The Development of a Corn Crop Response Model for an Economic Irrigation Scheduling Model+

by

Thomas H. Morgan*

Irrigation scheduling is becoming a more crucial management function because of the increasing costs associated with irrigation and the increasing shortages of irrigation water. Jensen [6] and others have developed irrigation scheduling models, but such models estimate only physical conditions, such as daily soil moisture depletion, but ignore economic factors.

The objective of this project is to develop a corn crop response relationship that can be used in an economic irrigation scheduling model. This paper covers a brief discussion of corn growth as it relates to this project, the development of the corn crop response model with water as the input and grain yield as the product, and the testing of the model.

Growth and Development of the Corn Plant as it Effects Grain Yield

The amount of grain produced by the corn plant will depend upon the rate of growth. Therefore, to estimate yield one needs to be able to estimate the rate of growth and any variable that effects the rate of growth. In our model all variables except water were held constant. To develop this model we needed to develop a growth function and estimate the relationship that water has with this growth function.

In this model we are interested in how water effects grain yield and plant growth. We are interested in the latter only to the extent that it

+Submitted to the Contributed Papers Session of the 1975 Summer AAEA Meeting

*Undergraduate in Agricultural Economics, Kansas State University.

effects grain yield. For our model we used a mid season hybrid that silks at 66 days after emergence (Figure 1). All normal corn plants follow the same general pattern of development though specific times of silking, maturity and other stages may vary between different hybrids [5].

It can be noted from Figure 1 that by silking all the accumulated leaf dry weight has occured. This indicates that at this point the plant has developed nearly all of its leaf area or photosynthetic capacity. All grain development starts at this point. We propose that ear development and grain yield are a function of plant development prior to silking and water available for plant uptake during ear development. Therefore we have divided the growth process into two functions using the accumulated dry matter curves in Figure 1 to estimate these functions.

The Exponential Vegetative Growth Function

The vegetative growth function is the growth curve for the corn plant prior to silking (the first 60 days after emergence) at which time it is building the photosynthetic factory from which it will produce the grain following silking. The corn plant during this period develops slowly at first, but as more leaves are exposed to sunlight the rate of growth increases gradually. By the 40th day enough leaves are exposed so that growth is rapid.

The exponential equation was used to estimate the vegetative growth function because it is of compatible shape (Figure 2). The exponential equation is

$$G_D = e^{\gamma+\delta D} \qquad (1)$$

where G_D is growth at day D ($0<D\leq 60$) and γ and δ are unknown constants. γ and δ were estimated from Hanway's data (Figure 1) by taking the log transformation of (1) and using least squares regression. The least squares

estimate for γ and δ are -1.7 and 0.094, respectively with an $R^2 = .947$.

The graph of the vegetative growth function using Hanway's data and the graph of equation (4) can be seen in Figure 2. In view of their compatibility, equation (4) appears to be a good approximation of the vegetative growth function.

To obtain the rate of growth we take the first derivative of equation (1) and substitute (1) into the result to obtain

$$\frac{dG_D}{dD} = \delta G_D \qquad (2)$$

where δ is the percent growth rate which is constant over time. So if water stress effects growth rate, it has equal effect over the whole vegetative growth stage.

Since growth in period D is dependent on accumulated growth in the previous period, D-1, we used a recursive form of equation (1),

$$G_D = e^{\gamma+\delta D} = e^{\delta}\, e^{\gamma+\delta(D-1)} = e^{\delta}\, G_{D-1} \qquad (3)$$

In the recursive form γ is no longer the y-intercept (since γ is lost in the development of the recursive form), but the value for G_{D-1} when D=1 becomes the y-intercept. In this function it is a very small number less than one since there is very little dry matter accumulation at emergence.

Using equation (3) as the growth function, we developed the following as the growth relationship between water and vegetative growth for the first 60 days after emergence:

$$G_D = \left[e^{\delta}\right]^{\alpha_D} G_{D-1} \qquad \text{for } 1<D\leq 60 \qquad (4)$$

where α_D indicates the plants ability to absorb water as soil moisture stress varies. α_D, which equals one at field capacity and zero at permanent wilting point, was proposed by Jensen as

$$\alpha_D = \mathrm{Ln}(AM_j + 1)/\mathrm{Ln}(101)$$

where AM_j is percent of available soil moisture. He proposed this function to reflect plants ability to absorb soil moisture. In our formulation growth is unrestricted with 100 percent available water, that is at field capacity, and growth stops at zero percent available water, that is at permanent wilting point.

We tested this function by computer simulation. We used the recursive growth relationship in equation (4) and 0.094 for δ. α_D was allowed to take on only three values in each simulation. This was done by breaking the 60 day vegetative growth period into three periods of 20 days each. We did this in order to simplify the testing of this function.

The following simulations were made:

		α's for the first period	α's for the second period	α's for the third period
Run 1				
Simulation	1	1.00	1.00	1.00
	2	.20	1.00	1.00
	3	1.00	.20	1.00
	4	1.00	1.00	.20
Run 2				
	1	1.00	1.00	1.00
	2	.80	1.00	1.00
	3	1.00	.80	1.00
	4	1.00	1.00	.80
Run 3				
	1	1.00	1.00	1.00
	2	.80	.80	.80
	3	.40	1.00	1.00
	4	1.00	.60	.80

The results of these simulations can be seen in Figures 3, 4 and 5.

It can be readily seen that limiting water to the point that α is .20 for 20 days has a significant effect on vegetative growth (Figure 3). It can also be seen that it does not matter when water is limiting if it is limiting for the same length time and to the same extent (Figures 3 and 4), or if the total amount of water that is available during the vegetative growth stage is the same, the distribution of it during this stage is not crucial (Figure 5). These latter properties are a result of the constant percent growth rate. This property appears to be consistent with agronomic studies [12].

<u>The Modified Logistics Ear Development Function</u>

For the ear development function we propose that the development of the cob, silks and grain are indicative of yield since if water is withheld at the time the silks are to be produced it will reduce the number of silks, result in poor pollination of the ovules, and restrict the number of kernels that develop. To estimate this function we used the combined dry matter accumulation curves for the cobs, silks, and grain parts. The function goes from the beginning of the development of these parts (60 days after emergence) to maturity (126 days after emergence). This function is dependent on the accumulated development of the vegetative growth function at day 60 and available water thereafter. It takes the form of first increasing at an increasing rate and then as it matures it increases at a decreasing rate until maturity. We used a modified logistics equation to estimate this function.

The logistics equation says growth slows as the plant approaches some maximum amount of development. The differential equation for grain development, H_t, assuming a logistic curve would be

$$\frac{dH_t}{dt} = \delta' H_t \left(\frac{K - H_t}{K}\right) \tag{5}$$

where t=D-60. The term $\left(\frac{K - H_t}{K}\right)$ says that as H_t, accumulated ear develop- at day t, approaches K, the maximum amount of ear development attainable, the growth rate slows.

We suggest that a truer relationship would be that growth slows as the plant approaches maturity.

By substituting T, date of maturity, for K and t, time in ear development stage, for H into the term $\left(\frac{K - H_t}{K}\right)$ in equation (5) we have

$$\frac{dH_t}{dt} = \delta' H_t \left(\frac{T-t}{T}\right) \tag{6a}$$

where $\left(\frac{T-t}{T}\right)$ says that the rate of ear development slows as the plant nears maturity.

