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APPLICATION OF AN EXPECTED RETURN-RISK CRITERION IN CROP PLANNING

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Introduction

Farm planning under less than perfect knowledge is receiving increasing attention. However, one of the commonly used methods for farm planning, linear programming, does not directly allow the introduction of risk or uncertainty. This paper discusses a modification of the usual linear programming modeling approach, which permits stochastic outcomes to be considered. The decision criterion discussed is the minimization of total absolute deviations [Markowitz, 1959, p. 187; Hazell, 1971]. This criterion is applied to a yearly crop planning model, which has received extensive use and gained acceptance among farmers. The lack of satisfactory data availability for a wider application of the decision criterion incorporating risk considerations is noted, and the implications for further research of an increased usage of such a criterion in farm planning are discussed.

The Nature of the Problem

Agricultural production is inherently stochastic, due to the effect on yields of such phenomena as weather and disease outbreaks. In addition to this yield uncertainty, the prices of inputs and outputs are not known with certainty. The combination of price and yield uncertainty gives rise to profit or income uncertainty.

However, in linear programming farm planning models it is commonly assumed that profit or income is to be maximized and that prices, costs and yields, which underlie the income measure, are known with certainty. The validity of these assumptions may be questioned. It is sometimes desirable that the income uncertainty is taken into account in the model, and that the maximization objective refer to a utility function including uncertainty considerations instead of income or profit only.

The problem then becomes one of how to model farm planning procedures under such assumptions. Markowitz [1959, p. 187] suggested and Hazell [1971] showed a linear approximation of the quadratic programming formulation of the problem of farm planning under uncertainty. The quadratic programming problem includes a couple of crucial assumptions. These assumptions are that the decision maker's utility can be described as a function of the expected value and the variance of income distribution, and that these two moments of the distribution exist and are finite.

Given historical observations on the income or profit from each individual activity in the total farm plan, expected individual profits and the accompanying variances and covariances may be estimated. Using a particular kind of objective function, where aversion to risk is assumed, a quadratic programming problem may be formulated. The quadratic property of the problem arises when variance and covariance considerations, indicating the risk of an activity, are introduced into the objective function.

Hazell's [1971] approach is very similar to the procedure described. The difference is that he assumes that utility is derived from expected income and the summed mean absolute deviation of income. This requires that the distribution of total income is described by its expected value and its mean absolute deviation. Thus, the measure of risk in this case is mean absolute deviation rather than variance.

Mean absolute deviation of income is estimated from historical observations in the same fashion as variance. Using Hazell's approach, the problem is formulated in a linear programming format, given that the utility maximizing objection function is linear.

Model Formulation

The ordinary income maximizing farm planning problem can be modelled as follows.

$$\max_{X} \quad C'X \tag{1}$$

$$\text{subject to} \quad AX \leq b \tag{2}$$

$$X \geq 0 \tag{3}$$

where

- C is an r x 1 vector of income contributions
- X is an r x 1 vector of activities
- A is an s x r matrix of resource usages
- b is an s x 1 vector of resource endowments.

This is the formulation presented empirically in Figure 2 below.

The formulation of the problem of minimization of total absolute deviation subject to a minimum level of expected income is then the following:

$$\min_{X,d} \quad \sum_{i=1}^{n} d_i^{+} + \sum_{i=1}^{n} d_i^{-} \tag{4}$$

subject to

$$C'X = \lambda \quad (5)$$

$$AX \leq b \quad (6)$$

$$DX - Id^{+} + Id^{-} = 0 \quad (7)$$

$$X, d^{+}, d^{-} \geq 0 \quad (8)$$

where A, b, C, X are as above.

$d^{+} = [d_i^{+}]$ and $d^{-} = [d_i^{-}]$ is an n x 1 vector of activities representing, respectively, the negative deviations that the income data for year i exhibits from the mean income.

$D = [D_{ij}]$ is an n x r matrix where D_{ij} is the deviation of income in year i for activity j from the mean income for activity j.

λ is an income level parmetrically varied from zero to the riskless linear programming maximum.

I is an n x n identify matrix.

The formulation, called the E-A formulation, is shown in Figure 1. A, b, C. and X are the same as in the linear programming model. λ is as above. The lower left partition is the D matrix, where positive and negative signs indicate income deviations observed in the sample.

