Statistics - C

1975

RECURSIVE MODELS AND STABILIZATION POLICIES: SOME PROBLEMS AND TECHNIQUES IN EVALUATION

S. Chin, D. Hedley, G. MacAulay, B. Paddock*

*Authors listed alphabetically, S. Chin, D. Hedley, G. MacAulay, B. Paddock are economists with the Research Division, Economics Branch, Agriculture Canada, Ottawa. This paper was submitted as a contributed paper to the A.A.E.A. Annual Meeting, Ohio, August, 1975.

ABSTRACT

Session Number _____ Session Title ____

Recursive Models and Stabilization Policies: Some Problems and Techniques in Evaluation

S. Chin, D. Hedley, G. MacAulay, B. Paddock, Research Division, Economics Branch, Agriculture Canada

The study of stabilization programs, such as a deficiency payments scheme, is made difficult when expectations of a payment are involved. An approach to modelling a minimum guaranteed price which is announced forward in time is outlined. Some useful measures for evaluating programs follow from the analysis.

RECURSIVE MODELS AND STABILIZATION POLICIES: SOME PROBLEMS AND TECHNIQUES IN EVALUATION

Introduction

Considerable theoretical work has been published on stabilization and its welfare implications (for example, Massell, 1969; Massell, 1970; Oi; Samuelson; Tisdell; and Waugh). However, very little work appears to have been done on how to incorporate stabilization programs into dynamic economic models (an exception is Evans), and as to how the effectiveness of these programs might be compared and evaluated. This note considers some of the problems encountered in attempting to make operational in an economic model a deficiency payments scheme. A major problem is that such programs will impinge on the process of forming expectations. The implications of these effects on expectations are examined in an applied context. Also some of the approaches to evaluating alternative stabilization programs are examined.

Policy Analysis - Objectives

The evaluation of simulated policies should take place in terms of the overall objectives of the policy makers. Frequently, the objectives are straight forward, such as the stabilization of prices and/or incomes of a particular agricultural industry. A related objective, but not often stated, is that the "preferred" policy will bring farmers' expectations *ex ante* closer into line with the actual market outcome *ex post*. It is producers' expectations which ultimately determine the supply. Therefore, if a policy affects producers' expectations so that the supply produced at some subsequent date is closer to that which gives a stable price (or income), then it can be presumed that a better allocation of resources has been made (Houck, p. 1115).

Policy Analysis - Methods

For the purpose of evaluating industry stabilization schemes dynamic market models are necessary to capture the interactions between supply and demand, the determination of the time paths of prices and quantities and any associated feedback (King, p. 816; also see Naylor for a review of policy simulation). In addition, if regional differences exist and trade between the regions is significant, then such elements should also be incorporated into the models to obtain a structural representation of the industry that is as realistic as possible (Martin and Zwart; and Lee and Seaver).

Typically, supply functions for dynamic models have incorporated some type of lagged or distributed lag response. Often the assumed lag distribution is that implied by an adaptive expectations hypothesis such as that used by Nerlove. For policy analysis a major difficulty with the Nerlovian formulation occurs when expectations are affected by the policy. This is likely to happen when the level of a policy instrument is made known in sufficient time to influence production decisions.[1/] Such a situation can be represented by comparing the following supply functions:

(1) $Y(t) = f[P^{*}(t), X]$,

(2) $Y(t) = f[P^{*\prime}(t), X]$,

where $Y(t)$ is the supply in period t, $P^{*}(t)$ is the price producers expect in period t, $P^{*t}(t)$ is the price producers expect in period t after they have utilized any information they have on policy instruments (for example, an announced support level for period t), and X represents a set of other variables which may or may not be expectational in nature.

To develop the operational form for equations (1) or (2), two alternative interpretations may be given to the X variables. First, they may be viewed as expectational. Second, they may be viewed as shifter variables. Examples of such in a supply function for hogs might include feed costs (expectational) and sow numbers on farms (non-expectational).