Equation (6a) is a derivative or the growth rate. To obtain the growth function we rewrite equation (6a) in the form of partial fractions,

$$\frac{dH_t}{H_t} = \delta' \left(\frac{T-t}{T}\right) dt = \delta' d_t - \delta' \frac{t}{T} dt \qquad (6b)$$

and integrate both sides to obtain

$$\log H_t = \delta' t - \frac{\delta' t^2}{2T} + C \qquad (7a)$$

where C is the constant of integration. Rewriting equation (7a) we have

$$\log H_t = C + \delta' \left(t - \frac{t^2}{2T}\right) \qquad (7b)$$

where H_t is accumulated ear development at day t, t=D-60, $0<t\leq 66$, $0<H<1$, C and δ' are constants. Using Hanway's data and equation (7b) we obtained regression estimates for C and δ' of -3.573 and 0.109, respectively and an $R^2 = .994$. To get the growth function we take the exponential of both sides to obtain

$$H_t = e^{C+\delta'(t-t^2/2T)} = e^{-3.573 + 0.109(t-t^2/2T)} \qquad (8)$$

where $0<t\leq 66$ and T=66.

The graph of the ear development function and the above estimate are in Figure 6. From Figure 6 it can be seen that equation (8) is a good approximation of this function. If the estimate was shifted down about three values it would be an even better estimate. This in fact happens when the estimate is used in recursive form since C, the y-intercept, is dropped.

The development of equation (8) into recursive form follows:

$$H_t = e^{C+\delta' t - (\delta'/2T)t^2} = e^{C+\delta' t - (\delta'/2T)((t-1)+1)^2}$$

$$H_t = e^{C+\delta' t - (\delta'/2T)((t-1)^2+2t-1)}$$

$$H_t = e^{C+\delta' t - (\delta'/2T)(t-1)^2 - (\delta'/2T)(2t)+\delta'/2T}$$

$$H_t = e^{C+\delta'(t-1) - (\delta'/2T)(t-1)^2}\, e^{\delta'-\delta' t/T + \delta'/2T} \qquad (9)$$

Since ear development for the previous day is

$$H_{t-1} = e^{C+\delta'(t-1) - (\delta'/2T)(t-1)^2} \tag{10}$$

we can substitute H_{t-1} from equatin (10) into equation (9) to obtain

$$H_t = H_{t-1}\, e^{\delta'-\delta' t/T + \delta'/2T} \tag{11a}$$

Rewriting equation (11a) we have

$$H_t = e^{\delta'+\delta'/2T}\, e^{-\delta' t/T}\, H_{t-1} \tag{11b}$$

Equation (11b) was used for the ear development function in the growth relationship for $0<t\leq 66$,

$$H_t = \left[e^{\delta'+\delta'/2T}\, e^{-\delta' t/T}\right]^{\alpha_t} H_{t-1} \tag{12}$$

When t=1 we used a very small number for H_{t-1} in testing this relationship separately but when we combined it with the exponential function we used the ratio of actual vegetative growth over optimum vegetative growth (i.e. water not limiting) to indicate the relative amount of photosynthetic capacity available to provide for ear development.

The growth relationship in equation (12) with 0.109 and 66 being used for δ' and T respectively was tested by computer simulation. α_t was allowed to take only three different values for each simulation for the purpose of simplification. Each period consisted of 22 days. The following is a list of the simulations and α_t's used:

Simulation	α's for the first period	α's for the second period	α's for the third period
1	1.00	1.00	1.00
2	.20	1.00	1.00
3	1.00	.20	1.00
4	1.00	1.00	.20

The results can be seen in Figure 7.

It is apparent from Figure 7 that the most critical time for water during ear development is early in this process, and the least critical period is the last days during ear development. Both properties correspond closely to what is already known about the relationship between water and corn grain yield [12]. At this point the properties of this function appear to be quite desireable.

The Model

Involved in this model are the interrelationships between water, plant growth, grain yield and evapotranspiration. The following is a brief discussion of the interrelationships between these factors and how they are used in this model.

The percentage of available soil moisture, AM_j, was used as the independent variable. Available soil moisture is the amount of water held in the soil which is available to the plant for uptake. It is the amount of water held in the soil between permanent wilting point, the soil moisture tension at which the plant first undergoes complete wilting without recovery in a saturated atmosphere, and field capacity, the maximum amount of water a soil will hold against gravitational forces after a soil has been saturated. AM_j was simplified by limiting it to nine periods of 14 days each where $j = 1, \ldots, 9$.

To estimate water use by corn plants we used daily evapotranspiration, ET_D. ET_D was estimated by

$$ET_D = \alpha_D \beta_D ET_D^* \qquad (13)$$

where ET_D^*, the evaporative potential of the atmosphere, was estimated from meteorological data, α_D is as described before, β_D is the crop coefficient which indicates the relationship between plant size and evapotranspiration. Two plant factors directly influencing evapotranspiration are leaf area and root mass. For the purpose of this model we used empirical data from 1970 tests at Scandia, Kansas for β_D. The general form of β_D for corn is given in Figure 8.

For the growth relationship in the model we used the following three equations:

$$X_D = \left[e^{0.094}\right]^{\alpha_D} X_{D-1} \tag{14a}$$

$$X'_{60} = \left[X_{60/0.286}\right] \tag{14b}$$

$$X_D = \left[e^{0.109}\ e^{-0.00165t}\right]^{\alpha_D} X_{D-1} \tag{14c}$$

Equation (14a) was obtained from equation (4) by substituting 0.094 for δ', X for G where $0<D\leq 60$ and X_{D-1} was a very small number when D=1. To make the transition from the vegetative growth relationship in equation (14a) to the ear development relationship in equation (14c) we used X'_{60} (equation (14b)) for X_{D-1} in equation (14c) when D=61 where is the ratio of X_{60}, actual vegetative growth at day 60 over 0.286, the value for optimum vegetative growth at day 60 (equation (14b)). For the last 66 days equation (14c) was used. It was obtained from equation (12) by substituting 0.109 for δ', 66 for T, X for H, and D for t, where $60<D\leq 126$ and t=D-60. At maturity, D=126, X_{126} was expressed as the percentage of optimum yield, X'_{126}, where optimum yield is the value for X_{126} when water is not limiting over the whole growth period, $0\leq D\leq 126$. Water use efficiency, WUE, was calculated by dividing percent of optimum yield, X'_{126}, by accumulated evapotranspiration, $\Sigma_{\forall D} ET_D$, as follows:

$$WUE = X'_{126} \Big/ \Sigma_{\forall D} ET_D \tag{15}$$

<u>The Testing of the Model</u>

The model was tested by computer simulation at various levels of available soil moisture (Tables 1 and 2). The most critical period is period five, the period surrounding silking. Past observations indicate that this property is correct [12]. Grain yield is lowest when water is withheld during the last five periods, the ear development periods. This also corresponds with past observations [12]. Yields tend to be higher when the water is distributed over the growing season instead of applying all needed during some periods and none during others. Another property of this model, as a result of the exponential function, is that water distribution is not critical during the vegetative growth periods.

The most efficient combination is when water is not limiting throughout the growing season. If this were true there would not be an economic problem, the decision would be to apply all needed water or to apply no water. We know that is not the experience of irrigators.

To check this we used the method of Lagrange multipliers to maximize a simplified version of the growth relationship,

$$X_j = \left[V_j\right]^{\alpha_j} X_{j-1} \qquad j = 1, \ldots, 9. \tag{16}$$

where V_j is the appropriate growth function (equations (14a)(14b) and (14c)) and each j represents a 14 day period, subject to water available to be used, S, equaling accumulative evapotranspiration (water use), $\sum_{\forall j} \alpha_j \beta_j ET_j^*$, as follows:

$$S = \sum_{\forall j} \alpha_j Z_j \qquad j = 1, \ldots, 9. \tag{17}$$

where j is as above, and since β_j and ET_j^* are not allowed to vary we combined β_j and ET_j^* to become Z_j thus simplifying the formulation without effecting the solution.