The Empirical Model

A linear programming crop planning model complete with input forms, matrix generator, and report writer has been extensively used at Purdue University [Candler _et al_., 1970; Doster and McCarl, 1974]. The model has gone through several revisions and has been successively improved for farmers and extension personnel. This applied model was adapted for use in the present experiment.

Activities

X	d^+	d^-	
	111 ... 111	111 ... 1	Min.
C			= λ
A			≤ b
+ + - + ... +	-1	1	= 0
- + + + ... +	-1	1	= 0
+ - - - ... -	-1	1	= 0
. . . .	.	.	.
+ + + + ... -	-1	1	= 0

Figure 1. The E-A Formulation of the Crop Planning Model.

A simplified schematic of the adapted model is presented in Figure 2. The model emphasizes the timeliness aspect of crop production as is depicted by the classification of the restraints. Labor and field time is disaggregated into six periods during the spring, one period during the summer, three periods during the harvesting season, etc. There are three harvest periods available for each of the corn and soybean production activities. The objective of this model is to maximize net income, i.e., total returns less variable costs. (The model, as formulated in Figure 2 actually minimizes negative net income.)

The risk portion of this linear programming model was formulated as the E-A formulation described above. The E-A efficient frontier was derived by parametrically varying the prespecified level of expected income λ. An efficient frontier is defined as the locus of maximum expected income for each level of riskiness or, alternatively, the minimum amount of risk associated with each level of expected returns.

Data Requirements and Availability

There were two major data requirements for operation of the model in this study. The first concerned the specification of restraints for the case farm. The second (and most important for purposes of this study) related to the deviations of activity income levels from their means. The deviations were required for estimates of the trade-off between expected income and variation of return.

The case farm was 550 acres in size. It was a one-man farming operation in Central Indiana. All the restraints inherent in the Purdue Crop Budget basic data set [Doster and McCarl, 1974] were assumed to apply to this situation.

	Corn Land Plowing	Corn Production Period 1	Corn Production Period 2	Corn Production Period 3	Corn Production Period 4	Corn Production Period 5	Corn Production Period 6	Soybean Land Plowing	Soybean Production Period 1	Soybean Production Period 2	Soybean Production Period 3	Soybean Production Period 4	Soybean Production Period 5	Soybean Production Period 6	Wheat Production Early	Wheat Production Late	
Objective		–	–	–	–	–	–		–	–	–	–	–	–	–	–	Min
Acreage	+							+							+	+	≤ +
Labor - Spring	+	+	+	+	+	+	+	+	+	+	+	+	+	+			≤ +
Labor - Summer		+	+	+	+	+	+		+	+	+	+	+	+	+	+	≤ +
Labor - Fall	+							+							+	+	≤ +
Plowing Field Time Spring	+							+									≤ +
Plowing Field Time Fall	+							+							+	+	≤ +
Planting Field Time Spring		+	+	+	+	+	+		+	+	+	+	+	+			≤ +
Cultivating Field Time, Spring & Summer		+	+	+	+	+	+		+	+	+	+	+	+			≤ +
Tractor Time-Spring	+	+	+	+	+	+	+	+	+	+	+	+	+	+			≤ +
Tractor Time-Fall	+							+							+	+	≤ +
Corn Harvest Time		+	+	+	+	+	+										≤ +
Soybean Harvest Time									+	+	+	+	+	+			≤ +
Wheat Harvest Time															+	+	≤ +
Plowed Corn Land	–	+	+	+	+	+	+										≤ +
Plowed Soybean Land								–	+	+	+	+	+	+			≤ +

Figure 2. Income Maximizing Crop Planning Model.[1/]

Ten years of data on gross returns and variable costs were secured for each of the corn, soybean and wheat producing activities. Data on average yields, average selling prices, and average variable costs were obtained from The Summaries of Illinois Farm Business Records [Cooperative Extension Service, 1963-1972]. These data represented a sample of from 73 Northern Illinois farms in 1963 to 315 Northern Illinois farms in 1972. The average size of farm in this sample ranged from 518 acres in 1964 to 564 acres in 1967.

The net income for each planting date for corn, soybeans, and wheat production was computed for the years 1963-1972 from data in the Summaries of Illinois Farm Business Records (see Tables A.3 and A.5). The net income consists of gross return less variable costs. Variable costs included seed, fertilizer, machinery operation (less depreciation), other cropping expenses, hired labor, insurance, and miscellaneous costs (Table A.1). Land taxes were not included since they represented a fixed cost over the production period. The yearly net income computed in this way was adjusted for diverted acres and for hay and pasture acreage. These net incomes were further adjusted in order to achieve the same relative magnitudes of margins for each crop as in the crop budgets prepared by the U.S. Department of Agriculture for East Central Illinois (U.S. Department of Agriculture, 1971].

