportation costs were incorporated into the model objective function. Resource shadow prices were developed and model solution costs were indicated under a variety of model assumptions.

The model could also have been used to simulate the effects on the hospital services system of various changes in costs, resource use coefficients, resource and capacity constraints, and population changes or demand coefficient changes. Planners could have adjusted demand coefficients to reflect a lack of transportation or inability to pay for health care services.

The model was constructed to utilize readily available data and provide problem solution inexpensively. It was devised for use by multi-county health planning groups.

With slight adaptation, it could be used at a multiplanning area or state wide level to investigate other health and public service questions.

REFERENCES

1. Agrawal, R. C., and Earl O. Heady. 1972. Operations research methods for agricultural decisions. Ames: Iowa State University Press, pp. 54-78.

2. Comprehensive Health Planning Council. 1974. Fact sheet on partnership for health, CHPC working paper. Des Moines. Office of Planning and Programming, State of Iowa.

3. Feldstein, M. S. 1968. Cost benefit analysis and health program planning for developing countries. Paper presented at Second Conference on the Economic Health, Baltimore, Maryland.

4. __________. 1967. Economic analysis for health service efficiency. Amsterdam: North Holland Publishing Company, pp. 168-183.

5. Gurfield, B. 1968. Planning hospital services. Santa Monica, Calif.: Rand Corporation.

6. Isard, Walter. 1960. Methods of regional analysis: an introduction to regional sciences. Cambridge, Mass.: The M. I. T. Press, pp. 413-492.

7. Revelle, W. and F. Feldmann. 1967. Mathematical models for the economic allocation of tuberculosis control activities in developing countries. Technical Information Bulletin 59. Geneva: World Health Organization.