For the last period of the growth relationship we have the following:

$$X_J = \left[V_J\right]^{\alpha_J} X_{J-1} \tag{18a}$$

$$X_J = \prod_{\forall j} V_j^{\alpha_j} X_0 \tag{18b}$$

$$\log X_J = \sum_{\forall j} \alpha_j (\log V_j) X_0 \tag{18c}$$

Since X_0 is a constant we dropped it when we formed the maximization criterion which follows with the constraint:

$$\text{Maximize} \qquad \sum_{\forall j} \alpha_j \log V_j \tag{19a}$$

$$\text{Subject to} \qquad S = \sum_{\forall j} \alpha_j Z_j \tag{19b}$$

where j = 1, ..., 9.

Using the method of Lagrange multipliers we first construct the auxiliary function, and its partial derivatives

$$\Lambda = \sum_{\forall j} \alpha_j \log V_j - \lambda(S - \sum_{\forall j} \alpha_j Z_j) \qquad (20)$$

$$\frac{\partial \Lambda}{\partial \alpha_j} = \log V_j - \lambda Z_j \qquad (21a)$$

$$\frac{\partial \Lambda}{\partial \lambda} = S - \sum_{\forall j} \alpha_j Z_j \qquad (21b)$$

When the partial derivatives (equations (21a) and (21b)) are set equal to zero the constraint is met and we have maximized the function resulting in

$$\lambda = \log V_j/Z_j \qquad \text{for all } j. \qquad (22)$$

In analyzing this function we would expect the following relationship should exist:

$$\frac{\log V_j}{Z_j} = \lambda = \frac{\log V_{j'}}{Z_{j'}} \qquad (23)$$

where $j \neq j'$. Therefore the following should be true:

$$\frac{\log V_j}{\log V_{j'}} = \frac{Z_j}{Z_{j'}} = \frac{\beta_j \; ET^*_j}{\beta_{j'} \; ET^*_{j'}} \qquad (24)$$

This relationship would hold true if both V_j and Z_j were not predetermined independently of each other. The case of our model is that V_j is determined by one function and both β_j and ET^*_j, components of Z_j, are both empirical functions determined before hand. The result is that equation (22) says to apply as much water as needed to the period with the highest λ and then do the same to each next highest λ until there is no more water available.

If either β or ET* were related to growth V_j then there would not be the above problem, ET* is not related to growth because it is the evaporative potential of the atmosphere and is measured from meteorological data. The β values used in this model were empirical data taken from experiments in Scandia, Kansas. This in effect made β a function of time, but β should actually be a function of growth as described earlier.

If a function could be developed that would represent β as a function of plant growth then this problem could be corrected. We propose to replace our current β's with one that would be a function of growth. We propose that a constant times the first derivative of the growth function as estimated by using a modified logistics as described before would result in a function of the same general shape as the one we are now using but will also be a function of growth.

Conclusion

At this point our model appears to give a realistic grain yield response to changes in available water. Further testing and comparisons to actual field trials will be useful in determining this models reliability and its limitations. Of immediate concern will be the development of a realistic β function as discussed earlier. With a useable β function we will be able to begin work on testing this models usefulness in economic irrigation scheduling.

We believe that a model such as this will be useful to the irrigation researcher in planning his field trials, since many computer simulations can be run quickly at a relatively low cost, allowing him to check only those most critical to his project. We also hope that this model will be of use to irrigation service groups in scheduling their constituents irrigation. Most important could be the models use in making policy decisions in areas where mining of non-renewable sources of irrigation water is taking place.

References

1. Biere, Arlo W. "Argonomic Production Functions with Water as an Input: A Review of the Arts" unpublished.

2. Biere, Arlo W. and Jong-i Perng "The Measurement of the Demand for Irrigation Water with Special References to Southwest Kansas," Project Completion Report for the Kansas Water Resources Research Institute, 1971.

3. Bonner, J. "Water-transport: This Classical Problem in Plant Physiology is Becoming Amenable to Mathematical Analysis," Science 129: 447-448, 1958.

4. Denmead, O.T. and R.H. Shaw, "Availability of Soil Water to Plants as Affected by Soil Moisture Content and Meteorological Condition," Agronomy Journal 54: 385-389, 1962.

5. Hanway, John J. "How a Corn Plant Develops" Iowa State University Extension Bulletin, June, 1971.

6. Jensen, M.E., J.L. Wright, and B.J. Pratt, "Estimating Soil Moisture Depletion from Climate and Soil Data," paper presented at 1969 Winter Meeting of American Society of Agricultural Engineers, Paper No. 69-941, 22 pp.

7. Jensen, M.E., "Water Consumption by Agricultural Plants," in Water Deficits and Plant Growth, T.T. Kozlowski (ed.) New York, Academic Press, 1968.

8. Moore, C.V., "A General Analytical Framework for Estimating the Production for Crops Using Irrigation Water," Journal of Farm Economics 63: 876-888, November, 1961.

9. Pielou, E.C., An Introduction to Mathematical Ecology, New York Wiley-Interscience, 1969.

10. Stewart, J. Ivan and Robert M. Hagan, "Predicting Effects of Water Shortage on Crop Yield," Proc. Soc. Civil Engrs. J. of Irrigation and Drainage Div., 55: 91-104, March, 1969.

11. Vanderlip, R.L., "How a Sorghum Plant Develops," Kansas Cooperative Extension Bulletin, Kansas State University, May, 1972, 19 pp.

12. Wormley, George W., "Moisture Short? Precondition Corn or Grow Milo," Farm Journal, Vol. 99, Number 2, February, 1975.