The model, as illustrated in Figure 2, emphasizes the timeliness aspects of planting and harvesting activities. Therefore, it was necessary to procure data relating to yield differentials by planting and harvesting dates. Several data sources were utilized to estimate the effect of planting date on yields of each of the crops (see Table A.2). The effect of harvesting data on yields were estimated on the basis of discussion with agronomists.[2/] There are three harvesting periods in the model (only data for the median period is shown). The yield advantage and penalty for the earlier and later harvesting periods were 2 and 3 percent for corn and soybeans, respectively. The wheat yields were assumed to be 5 percent above and 5 percent below average annual yields for the early and late planted activities.

The mean incomes (Tables A.3 and A.5) further adjusted for the timeliness factors noted above, constituted the set of expected incomes (i.e., the C vector) for the operation of the model.

The computed absolute deviations (Tables A.4 and A.5) were utilized in the E-A formulation of risk.

Results

The maximum expected income for the farm represented in this study was $28,461. This expected income corresponds to the maximum expected income in the ordinary LP solution which does not consider risk.

Selected points of the expected income-risk trade-off function are presented in Table B.1.

Though it is not obvious from the results presented, decreased levels of income (below $28,000) resulted in some unused land acreage. The objective of the formulation is to secure minimum risk solutions for each given level of profit. Therefore, the model would bring the riskless slack activities for unused land into the solution, once the predetermined income level was achieved.

The optimum crop plans for selected levels of income are presented in Table B.2. The E-A formulation exhibits a pronounced shift from soybean acreage to corn acreage with a decrease in risk (and profit level).

Note that only a $61 decrease from the riskless optimum increases corn acreage by approximately 33 acres, increases wheat acreage by approximately 4 acres, and decreases soybean acreage by 37 acres. At the income level of $27,900, 109 acres have been transferred out of soybean production, of which 99 were added to corn production and 10 to wheat production.

Conclusion

The focus of this paper was to present a formulation of the expected return--risk trade-off which could have application for extension purposes. The formulation showed risk averse behavior through shifting cropping patterns.

The degree of risk aversion exhibited by individual decision makers is an empirical question. If the decision maker's utility function were known to the analyst, it would be possible to obtain the utility maximizing crop plan. Alternatively, it would be possible to settle on the utility maximizing crop plan by confronting the decision maker with the efficient frontier in the form of a graph or a table. This assumes that the decision maker actually quantifies risk in terms of the same concepts as in the analysis, viz., total absolute deviation.

The results generated were, of course, only demonstrative of what could be done with actual farmers in a workshop environment. It is clear that even small degrees of risk aversion can result in some rather substantial shifts in production plans.

There are several areas which ought to be investigated prior to any large scale use of models incorporating risk. This paper discussed the requirements for specific types of data needed to incorporate risk into a farm planning model. The availability of this data and the way the farmer thinks about the problem needs to be studied. Certainly in a mass audience setting it would be necessary to determine measurements of farmer risk aversion which are robust enough to be included in a generally available model.[3/]

An additional consideration for large scale use would be to include the risk term in the objective function with the profit coefficients. This would permit the interpretation of shadow prices as contributions towards profit discounted for risk.

APPENDIX A

Table A.1 Average Prices and Variables Costs, Northern Illinois Farms, Soil 76-100, 500-600 Tillable Acres, 1963-1972.

	Prices			Variable Costs		
Year	Corn	Soybeans	Wheat	Corn	Soybeans	Wheat
1963	$ 1.14	$ 2.52	$ 1.86	$ 55.95	$ 30.38	$ 44.33
1964	1.12	2.58	1.43	56.87	30.87	45.04
1965	1.13	2.59	1.40	59.43	32.28	47.16
1966	1.19	2.83	1.81	60.28	32.74	47.79
1967	1.16	2.66	1.46	55.19	30.35	44.32
1968	1.00	2.53	1.24	58.44	31.73	46.35
1969	1.12	2.43	1.19	62.64	34.02	49.68
1970	1.18	2.55	1.27	61.01	33.16	48.40
1971	1.26	2.91	1.40	68.54	37.19	54.27
1972	1.13	3.21	1.56	77.16	41.90	61.18

Source: "Summary of Illinois Farm Business Records, Commercial," 1963-1972, Cooperative Extension Service, University of Illinois at Urbana-Champaign.

