Staff Paper Series

PAPER PRESENTED AT AAEA ANNUAL MEETING
COLUMBUS, AUG. 10-13, 1975

FOOD AND RESOURCE ECONOMICS DEPARTMENT

Institute of Food and Agricultural Sciences

University of Florida

Gainesville, Florida 32611

INCOMMENSURABLES, TRADEOFFS AND THE MULTIPLE PRODUCT MODELS: WHY THE CONFUSION?

by

Gary D. Lynne

Staff Paper 14 August 1975

Food and Resource Economics Department
Institute of Food and Agricultural Sciences
University of Florida
Gainesville, Florida 32611

Paper presented at AAEA annual meeting, Columbus, Aug. 10-13, 1975.

INCOMMENSURABLES, TRADEOFFS AND THE MULTIPLE PRODUCT MODEL: WHY THE CONFUSION?

Gary D. Lynne
Natural Resource Economist
Food and Resource Economics Department
University of Florida

The ultimate goal of any society is to maximize social welfare. The proper measure of the achievement of this lofty goal, however, has and will continue to be, the subject of considerable debate. The problem is this: there are many "lesser goals" or objectives that society finds important which are not commensurable. Such diverse components of welfare as health and illness, social mobility, physical environment, income and poverty, public order and safety, participation and alienation, and learning, science and art (U.S. Department of Health, Education and Welfare) lead to what is commonly referred to as the "multiple objective problem". Stated somewhat differently, various measures of achievement of the objectives related to these components of welfare cannot at the present time be compressed into one measure of achievement such as dollars, utils, or "welfare". This problem should not cause the economist to despair, however, as the mere existence of money valued and non-money valued goods within the same objective function just means our task as economists is not done (Castle, p. 730) and, I might add, just makes our job a little more difficult, but not impossible.

The outline of this paper is as follows. First, a conceptual model is presented that is useful for dealing with the multiple objective problem. The model is very familiar to, I hope, all production economists and those economists who find the study of welfare economics a useful endeavor. Second, it will be shown where several attempts to deal with the multiple objective problem have gone awry. Third, a simplified example will be presented which illustrates an attempt at applying the conceptual lesson that must be learned by all those analysts concerned with multiple objective planning.

The Conceptual Basis for "Tradeoffs"

The notion of a "tradeoff" is much used and much abused. I'm sure that everyone here today has used the notion at some time or another. One of the most frequent uses is by an individual who is trying to decide on a particular job offer, where he is forced to make various "tradeoffs" regarding salary, location of job, and potential for advancement. One also finds the notion used quite freely in the media and among politicians. Unfortunately, this intuitive notion of a tradeoff which is usually used in the context of a sacrifice of something to gain something else is inadequate to the task of dealing with a multiple objective planning function in water resource development.

A case in point is the recently approved Federal document that is to guide the planning process for water and related land development (U.S. Water Resources Council (WRC), 1973). The notion of a tradeoff is used freely throughout the document, even to the point where the planner is told how to calculate a tradeoff, but nowhere in that document is the concept defined. Several authors writing on the topic of multiple objective planning in professional journals have also used very intuitive notions of tradeoffs, with no apparent concern over the meaning, the conceptual basis, or the interpretation of such calculated values. As a result, as will be shown later, errors in use and interpretation have and will continue to be made. So, what is a "tradeoff"?

The conceptual basis for calculating tradeoffs rests in the multiple product production model and the theory of welfare economics. The essence of this body of theory can be explained with reference to Figure 1. Assume, for the moment, perfect competition in all product and factor markets. Given those "perfect" prices, assume a particular level of expenditure is made on land, labor, capital, and management resources to produce three products, identified as q_1, q_2, and q_3. The relationships among these three products is illustrated in Figure 1. All of the points on the surface represent feasible