Expectational Shifters

In cases in which all variables in the supply equation are expectational a useful form of the model for estimation can be obtained by applying the usual Koyck transformation and/or the adaptive expectations hypothesis (see Johnston). The supply function in linear form may be written as:

(3) $Y(t) = \alpha + \beta(1)\ P^*(t) + \beta(2)\ X^*(t) + \varepsilon(t),$

where α is the intercept term, $\beta(1)$ and $\beta(2)$ are slope coefficients, $P^*(t)$ is the expected price, $X^*(t)$ represents other expected variables, and $\varepsilon(t)$ the error term which has a normal distribution with mean of zero and constant variance. To transform equation (3) to a statistically operational form the following equations are substituted into (3):

(4) $P^*(t) = [1-\lambda]\ P(t-i) + \lambda\ P^*(t-1),$

(5) $X^*(t) = [1-\lambda]\ X(t-i) + \lambda\ X^*(t-1),$

where λ is the coefficient of expectations (in this case assumed to be the same in both equations), and $P(t-i)$ and $X(t-i)$ are the actual price and shifter variables lagged i periods. After substitution and some algebraic simplification equation (3) becomes:[2/]

(6) $Y(t) = \alpha(1-\lambda) + \beta(1)(1-\lambda)\ P(t-i) + \beta(2)(1-\lambda)\ X(t-i) + \lambda Y(t-1) + \mu(t)$,

where $\mu(t) = \varepsilon(t) - \lambda\varepsilon(t-1)$.

Non-Expectational Shifters

The case in which the X variables can no longer be considered expectational in nature raises the difficulty of how to obtain a satisfactory operational form. Two approaches are examined in this note. The first treats the X variables as a part of the initial hypothesis about the factors determining supply. The second treats the X variables as being added to the supply equations after an operational form has been obtained to deal with the expectations on the price variable.

The supply function in the first case may be written as:

(7) $Y(t) = \alpha + \beta(1)\ P^*(t) + \beta(2)\ X(t-i) + \varepsilon(t)$.

Substitution of equation (4) into equation (7) yields the following operational form:

(8) $Y(t) = \alpha(1-\lambda) + \beta(1)(1-\lambda)\ P(t-i) + \beta(2)\ X(t-i) - \lambda\beta(2)\ X(t-i-1)$
$+ Y(t-1) + \mu(t)$.

Equation (8) is non-linear in the parameters and if estimated in the usual way by ordinary least squares is subject to the problems indicated in footnote 2. In addition difficuties are likely to arise with multicollinearity in the X variables.

The supply function in the second case may be written as:

(9) $Y(t) = \alpha + \beta(1)\ P^*(t) + \varepsilon(t)$,

where we assume that,

(10) $\varepsilon(t) - \lambda\varepsilon(t-1) = \beta(2)\ X(t-i) + \mu(t)$.

In effect the supply function has only been partially specified in equation (9) and requires the addition of equation (10) for completion. Substitution of equation (4) into equation (9) yields (11):

(11) $Y(t) = \alpha(1-\lambda) + \beta(1)(1-\lambda)\ P(t-i) + \lambda\ Y(t-1) + \varepsilon(t) - \lambda\varepsilon(t-1).$

Substitution of equation (10) into (11) then provides the operational form of the supply function as:

(12) $Y(t) = \alpha(1-\lambda) + \beta(1)(1-\lambda)\ P(t-i) + \beta(2)\ X(t-i) + \lambda\ Y(t-1) + \mu(t).$

Equation (12) is the form of the supply function often used, but with little explicit recognition of the way in which it must be derived.

Seasonal Dummy Variables - A Special Case

When using either of the two procedures outlined above, and equations for which seasonal dummy variables have been included, special problems arise as to how to calculate the seasonal parameters. To illustrate, we re-write equation (3) so as to include the seasonal dummy variables:

(13) $Y(t) = \sum_j \alpha(j)\ S(j) + \beta(1)\ P^*(t) + \beta(2)\ X^*(t) + \varepsilon(t), \quad (j = 1,\ldots,4),$

where $S(j)$ are the seasonal dummy variables and $\alpha(j)$ their coefficients. Let $P^*(t)$ and $X^*(t)$ be generated as shown in equations (4) and (5). After substitution of equations (4) and (5) into equation (13) and some algebraic simplification equation (13) can be written as:

(14) $$Y(t) = \sum_j \alpha(j)\ S(j) - \lambda \sum_j \alpha(j)\ S(j-1) + \lambda\ Y(t-1) + (1-\lambda)\beta(1)\ P(t-i) + (1-\lambda)\beta(2)\ X(t-i) + \mu(t)$$

Because of the repetitive nature of seasonal dummy variables the following relationship holds:

(15) If $S(1) = S(j) = [1\ 0\ 0\ 0]'$

then $S(2) = S(j-1) = [0\ 1\ 0\ 0]'$.