TABLE 1. MODEL TEST RESULTS ORGANIZED BY PERCENT AVAILABLE MOISTURE, AM(J) WHERE J = 1, . . ., 9

YIELD	EVAPOTRANSPIRATION	WATER USE EFFICIENCY	AM(1)	AM(2)	AM(3)	AM(4)	AM(5)	AM(6)	AM(7)	AM(8)	AM(9)
26.712	21.073	1.268	0.	100.	100.	100.	100.	100.	100.	100.	100.
26.712	20.869	1.280	100.	0.	100.	100.	100.	100.	100.	100.	100.
26.712	20.358	1.312	100.	100.	0.	100.	100.	100.	100.	100.	100.
26.712	19.570	1.365	100.	100.	100.	0.	100.	100.	100.	100.	100.
24.998	18.766	1.332	100.	100.	100.	100.	0.	100.	100.	100.	100.
32.119	18.062	1.778	100.	100.	100.	100.	100.	0.	100.	100.	100.
44.383	17.702	2.507	100.	100.	100.	100.	100.	100.	0.	100.	100.
61.328	18.046	3.398	100.	100.	100.	100.	100.	100.	100.	0.	100.
84.744	18.630	4.549	100.	100.	100.	100.	100.	100.	100.	100.	0.
1.852	4.668	0.397	100.	100.	100.	100.	0.	0.	0.	0.	0.
7.409	7.536	0.983	100.	100.	100.	100.	100.	0.	0.	0.	0.
23.066	11.109	2.076	100.	100.	100.	100.	100.	100.	0.	0.	0.
51.972	15.042	3.455	100.	100.	100.	100.	100.	100.	100.	0.	0.
67.003	16.906	3.963	100.	100.	100.	100.	100.	100.	100.	10.	0.
71.754	17.409	4.122	100.	100.	100.	100.	100.	100.	100.	20.	0.
74.776	17.712	4.222	100.	100.	100.	100.	100.	100.	100.	30.	0.
77.024	17.929	4.296	100.	100.	100.	100.	100.	100.	100.	40.	0.
78.826	18.099	4.355	100.	100.	100.	100.	100.	100.	100.	50.	0.
80.336	18.238	4.405	100.	100.	100.	100.	100.	100.	100.	60.	0.
81.638	18.356	4.448	100.	100.	100.	100.	100.	100.	100.	70.	0.
82.785	18.458	4.485	100.	100.	100.	100.	100.	100.	100.	80.	0.
83.813	18.549	4.518	100.	100.	100.	100.	100.	100.	100.	90.	0.
84.744	18.630	4.549	100.	100.	100.	100.	100.	100.	100.	100.	0.
92.355	20.191	4.574	100.	100.	100.	100.	100.	100.	100.	100.	10.
94.522	20.612	4.586	100.	100.	100.	100.	100.	100.	100.	100.	20.
95.851	20.865	4.594	100.	100.	100.	100.	100.	100.	100.	100.	30.
96.817	21.047	4.600	100.	100.	100.	100.	100.	100.	100.	100.	40.
97.578	21.190	4.605	100.	100.	100.	100.	100.	100.	100.	100.	50.
98.207	21.306	4.609	100.	100.	100.	100.	100.	100.	100.	100.	60.
98.743	21.405	4.613	100.	100.	100.	100.	100.	100.	100.	100.	70.
99.211	21.491	4.616	100.	100.	100.	100.	100.	100.	100.	100.	80.
99.626	21.566	4.619	100.	100.	100.	100.	100.	100.	100.	100.	90.
100.000	21.634	4.622	100.	100.	100.	100.	100.	100.	100.	100.	100.
9.379	10.450	0.897	90.	60.	60.	50.	30.	15.	10.	0.	0.
18.858	13.736	1.373	90.	90.	90.	60.	35.	25.	10.	5.	5.
47.522	18.664	2.546	90.	60.	55.	80.	85.	90.	60.	35.	15.
67.944	19.962	3.404	95.	90.	90.	90.	80.	80.	65.	55.	50.
77.359	20.901	3.701	90.	90.	90.	90.	85.	85.	85.	90.	75.
5.657	9.248	0.612	85.	55.	40.	35.	25.	10.	5.	0.	0.
28.010	17.958	1.560	80.	55.	40.	35.	55.	80.	65.	30.	20.
37.133	18.875	1.967	80.	50.	40.	65.	75.	80.	65.	40.	30.
53.771	20.495	2.624	80.	55.	55.	85.	85.	80.	75.	90.	75.

TABLE 2. MODEL TEST RESULTS ORGANIZED BY YIELD

YIELD	EVAPOTRANSPIRATION	WATER USE EFFICIENCY	AM(1)	AM(2)	AM(3)	AM(4)	AM(5)	AM(6)	AM(7)	AM(8)	AM(9)
1.852	4.668	0.397	100.	100.	100.	100.	0.	0.	0.	0.	0.
5.657	9.248	0.612	85.	55.	40.	35.	25.	10.	5.	0.	0.
7.409	7.536	0.983	100.	100.	100.	100.	100.	0.	0.	0.	0.
9.379	10.450	0.897	90.	60.	60.	50.	30.	15.	10.	0.	0.
18.858	13.736	1.373	90.	90.	90.	60.	35.	25.	10.	5.	5.
23.066	11.109	2.076	100.	100.	100.	100.	100.	100.	0.	0.	0.
24.998	18.766	1.332	100.	100.	100.	100.	0.	100.	100.	100.	100.
26.712	21.073	1.268	0.	100.	100.	100.	100.	100.	100.	100.	100.
26.712	20.869	1.280	100.	0.	100.	100.	100.	100.	100.	100.	100.
26.712	20.358	1.312	100.	100.	0.	100.	100.	100.	100.	100.	100.
26.712	19.570	1.365	100.	100.	100.	0.	100.	100.	100.	100.	100.
28.010	17.958	1.560	80.	55.	40.	35.	55.	80.	65.	30.	20.
32.119	18.062	1.778	100.	100.	100.	100.	100.	0.	100.	100.	100.
37.133	18.875	1.967	80.	50.	40.	65.	75.	80.	65.	40.	30.
44.383	17.702	2.507	100.	100.	100.	100.	100.	100.	0.	100.	100.
47.522	18.664	2.546	90.	60.	55.	80.	85.	90.	60.	35.	15.
51.972	15.042	3.455	100.	100.	100.	100.	100.	100.	100.	0.	0.
53.771	20.495	2.624	80.	55.	55.	85.	85.	80.	75.	90.	75.
61.328	18.046	3.398	100.	100.	100.	100.	100.	100.	100.	0.	100.
67.003	16.906	3.963	100.	100.	100.	100.	100.	100.	100.	10.	0.
67.944	19.962	3.404	95.	90.	90.	90.	80.	80.	65.	55.	50.
71.754	17.409	4.122	100.	100.	100.	100.	100.	100.	100.	20.	0.
74.776	17.712	4.222	100.	100.	100.	100.	100.	100.	100.	30.	0.
77.024	17.929	4.296	100.	100.	100.	100.	100.	100.	100.	40.	0.
77.359	20.901	3.701	90.	90.	90.	90.	85.	85.	85.	90.	75.
78.826	18.099	4.355	100.	100.	100.	100.	100.	100.	100.	50.	0.
80.336	18.238	4.405	100.	100.	100.	100.	100.	100.	100.	60.	0.
81.638	18.356	4.448	100.	100.	100.	100.	100.	100.	100.	70.	0.
82.785	18.458	4.485	100.	100.	100.	100.	100.	100.	100.	80.	0.
83.813	18.549	4.518	100.	100.	100.	100.	100.	100.	100.	90.	0.
84.744	18.630	4.549	100.	100.	100.	100.	100.	100.	100.	100.	0.
92.355	20.191	4.574	100.	100.	100.	100.	100.	100.	100.	100.	10.
94.522	20.612	4.586	100.	100.	100.	100.	100.	100.	100.	100.	20.
95.851	20.865	4.594	100.	100.	100.	100.	100.	100.	100.	100.	30.
96.817	21.047	4.600	100.	100.	100.	100.	100.	100.	100.	100.	40.
97.578	21.190	4.605	100.	100.	100.	100.	100.	100.	100.	100.	50.
98.207	21.306	4.609	100.	100.	100.	100.	100.	100.	100.	100.	60.
98.743	21.405	4.613	100.	100.	100.	100.	100.	100.	100.	100.	70.
99.211	21.491	4.616	100.	100.	100.	100.	100.	100.	100.	100.	80.
99.626	21.566	4.619	100.	100.	100.	100.	100.	100.	100.	100.	90.
100.000	21.634	4.622	100.	100.	100.	100.	100.	100.	100.	100.	100.