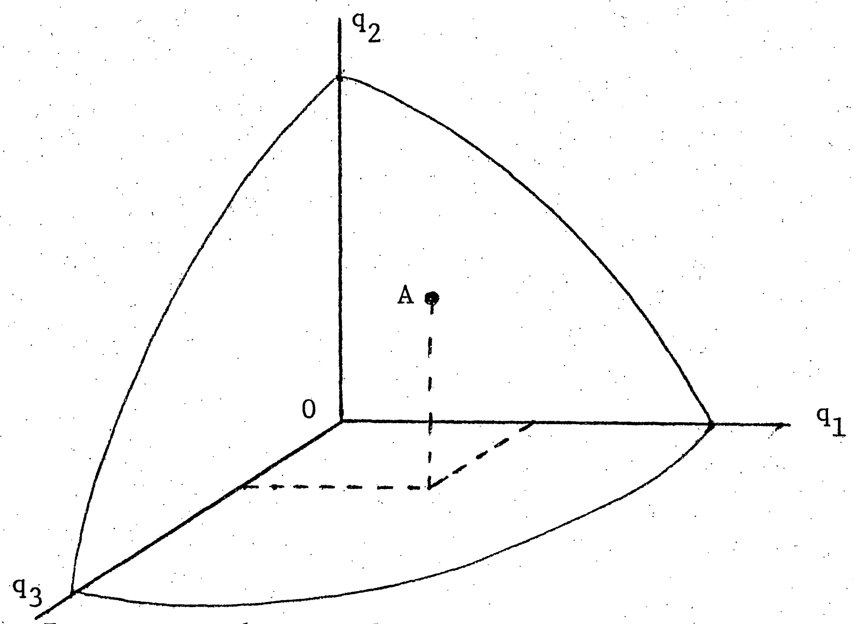

Figure 1. Iso-cost product surface

product combinations for the given, constant cost level. Different levels of expenditure would yield different iso-cost surfaces, some higher and some lower than that depicted in Figure 1. Now, visualize a social welfare, or social indifference, surface lying above and tangent to this particular production surface. Assume this surface was tangent at point A; i.e., the product mix at A is the socially desirable combination. At point A, then, all the proper price ratios are equal to the various rates of product and factor substitution in consumption and production. Also, assuming all the marginal revenues were equal to the marginal costs for each product at point A, we would conclude that society had also chosen the proper scale of this particular productive enterprise.

There are at least two different tradeoffs of concern here, namely the "price tradeoffs" and the "expansion tradeoffs". The price tradeoffs are represented at the point of tangency of the social indifference surface and the iso-cost production surface (at points such as A). These can be more accurately referred to as product-product tradeoffs. Implicit in the selection of point A, of course, is that all factor-factor and factor-product tradeoffs are also satisfied. Stated somewhat differently, at all such tangency points A, the rates of commodity substitution in consumption must equal the rates of product transformation, rates of technical (factor) substitution, and the marginal conditions must equal their proper price ratios.

So-called "expansion tradeoffs" would involve sacrifices in movements between and among surfaces. Assume, for example, that at point A all the marginal revenue--marginal cost relations were satisfied. Consider moving to some point on a higher or lower production surface. The tradeoff is the change in net dollar returns due to the change in product mix. Note that this change in net dollar returns would give us absolutely no indication of the relative values of each product concerned.

Now, this is a pure model and can be criticized easily with regard to the real world. But, there is a conceptual lesson that rings through clearly. Assume, for example, that each of these products represented a proxy for the level of achievement of a particular objective in a multiple objective function for water resources development. Assume, further, that the price of q_1 was not known, but the prices of q_2 and q_3 were market determined. It becomes apparent, then, that the relative value of q_1 at such points as A is given by movements along an iso-cost surface and not from movements between and among surfaces. This quite simple theoretical fact has been missed by many authors writing on the subject of tradeoffs and incommensurables in professional journals. Also, and probably even more unfortunate, the Federal document which is to guide water resource planners actually encourages the calculation of tradeoffs between and among surfaces.

Other Approaches and the Federal Planning Document

The tradeoff calculation process outlined by several authors, including Major (1969, 1970), Maass (1966), Marshall (1974), Marglin (1967), Miller and Byers (1973), Cohen and Marks (1973), Freeman (1969) and recommended in the Federal water planning document (WRC, 1973) could lead to some real distortion in relative valuations of non-commensurable outputs (and inputs). The type of approach taken by these authors is typified by an example provided by Freeman (1969). The data from that article is reporduced here in Table 1. Freeman emphasizes the felt need to use <u>net</u> benefits in this tradeoff approach (p.570). He does not, however, justify this need. Using these data, the argument goes as follows. Decision makers should be provided with data such as that in Table 1.