After factoring the $S(j)$'s using relationship (15) equation (16) results:

(16) $Y(t) = \sum_{j=1}^{4}[\alpha(j) - \lambda\alpha(k)]\ S(j) + \lambda\ Y(t-1) + (1-\lambda)\beta(1)\ P(t-i)$
$+ (1-\lambda)\beta(2)\ X(t-i) + \mu(t)$, (when $j = 1, 2, 3, 4$, then $k = 4, 1, 2, 3$).

If the form of equation (16) that is estimated is:

(16') $Y(t) = \sum_{j=1}^{4} \hat{a}(j)\ S(j) + \hat{\lambda}\ Y(t-1) + \hat{c}\ P(t-i) + \hat{d}\ X(t-i)$,

then the following relationships hold:

(17) $c = (1-\lambda)\beta(1)$,

(18) $d = (1-\lambda)\beta(2)$,

(19) $a(j) = \alpha(j) - \lambda\alpha(k)$, (when $j = 1, 2, 3, 4$, then $k = 4, 1, 2, 3$).

To compute the values for the β's the relationships (17) and (18) may be used. The solutions for the α's can be obtained by solving the system of equations in (19). The four equations in (19) can be written in matrix form as:

(20) $A = \Lambda\ \Psi$,

where:

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & -\lambda \\ -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \end{bmatrix},$$

$$\Psi = [\ \alpha(1)\ \alpha(2)\ \alpha(3)\ \alpha(4)]',$$

$$A = [\ a(1)\ a(2)\ a(3)\ a(4)]'.$$

The solution for the original $\alpha(j)$ coefficients is obtained by inverting the matrix Λ so that:

(21) $\Psi = \Lambda^{-1}A$.

With the solution to equation (19) given in (21) it is possible to use equations such as (3) in a policy experiment in which the expected variables $P^*(t)$ and $X^*(t)$ are calculated according to equations (4) and (5).

As a conclusion to this section it is clear that the use of seasonal shifter variables, in equations such as (6), with lagged dependent variables is quite legitimate in that no notion of expectation need be applied to them. This result holds because of the repetitive nature of the variables which take the values zero or one. The same result would not hold for a set of dummy variables whose values changed through time, as then the relationships expressed in equation (15) would fail to hold.

Specification of a Minimum Price Guarantee

To make a minimum price guarantee operational in a recursive simulation model it is hypothesized that producers will incorporate the deficiency payments actually made into the formation of their market price expectations. In addition, it is assumed they will form an expectation of a deficiency payment and whether or not such a payment will be made.[3/] As a part of the deficiency payment program, and to illustrate the effects of expectations, it is further assumed that the minimum guaranteed price is announced forward so that producers may use this information in formulating production decisions.

Calculation of the deficiency is made by using a long-run moving average price, say T, and subtracting from it the current market price P(t), so that:

(22) $DP(t) = \delta(1,t)\ [T(t) - P(t)]$,

where DP(t) is the deficiency payment and $\delta(1,t) = 1$ if the market price is below the moving average or $\delta(1,t) = 0$ if the market price is above the moving average. In the latter case no deficiency payment is made.

In addition to the actual deficiency payment, an expected deficiency payment $DP^*(t)$, can be specified in a similar manner:

(23) $DP^*(t) = \delta(2,t)\,[T(t-4) - P^*(t)]$,

where $\delta(2,t)$ takes the value zero if the value of the terms in the brackets is negative and one if it is positive. The guaranteed price is lagged (say four quarters) to take into account the effect of being announced forward.

To incorporate the actual and expected deficiency payments into the supply function it is useful to define a response price, $RP(t)$:

(24) $RP(t) = P^*(t) + DP^*(t)$.

Using the adaptive expectations hypothesis, the expected price $P^*(t)$ is now defined as:[4/]

(25) $P^*(t) = (1-\lambda)\,[P(t-i) + \delta(1,t-i)\,DP(t-i)] + \lambda P^*(t-1)$.

By substitution of equation (25) and (23) into (24) an expression may be obtained for the response price:

(26) $RP(t) = (1-\lambda)\,[P(t-i) + \delta(1,t-i)\,DP(t-i)] + \lambda P^*(t-1) + \delta(2,t)\,T(t-4)$
$\quad - \delta(2,t)(1-\lambda)[[P(t-i) + \delta(1,t-i)\,DP(t-i)] + \lambda P^*(t-1)]$.