ERS, USDA, Selected U.S. Crop Budgets, Volume II, North Central Region, ERS 458, 1971.

Table A.2 Average Yields by Planting Period, Corn and Soybeans, 1963-1972.

	Planting Period 1	Planting Period 2	Planting Period 3	Planting Period 4	Planting Period 5	Planting Period 6
Corn						
1963	122.8	126.0	129.2	105.4	99.9	94.3
1964	116.2	115.4	114.4	114.0	94.3	74.5
1965	126.3	124.7	123.0	118.3	110.7	102.9
1966	106.8	101.2	95.6	100.3	100.1	99.8
1967	132.9	117.5	102.0	126.8	123.7	120.7
1968	110.4	108.9	107.3	86.9	86.7	86.5
1969	150.7	139.5	128.3	120.7	117.9	115.1
1970	99.6	101.1	98.6	109.8	88.0	66.7
1971	138.6	154.1	130.4	126.3	118.5	110.7
1972	134.3	130.7	130.0	129.4	136.4	118.7
Soybeans						
1963[4/]	41.5	39.9	38.3	37.5	34.3	31.2
1964[4/]	31.5	31.9	32.2	32.7	30.0	27.2
1965[4/]	38.1	37.1	36.1	32.1	32.9	33.7
1966	35.1	35.5	35.8	36.4	33.4	30.3
1967	42.2	41.2	40.1	34.4	33.0	31.7
1968	41.7	40.1	38.5	37.7	34.5	31.3
1969	46.1	44.9	43.7	38.9	39.9	40.9
1970	27.7	28.1	28.5	26.6	25.9	25.2
1971	43.0	42.9	42.9	43.0	43.0	42.9
1972[4/]	50.1	48.8	47.0	40.7	39.2	37.6

Sources: Griffith, D. R., O. W. Luetkemeier, and R. K. Stivers, "Effect of Planting Date and Variety on Soybean Yields," Research Progress Report 363, Purdue University, Lafayette, In. Nov., 1969.

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Table A.3 Computed Margins for Corn and Soybean Activities, Northern Illinois Farms, Soil 76-100, 500-600 Tillable Acres, 1963-1972.

	Planting Period 1	Planting Period 2	Planting Period 3	Planting Period 4	Planting Period 5	Planting Period 6
Corn						
1963	$ 84.08	$ 87.69	$ 91.29	$ 64.26	$ 57.95	$ 51.52
1964	73.30	72.36	71.30	70.84	48.71	26.58
1965	83.29	81.43	79.57	74.26	65.62	56.85
1966	66.86	60.15	53.45	59.07	58.84	58.48
1967	99.01	81.07	63.16	91.84	88.34	84.85
1968	51.98	50.41	48.85	28.50	28.26	28.02
1969	106.14	93.59	81.04	72.53	69.43	66.32
1970	56.57	58.23	55.35	68.54	42.81	17.74
1971	106.10	125.67	95.78	90.63	80.82	70.92
1972	74.59	70.47	69.74	69.01	76.94	56.96
Mean	80.19	78.11	70.95	68.95	61.72	51.82
Soybeans						
1963	74.14	70.12	66.10	64.05	56.10	48.15
1964	50.53	51.32	52.12	53.47	46.46	39.37
1965	66.35	63.72	61.09	50.85	52.93	55.11
1966	66.66	67.63	68.60	70.26	61.64	53.02
1967	81.95	79.19	76.43	61.03	57.43	53.93
1968	73.77	69.72	65.66	63.58	55.56	47.54
1969	78.08	75.09	72.10	60.46	62.87	65.30
1970	37.48	38.44	39.41	34.86	32.93	31.01
1971	87.94	87.81	87.56	87.94	87.94	87.81
1972	118.83	114.88	110.92	88.89	83.80	78.72
Mean	73.57	71.79	70.00	63.54	59.77	56.00

Table A.4 Computed Absolute Deviation from the Mean for Corn and Soybean Activities, Northern Illinois Farms, Soil 76-100, 500-600 Tillable Acres, 1963-1972.