Table 1

	Net money valued benefits	Measurable unvalued benefits
Project A		
Design No. 1	\$ 9,000	4 units
Design No. 2	\$10,000	6 units
Design No. 3	\$12,000	5 units
Design No. 4	\$13,000	4 units
Design No. 5	\$14,000	3 units

The relative value of the non-money valued benefits can then be determined by the choice made by the decision makers. If the decision body chooses design no. 3, this would imply that one unit of non-money valued benefits is worth at least \$1000; i.e., the decision body was willing to sacrifice \$1000 to increase the non-money valued benefit from 4 to 5 units. However, its relative value is less than \$2,000, because the decision body was unwilling to accept design no. 2. Therefore, it is concluded, the "price" of the non-valued unit, p_n, is in the range $\$1000 < p_n < \$2000$.

This approach is valid only if all five of the designs have the same cost. This is the only way, as argued in the previous section of this paper, that "pric tradeoffs" can be calculated. Now, more than likely, five different designs for a project will have different costs; therefore, the estimate of p_n in the range of \$1000 - \$2000 is distorted. The net benefit tradeoff is equivalent to the "expansion tradeoff" discussed earlier in this paper.

Now, of what use are the "expansion tradeoffs"; i.e., is knowledge of the net benefits gained or sacrificed in movements between and among surfaces useful? My response to this has to be a qualified no. If the purpose of tradeoff calculations and analysis is to give some indication of the relative valuation of non-money valued goods, calculation of an expansion tradeoff is a fruitless endeavor. As long as one or more prices are missing, estimated changes in net benefits are distorted. It seems appropriate to conclude that "expansion tradeoffs" are useful only when all prices are known. For that case, such calculations are equivalent to examining marginal revenue-marginal cost conditions at various points on the production surface.

The same type of error could be committed under the auspices of the Federal planning document. In fact, the error is encouraged as given in the statement (WRC, 1973, p.24830):

> To facilitate comparisons and tradeoffs of beneficial and adverse effects measured in nonmonetary terms with beneficial and adverse effects measured in monetary terms, one alternative plan should be formulated in which optimum contributions are made to the component needs of the national economic development objective. Additionally, during the planning process at least one alternative plan will be formulated which emphasizes the contribution to the environmental quality objective. Other alternative plans reflecting significant tradeoffs between the national economic development and environmental quality objectives may be formulated so as not to overlook a best overall plan.

It is doubtful that plans which maximize gains to national economic development in contrast to plans that provide for enhancement of environmental quality will have the same cost. Therefore, tradeoffs would have to be calculated among and between iso-cost surfaces, and would necessarily be in error, in terms of giving an indication of relative values.

Another problem is also of importance here. Many of the components of the environmental quality objective in water resource development are really inputs into the production of products from water. Consider, for example, a wilderness area or a wild, scenic, white water river that would be inundated by a dam-reservoir complex. These two components can be used directly as final consumption goods or used as inputs (by inundation) into the production of, say, water for irrigation or hydro power generation. Therefore, the relevant tradeoffs for these components are the factor-product and the factor-factor price tradeoffs. Again, costs would have to be held constant along the relevant factor-product curves in order to eliminate distortion in the estimates. Major (1970) did not remove this distortion in his development of a "net benefit transformation curve" between net dollar benefits and acres of ecological area. Also, the example used by Cohen and Marks (1971), where they developed a transformation curve between net dollar benefits and animals lost from building the Alaskan pipeline, is in error for the same reason. This failure to recognize the difference between products and factors is equivalent to the concern expressed by Castle and Youmans regarding different levels in the means-objective-goals scheme, where national income is viewed as an input to higher goals (p.1663).

Empirical Estimation of Tradeoffs in Dam-Reservoir Complexes

The procedure for estimation-quantification of a multiple output production surface descriptive of the production processes involved in dam-reservoir complexes is necessarily complex. Large quantities of information are needed to identify such surfaces. Water resource planning and action agencies will find the planning process more expensive as a result of specified needs to provide tradeoff estimates to Congress under the auspices of the Federal planning document. The following example of the estimation process serves to illustrate one approach that might be taken to establish production tradeoffs, where the non-money valued component is a product or output of a dam-reservoir complex (see Lynne, 1975, for more detail).