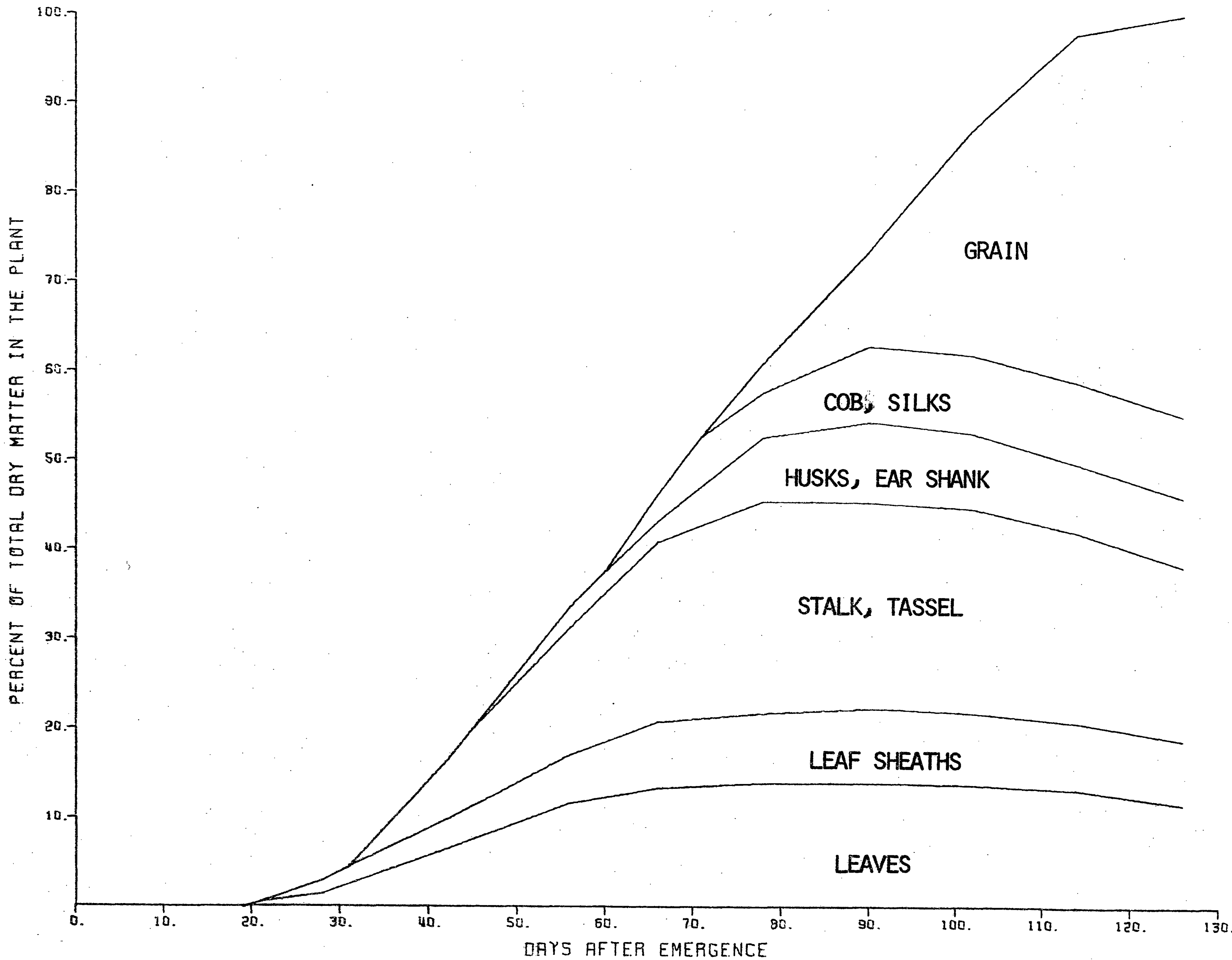

FIGURE 1. DRY MATTER ACCUMLATION IN THE CORN PLANT
REPRODUCED USING DATA FROM HANWAY

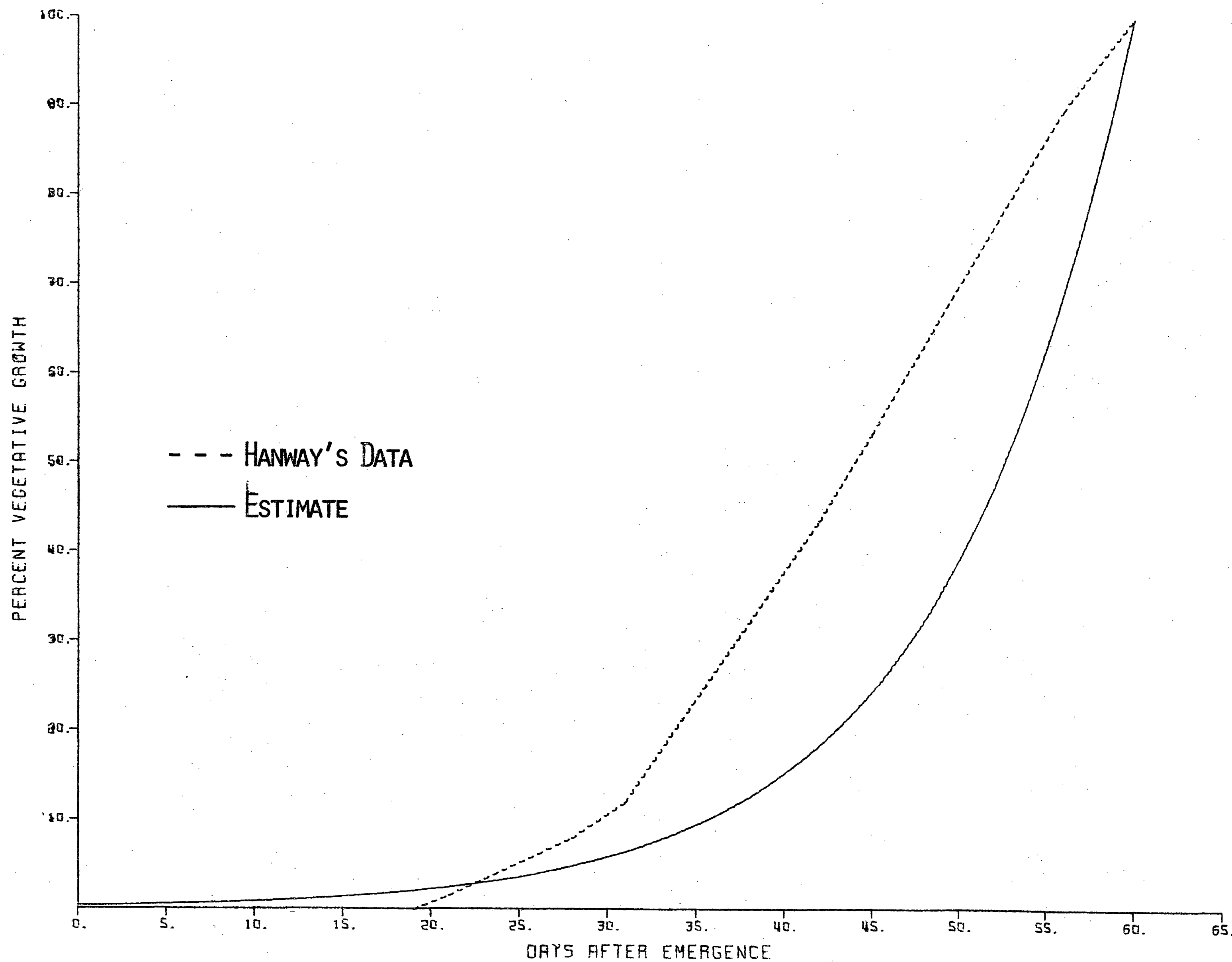

FIGURE 2. VEGETATIVE GROWTH FUNCTION FOR CORN AND ESTIMATE