From equation (26) it is apparent that there are four possible situations depending on the values of $\delta(1,t-i)$ and $\delta(2,t)$. If a deficiency payment is expected (i.e., $\delta(2,t) = 1$) and regardless of whether or not a deficiency payment was made in period t-i (i.e., $\delta(1,t-i) = 0$ or 1) the response price is the announced guaranteed price, $T(t-4)$. If no deficiency payment is expected or was actually paid (i.e., $\delta(2,t) = 0$ and $\delta(1,t-i) = 0$) then the response price is the expected price without consideration of deficiency payments in period t−i but with the effects of past payments captured in $P^*(t-1)$, thus:

(27) $RP(t) = (1-\lambda)\ P(t-i) + \lambda\ P^*(t-1)$.

For the case in which no deficiency payment is expected (i.e., $\delta(2,t) = 0$), but a deficiency payment is made in period t−i (i.e., $\delta(1,t-i) = 1$) then the response price is the expected price adjusted for the deficency payments, thus:

(28) $RP(t) = (1-\lambda)\ [P(t-i) + \delta(1,t-i)\ DP(t-i)] + \lambda\ P^*(t-1)$.

The above is one of a number of possible specifications for the response price. The response price may now be incorporated into the supply function and to do so requires returning to the alternative specifications of the supply function discussed earlier.

Response Price and the Supply Function

Suppose that the form of equation (12) to be estimated is:

(29) $Y(t) = a + b(1)\ P(t-i) + b(2)\ X(t-i) + \lambda\ Y(t-1) + \mu(t)$,

where a, b(1), b(2), and λ are the coefficients to be estimated. Using equation (4) and substituting for $P(t-i)$ in (29) the price variable in (29) can be transformed back to an expectations form, thus:

(30) $Y(t) = a + \frac{b(1)}{1-\lambda}\ [P^*(t) - \lambda\ P^*(t-1)] + b(2)\ X(t-i) + \lambda\ Y(t-1) + \mu(t)$

Equation (30) is now in a form which may be used in the simulation of alternative policies which directly affect expected prices, such as that illustrated earlier. In the case illustrated the response price in equation (26) may be substituted for $P^*(t)$ in equation (30).

The case of non-expectational shifters as well as expectational shifters is easily dealt with in a simulation model using the above procedure. In the case of expectational variables other than price which are also influenced by a policy then additional response equations paralleling equation (26) might be used. It is also apparent that unless the original parameters on the seasonal dummy variables of an equation such as (13) are required then seasonal shifters may be simply viewed as non-expectational shifters.

A Numerical Illustration

This section illustrates how a supply equation, such as equation (13), can be used in the evaluation of a stabilization plan with deficiency payments. Results which might be used to evaluate such a plan are also presented.

The supply equations shown below form part of a quarterly economic model of the Canadian hog industry. The model consists of supply equations for Eastern and Western Canada, a consumption demand for all of Canada, two equations describing demand for closing stocks of pork in Eastern and Western Canada, a supply-demand identity and an equation relating pork prices in Eastern Canada to Western Canada.

The two supply equations in the hog model correspond to equation (3), or equation (13), and the assumptions and definitions relating to the variables are as follows:

$$(31)\quad Y^{ec}(t) = \sum_{j=1}^{4} \alpha(j)\ S(j) + \beta(1)\ M^{*ec}(t) + \beta(2)\ GSP^{*}(t) + \varepsilon^{ec}(t),$$

$$(32)\quad Y^{wc}(t) = \sum_{j=1}^{4} \alpha(j)\ S(j) + \beta(1)\ M^{*wc}(t) + \beta(3)\ GS^{*}(t) + \varepsilon^{wc}(t),$$

$$(33)\quad M^{*}(t) = (1-\lambda)\ M(t-5) + \lambda\ M^{*}(t-1),$$

$$(34)\quad GSP^{*}(t) = (1-\lambda)\ GSP(t-5) + \lambda\ GSP^{*}(t-1),$$

(35) $GS^*(t) = (1-\lambda)\ GS(t-5) + \lambda\ GS^*(t-1)$,

(36) $M(t) = P(t) - \omega(t)\ FC(t)$,

where Y is the pork production (millions of pounds, cold dressed weight); M is the margin ($ per hundredweight, cold dressed weight); P is the price of pork ($ per hundredweight, cold dressed weight); $\alpha(j)$ is the coefficient of the jth seasonal dummy variable ($j = 1,\ldots,4$); $\omega(j)$ is the coefficient for feed used to produce one hundredweight of cold dressed pork; FC is feed cost; GS is stocks of wheat, oats and barley in Manitoba, Saskatchewan and Alberta (millions of bushels); GSP is stocks of wheat, oats and barley in Manitoba, Saskatchewan and Alberta plus production in Eastern Canada (millions of bushels); S(j) is the jth seasonal dummy variable ($j = 1,\ldots,4$); ec is Eastern Canada; wc is Western Canada; and * indicates a variable expressed in expected values.