The basic premise upon which the following approach rests is that a government planning-action agency can be viewed as a quasi-firm, which (at least in the planning stage) can organize the factors of production in a least cost manner to produce various "water products", such as water for irrigation, water for recreation, and water for municipal use. Further, it can be argued there are really only two dimensions of water stored in a reservoir that ultimately have economic value, namely volume and surface area of the water (at least at any given point in time). Given these two premises are representative of reality, the following functions become relevant, namely

$$V = V(C) \tag{1}$$

$$A = A(V) \tag{2}$$

where

C = annual dollar costs over a reasonable period of time (say, 50 years), which represents the total cost of the proposed project.

V = volume of the proposed reservoir, representing the maximum amount of water that could be stored at any given point in time.

A = surface area of the reservoir.

These fundamental relations can be estimated from engineering data.

The relations specified in (1) and (2) were estimated for a dam site in Western North Dakota to give (Lynne, 1975, p.15):

$$V = -17893.0 + 0.31070C + 0.00000050357C^2 \tag{3}$$
$$(-4.828) \quad (10.633) \quad (12.330)$$
$$R^2 = .99939 \qquad \text{d.f.} = 7$$

$$A = 749.33 + 0.03653V - 0.000000025712V^2 \tag{4}$$
$$(6.520) \quad (20.910) \quad (-5.295)$$
$$R^2 = .99267 \qquad \text{d.f.} = 24$$

The t-statistics (below each regression coefficient) are significant at acceptable levels.

The multiple product surface can now be developed in the following manner. Assume there were three water products of concern, namely, water for agricultural (irrigation) use (W_a), water for municipal use (W_m), and water for recreation use (W_r), where the latter product is measurable in surface area of the reservoir. Assume, further, that the price of W_r is not known. Therefore, information regarding tradeoffs on the physical production side would be useful. The multiple product (or iso-cost surface) function then becomes, using the coefficients of equation (4),

$$W_r = 749.33 + 0.03653(V-W_a-W_m) - 0.000000025712(V-W_a-W_m)^2 \tag{5}$$

where surface area (in acres) of the water in the reservoir is used as a measure of water available for recreation (W_r).

Given various levels of annual cost, each representing a particular size of structure at the site, the volume of storage available is given by equation (3). This volume estimate can then be used in equation (5). Assuming there would be sufficient water to fill the reservoir by the beginning of some particular use period (say a joint recreation, irrigation, and municipal use season), the equation can be used to determine the relevant isocost surface. The results from using this approach at a proposed dam site in the Knife River Basin, North Dakota, for a $200,000 annual cost level, are presented in Table 2.

Table 2. Combinations of water for irrigation, municipal (and industrial) uses and recreation, for $200,000 annual cost, Bronco dam site, Knife River Basin, North Dakota.

Water for Municipal and Industrial (W_m)	Water for Irrigation (W_a)	Water for Recreation (W_r)	$\frac{dW_a}{dW_r} = \frac{dW_m}{dW_r}$ *
(acre feet)	(acre feet)	(acres)	
0	38,634	1673	-28.41
0	32,929	1873	-28.64
0	27,176	2073	-28.89
0	21,372	2273	-29.14
0	15,519	2473	-29.40
0	9,612	2673	-29.66
0	3,653	2873	-29.94
0	0	2995	-30.10
10,000	28,634	1673	-28.41
10,000	22,929	1873	-28.64
10,000	17,176	2073	-28.89
10,000	11,372	2273	-29.14
10,000	5,519	2473	-29.40
10,000	0	2660	-29.60
20,000	18,634	1673	-28.41
20,000	12,929	1873	-28.64
20,000	7,176	2073	-28.89
20,000	1,373	2273	-29.14
20,000	0	2320	-29.20
30,000	8,634	1673	-28.41
30,000	2,929	1873	-28.64
30,000	0	1975	-28.80

* The rates of change (tradeoffs) in W_a and W_m for changes in W_r. Both are equal at every point. Also, $(dW_a/dW_m) = -1.0$ at every point on the surface.