USING EXPONENTIAL EQUATION AND DATA FROM HANWAY

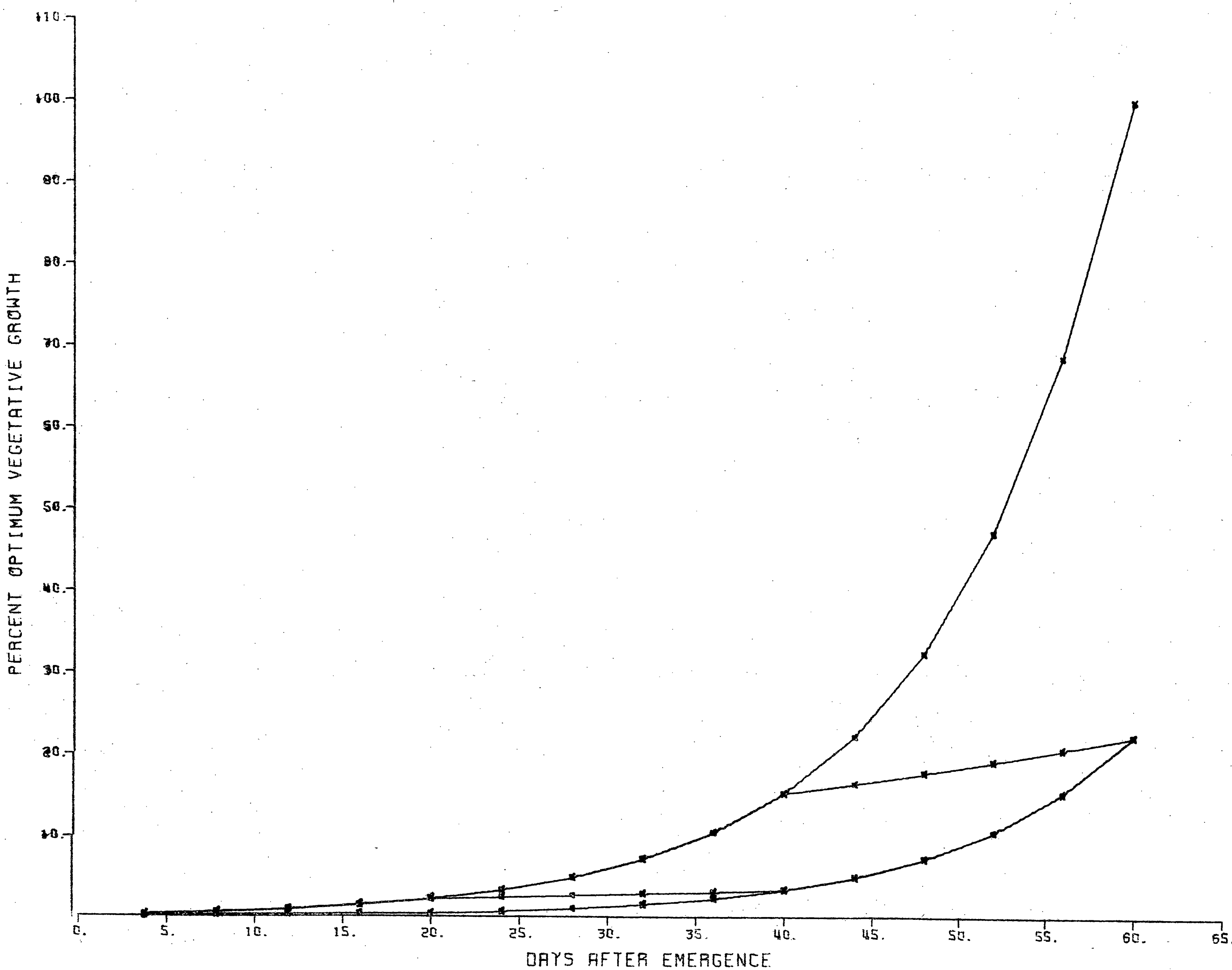

FIGURE 3. VEGETATIVE GROWTH FUNCTION TEST RESULTS, RUN 1

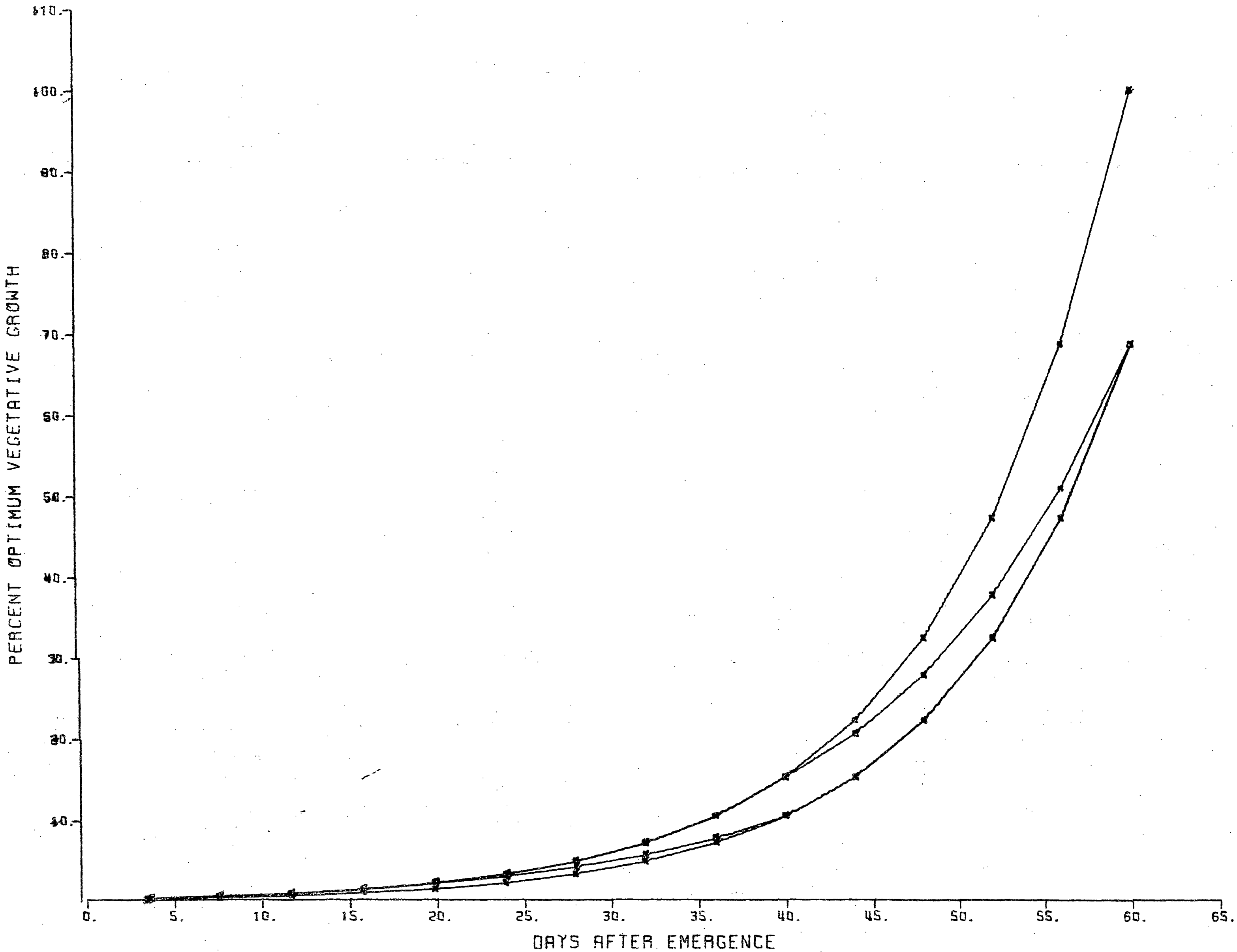

FIGURE 4. VEGETATIVE GROWTH FUNCTION TEST RESULTS, RUN 2