After transforming the supply equations to a form for statistical estimation, as in equation (16'), ordinary least squares was then used to estimate the coefficients. The estimated coefficients were then transformed into the expectations form, as in equation (3). The results of these transformations are:[5/]

(37) $Y^{ec}(t) = 23.285\ S(1) + 21.627\ S(2) + 20.926\ S(3) + 23.677\ S(4)$
$+ 3.329\ M^{*ec}(t) + 0.008\ GSP^*(t)$,

(38) $Y^{wc}(t) = 8.836\ S(1) + 9.777\ S(2) + 3.717\ S(3) + 5.533\ S(4)$
$+ 3.105\ M^{*wc}(t) + 1.120\ GS^*(t)$.

The stabilization plan to be illustrated is an annual deficiency payment program which guarantees pork producers the difference between average market prices in Canada and a minimum price guarantee. The minimum price guarantee is calculated as a percentage (say 90) of the five-year moving average of Canadian pork prices with adjustment for changes in cash input costs.

$$\text{Price guarantee} = 0.9 \times \begin{matrix}\text{5-year}\\ \text{average}\\ \text{price}\end{matrix} - \begin{matrix}\text{5-year}\\ \text{average}\\ \text{cash costs}\end{matrix} + \begin{matrix}\text{current}\\ \text{cash}\\ \text{costs}\end{matrix}$$

Simulation results for the period 1965 to 1974 are presented in Table 1 and show a number of the measures used to evaluate the program. The relative cost of the plan is indicated by the percent of gross revenue required to fund the program as a levy, i.e. 2.5 percent. The measure termed 'gain in expectations efficiency' relates to the percentage reduction in the standard error of a regression relating 'realized' net revenue per hundredweight of pork to 'expected' net revenue with and without the program in place. Substantial gains are indicated from the program for both Eastern and Western Canada. This measure indicates that when the price guarantee is known at the time production decisions are made producers tend to make decisions more in line with market conditions at the time of sale of their hogs.

Stability of a time series is a difficult concept to measure (see Firch; and Houck). One approach adopted in evaluating the various stabilization plans derived from the hog model was to use the standard deviation of the percentage change of the series on net revenue. The percentage gain in stability is shown in Table 1. This measure compares the standard deviation of the percentage change in net revenue per hundredweight of pork between corresponding quarters of one year to the next with and without the program in place. In the example given substantial gains in stability are indicated in response to the program.

Ratios of the change in the standard error and standard deviation to total program costs are shown in the third and fourth rows of Table 1. These two measures represent the relative cost for the particular program of acquiring additional efficiency in expectations or stability. The ratios may be used to compare different types of programs. For the plan presented in Table 1, the industry benefits over the period 1965 to 1974 by receiving \$176 million in additional revenue of which \$125 million is derived from deficiency payments and the remainder of \$51 million is generated within the market place.

Table 1. Stabilization Measures for a Price Margin Stabilization Plan Using the Canadian Hog Model, 1965-74

	Western Canada	Eastern Canada	Canada
Gain in expectations efficiency (percent)[a]	30	42	-
Gain in stability (percent)[b]	14	13	-
Gain in expectations efficiency per million dollars of program costs (ratio)	0.61	0.71	-
Gain in stability per million dollars of program costs (ratio)	0.26	0.21	-
Program costs ($ million)[c]	-	-	125
Levy on gross revenue (percent)	-	-	2.5
Gain in net revenue ($ million)[c]	-	-	176
Gain in net revenue per dollar of program costs (ratio)	-	-	1.41

[a] Gain in expectations efficiency is defined as the percentage reduction of the standard error of estimate of the regression of 'realized' net revenue on 'expected' net revenue per hundredweight of pork.

[b] Gain in stability is defined as the percentage reduction of the standard deviation of the percentage changes in net revenue per hundredweight of pork from a quarter in one year to the corresponding quarter in the previous year.

[c] All revenues and costs are expressed in present values.