	Planting Period 1	Planting Period 2	Planting Period 3	Planting Period 4	Planting Period 5	Planting Period 6
Corn						
1963	$ 3.89	$ 9.58	$ 20.34	$ -4.69	$ -3.77	$ -.30
1964	-6.89	-5.75	.35	1.89	-13.01	-25.24
1965	3.10	3.32	8.62	5.31	3.90	5.03
1966	-13.33	-17.96	-17.50	-9.88	-2.88	6.66
1967	18.82	2.96	-7.79	22.89	26.62	33.03
1968	-28.21	-27.70	-22.10	-40.45	-33.46	-23.80
1969	25.95	15.48	10.09	3.58	7.71	14.50
1970	-23.62	-19.88	-15.60	-.41	-18.91	-34.08
1971	25.91	47.56	24.83	21.68	19.10	19.10
1972	-5.60	-7.64	-1.21	.06	15.22	5.14
Soybeans						
1963	$ .57	$ -1.67	$ -3.90	$ .51	$ -3.67	$ -7.85
1964	-23.04	-20.47	-17.88	-10.07	-13.31	-16.63
1965	-7.22	-8.07	-8.91	-12.69	-6.84	-.89
1966	-7.91	-4.16	-1.40	6.72	1.87	-2.98
1967	8.38	7.40	6.43	-2.51	-2.34	-2.07
1968	.20	-2.07	-4.34	.04	-4.21	-8.46
1969	4.51	3.30	2.10	-3.08	3.10	9.30
1970	-36.09	-33.35	-30.59	-28.68	-26.84	-24.99
1971	14.37	16.02	17.56	24.40	28.17	31.81
1972	45.26	43.09	40.92	25.35	24.03	22.72

Table A.5 Computed Margins and Absolute Deviations from the Mean for Wheat Activities, Northern Illinois Farms, Soil 76-100, 500-600 Acres, 1963-1972.

	Wheat Margins[5]		Absolute Deviations	
	Early Planting	Late Planting	Early Planting	Late Planting
1963	$ 54.69	$ 45.26	$ 28.67	$ 26.37
1964	22.83	16.37	-3.19	-2.52
1965	16.61	10.56	-9.41	-8.33
1966	49.32	40.07	23.29	21.18
1967	25.89	19.21	-.13	.32
1968	10.55	5.31	-15.47	-13.76
1969	10.67	4.93	-15.35	-13.96
1970	7.61	2.27	-18.41	-16.62
1971	26.58	18.88	.56	-.01
1972	35.46	26.26	9.44	7.37
Mean	26.02	18.89		

APPENDIX B

Table B.1 Expected Return-Risk Trade-off from E-A Criterion--Selected Points

Expected Return (λ)	Total Absolute Deviation (E-A)
$ 28,461	$ 58,106
28,000	52,307
26,000	43,201
24,000	35,942
22,000	31,946
20,000	28,278
18,000	25,057
16,000	21,938
14,000	18,833
12,000	15,915
10,000	13,155

Table B.2. Minimum Dispersion Solutions, Selected Profit Levels

Profit	Corn Acres Planted						Soybean Acres Planted						Wheat Acres Planted	
	Period						Period							
Level	1	2	3	4	5	6	1	2	3	4	5	6	Early	Late
							E-A Formulation							
$28,461	72.3	52.5							84.7	31.1	31.0	36.2	242.2	
$28,400	72.3	72.3		13.0					62.0	15.4	32.9	36.2	246.0	
$28,300	72.3	72.3	6.6	26.4					54.4		33.2	36.9	247.8	
$28,200	72.3	72.3	18.7	26.4					40.6		33.9	36.9	248.9	
$28,100	72.3	72.3	30.8	26.4					26.7		34.5	36.9	250.1	
$28,000	72.3	72.3	42.3	26.4					13.5		35.1	36.9	251.1	
$27,900	72.3	72.3	52.8	26.4					1.5		35.7	36.9	252.1	

Footnotes

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1/ The positive signs in the main portion of the figure indicate resource usages and negative signs indicate resource supplies. Thus, corn land plowing supplies, plowed corn land and corn production uses it. The initial supplies of resources are denoted by positive signs in the right hand column.

2/ We are particulrly grateful to Harold Reiss and Russ Stivers, Agronomy Department, Purdue University, for their assistance in estimating these effects.

3/ James Wilkens at International Harvester has suggested this could be done through cash rental quotation. Others have suggested crop insurance premiums.

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