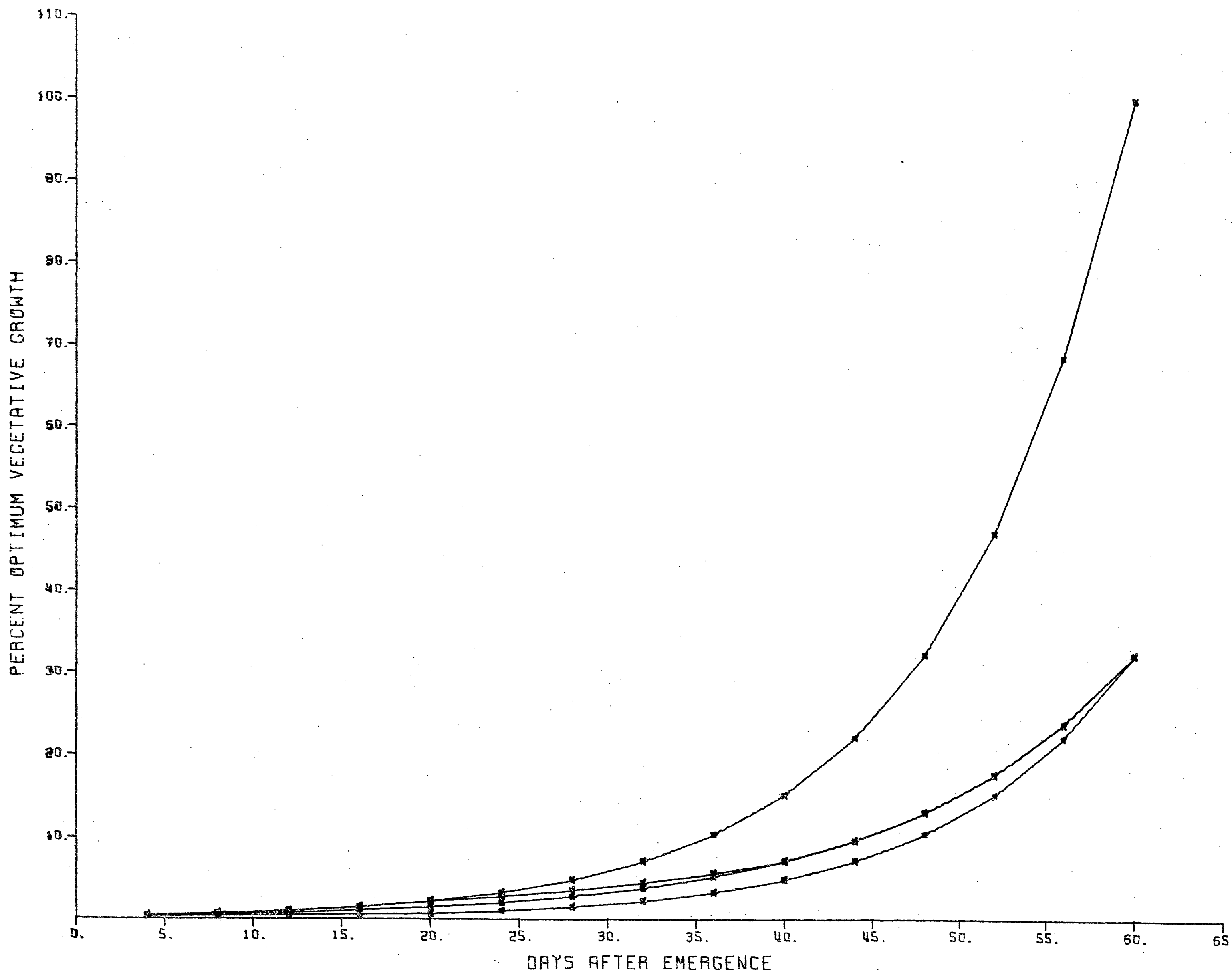

FIGURE 5. VEGETATIVE GROWTH FUNCTION TEST RESULTS, RUN 3

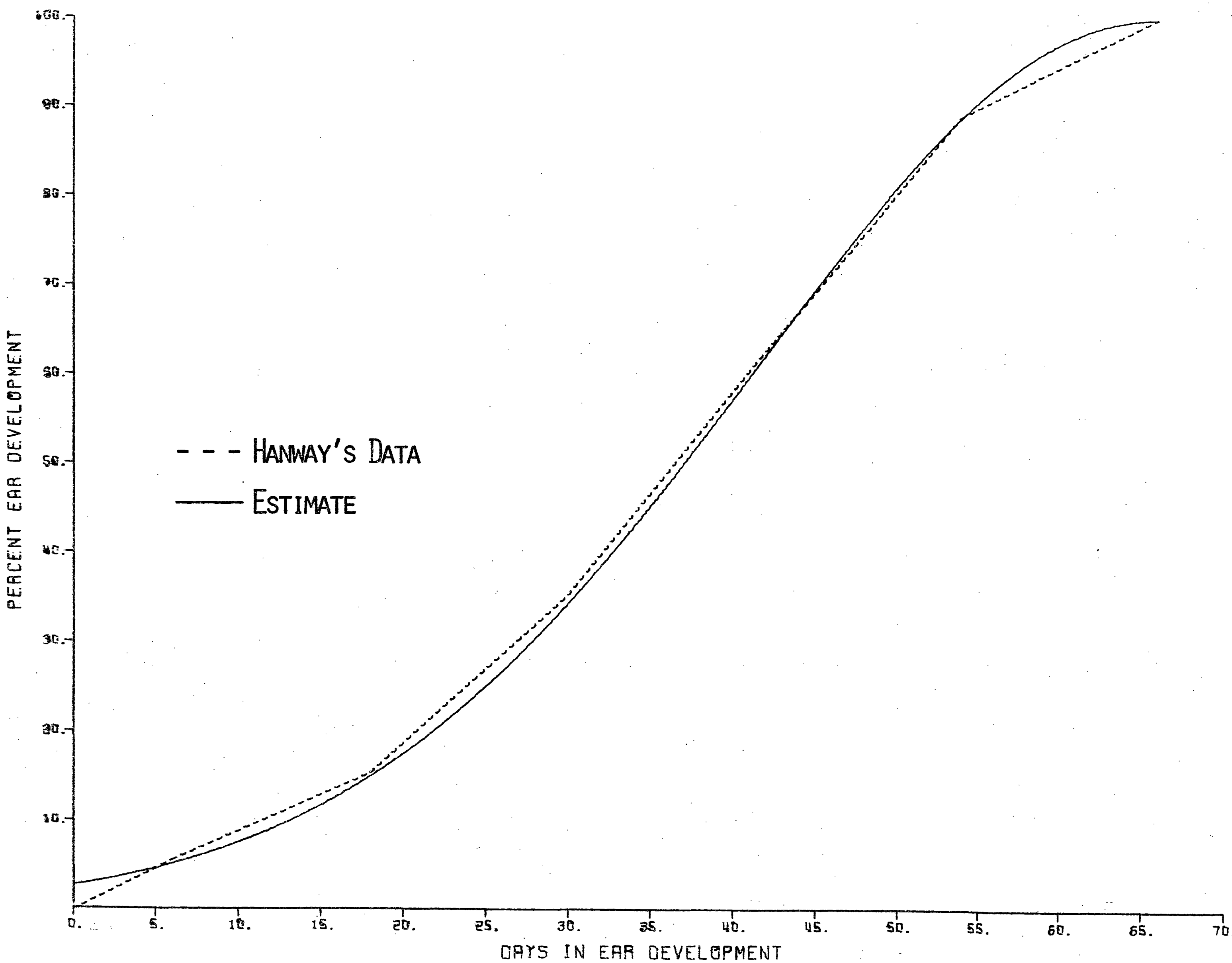

FIGURE 6. EAR DEVELOPMENT FUNCTION FOR CORN AND ESTIMATE USING MODIFIED