These types of tables would be useful to decision bodies concerned with evaluating tradeoffs. Assume, for illustrative purposes, the decision body selected the point where W_m = 10,000, W_a = 22,929, and W_r = 1873 (Table 1). Assume further, the prices of W_m and W_a were both \$30.00 per acre foot. The implicit value of W_r at that point is then (28.64)(\$30.00) = \$859.20 per acre.

A Caveat

A cautionary note is in order. The true marginal value, or price, of a product (or an input) can only be determined if the price is set in a perfectly functioning market situation. The calculation of relative values from production surfaces or along factor-product transformation curves is useful only to the extent that it provides some range of alternatives from which society, or some decision body which accurately reflects society's preferences, can pick the true value. In the real world of water planning, all the analyst can hope to do is to delineate some possibilities, some ranges on the production surface. If we can ascribe the power of price determination to Congress, the selection of a particular project or design of a project reflects society's valuation. Stated somewhat differently, if planners present an array of price ratios or tradeoffs to Congress, and Congress can accurately reflect society's valuation of particular product mixes, the choice of a particular project (or design of a project) will set at least minimum prices of the non-money valued goods.

Summary and Conclusions

It has been argued that the notion of a tradeoff has a conceptual base which should be well known to most economists, but has been misapplied by many. The purely theoretical multiple product production model should form the guiding framework for calculating tradeoffs. The primary error made by several analysts has been ignoring the fact that relative values of products and/or factors cannot be determined from the comparison of projects, or designs of projects, having different costs. The following conclusions can be highlighted:

1. the relevant tradeoffs are really approximations of price ratios. The relative values of non-money valued goods can then be estimated from the dollar value of money valued goods.

2. if the non-money valued component is a product in the production process of concern, then product-product tradeoffs should be determined.

3. if the non-money valued component is a factor, a factor-product and/or factor-factor tradeoff should be calculated.

The proper application of this model provides a considerable challenge to the empirical researcher. Hopefully, our empirical techniques and our data sources will improve to the point where we can calculate true, distortion-free, production tradeoffs in the near future. However, the conceptual lesson outlined in this paper must first be learned in order to remove the apparent confusion regarding incommensurables and tradeoffs.

References

Castle, Emery N. "Economics and the Quality of Life", AJAE, Vol. 54, No.5, December 1972, pp. 723-736.

Castle, Emery N. and Russell C. Youmans. "Economics in Regional Water Research and Policy". AJAE, Vol. 50, No. 5, December 1968, pp. 1655-1670.

Cohen, Jared L. and David H. Marks. "Multiobjective Screening Models and Water Resource Investment". Water Resources Research, Vol. 9, No. 4, August 1973, pp. 826-836.

Freeman, A. Myrick III. "Project Design and Evaluation with Multiple Objectives". The Analysis and Evaluation of Public Expenditures: The PPB System. U.S. Government Printing Office, Washington, D.C., 1969, pp. 565-578.

Lynne, Gary D. "Tradeoff Ratio Calculations in Water Resources Planning". Special Report 429, Ag. Exp. Sta., Oregon State University, January 1975.

Maass, Arthur. "Benefit-Cost Analysis: Its Relevance to Public Investment Decisions". Quarterly Journal of Economics. Vol. 80, No. 2, 1966, pp. 208-226.

Major, David C., et al, "Multiple Objective Redesign of the Big Walnut Project". Mimeo, Department of Civil Engineering, MIT, April 1970.

Major, David C. "Benefit-Cost Ratios for Projects in Multiple Objective Investment Programs". Water Resources Research. Vol. 5, No. 6, December 1969, pp. 1174-1178.

Marglin, Stephan A. Public Investment Criteria. The MIT Press, 1967, 103pp.

Marshall, Harold Emory. "Cost Sharing and Multiobjectives in Water Resource Development". Water Resources Research. Vol. 9, No. 1, February 1974, pp. 1-10.

Miller, W. L. and D. M. Byers. "Development and Display of Multiple Objective Project Impacts". Water Resources Research. Vol. 9, No. 1, February 1973, pp. 11-20.

U.S. Dept. of Health, Education, and Welfare. Toward a Social Report. Washington, U.S. Government Printing Office, 1969.

U.S. Water Resources Council, "Establishment of Principles and Standards for Planning". Federal Register. Vol. 38, No. 174, U.S. Government Printing Office, Washington, D.C., September 10, 1973. pp. 24778-24869.