FOOTNOTES

1/ Supply may also be influenced by policy instruments which have their value or levels made known after the major production decisions have been taken. In such a case the impact is one of the policy instruments changing the nature of the outcome of the market with subsequent feedback into the formation of expectations.

2/ It should be clear from the expression for $\mu(t)$ that even when $\varepsilon(t)$ has the desired classical properties the error term for the operational form of the distributed lag model is not well behaved. Estimation procedures adjusting for this problem are non-linear and give only desirable large sample properties for the parameter estimates. As a consequence ordinary least squares estimates, although obviously inconsistent, are most commonly employed in applied work (Johnston).

3/ The type of deficiency payment referred to is one based on an average price for an industry with an equal payment made to all producers regardless of the price the individual producer obtains.

4/ Equation (25) requires a starting value, $P^*(0)$, when used in a simulation. An approximation is:

$$P^*(0) = (1-\lambda)\ [P(t-i) + \lambda P(t-i-1) + \lambda^2 P(t-i-2) + \ldots + \lambda^n P(t-i-n)] + \lambda^{n+1} P(t-i-n).$$

This approximation is designed to handle a situation in which the number of available lagged prices, n, is relatively small. The last price is taken as representative of all earlier prices and weighted by λ^{n+1}. The approximation was suggested by J. Nash, Economics Branch, Agriculture Canada.

5/ The estimated coefficients on the seasonal dummy variables were: for Eastern Canada, 21.952, 16.556, 18.740, and 32.237; for Western Canada, 15.453, 11.610, 8.422, and 9.170. The value of λ for Eastern Canada was 0.750 and for Western Canada was 0.667.

REFERENCES

Evans, Martin. "Guaranteed Price Adjustment and Market Stability in the United Kingdom: The Case of Beef and Milk." Amer. J. Agr. Econ. 56 (1974):26-37.

Firch, R. S. "Stability of Farm Income in a Stabilizing Economy." J. Farm Econ. 46 (1964):323-340.

Houck, James P. "Some Economic Aspects of Agricultural Regulation and Stabilization." Amer. J. Agr. Econ. 56 (1974):1113-1124.

Johnston, J. Econometric Methods. 2nd edn., New York: McGraw-Hill Book Company, 1972.

King, Richard A. (Discussion) "Applied Welfare Economics." Amer. J. Agr. Econ. 54 (1972):815-816.

Koyck, L. M. Distributed Lags and Investment Analysis. Amsterdam: North-Holland Publishing Company, 1954.

Lee, Tsoung-Chao and Stanley K. Seaver. "A Simultaneous Equation Model of Spatial Equilibrium and Its Application to the Broiler Markets." Amer. J. Agr. Econ. 53 (1971):63-70.

Martin, Larry and Anthony C. Zwart. "A Spatial and Temporal Model of the North American Pork Sector for the Evaluation of Policy Alternatives." Amer. J. Agr. Econ. 57 (1975):55-66.

Massell, Benton F. "Price Stabilization and Welfare." Quart. J. Econ. 83 (1969):284-298.

Massell, Benton F. "Some Welfare Implications of International Price Stabilization." J. Polit. Econ. 78 (1970):404-417.

Naylor, Thomas H. "Policy Simulation Experiments with Macroeconometric Models: The State of the Art." Amer. J. Agr Econ. 53 (1970):263-271.

Nerlove, Marc. The Dynamics of Supply: Estimation of Farmers' Response to Price. Baltimore: John Hopkins Press, 1958.

Oi, Walter Y. "The Desirability of Price Instability Under Perfect Competition." Econometrica 29 (1961):58-64.

Samuelson, Paul A. "The Consumer Does Benefit from Feasible Price Stability." Quart. J. Econ. 86 (1972):476-493.

Tisdell, Vern. "Uncertainty, Instability, Expected Profit." Econometrica 31 (1963):243-247.

Waugh, Frederick V. "Consumer Aspects of Price Instability." Econometrica 34 (1966):504-508.

ACKNOWLEDGEMENTS

The authors acknowledge the helpful comments of S. R. Johnson, D. MacLaren and L. J. Martin on an earlier draft of this paper.

Footn

* Giannini Foundation Paper No. ____.

1. Ijiri uses the Moore-Penrose generalized inverse B^+. Such an inverse is unique, and it is unnecessarily restrictive to represent the existence conditions. For a detailed discussion of this point, see [6].

2. A search of the literature reveals an absence of aggregation studies cast in a duality framework.