LOGISTICS EQUATION AND HANWAY'S DATA

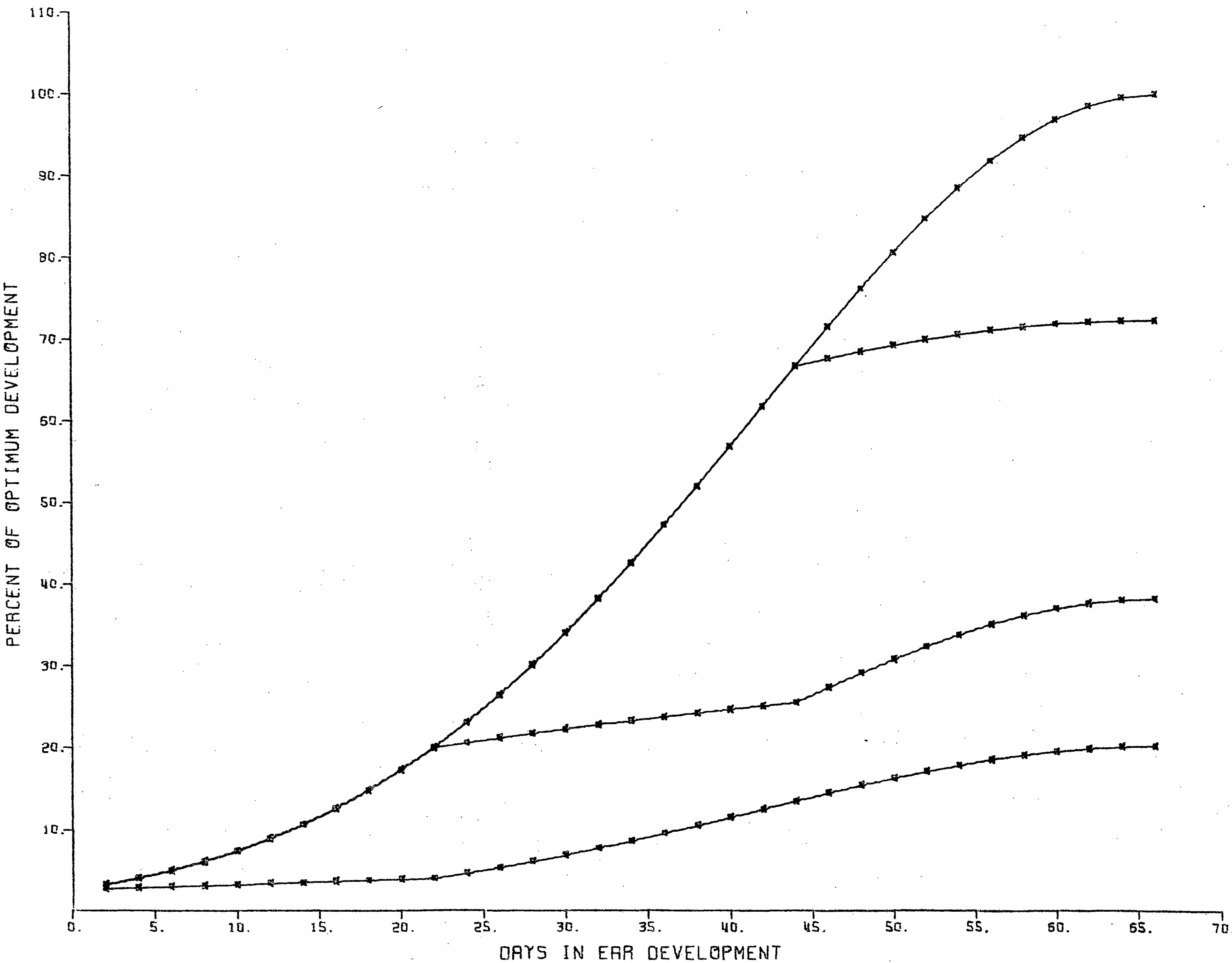

FIGURE 7. EAR DEVELOPMENT FUNCTION TEST RESULTS

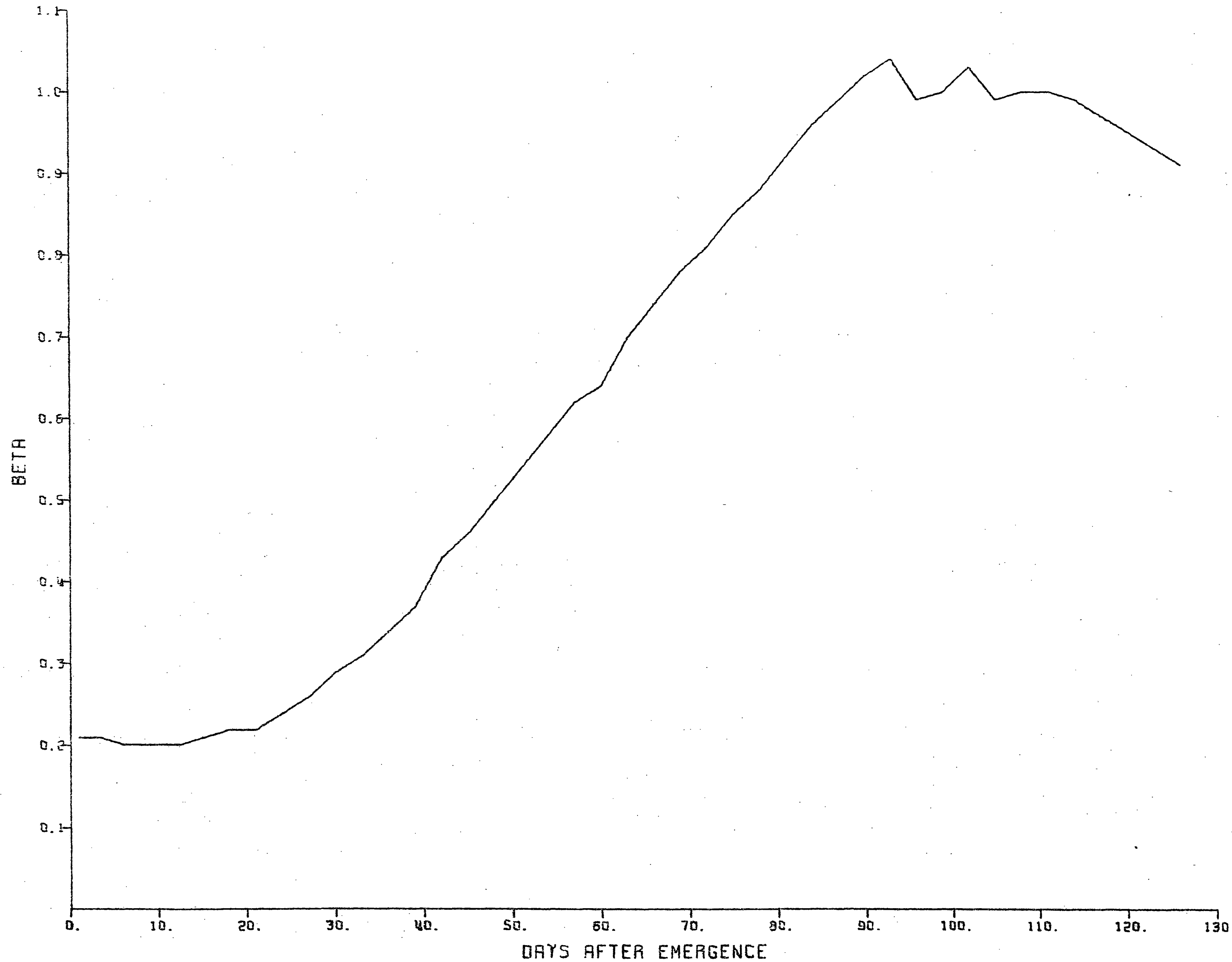

FIGURE 8. CRØP COEFFICIENT FØR CORN, SCANDIA, KANSAS, 1970