Risk

1977

DETERMINING OPTIMAL FERTILIZATION RATES UNDER VARIABLE WEATHER AND PRICE CONDITIONS: AN EXTENSION*

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ABSTRACT

This paper presents a theoretical framework for determining optimal fertilization rates under risk aversion. Three sources of risk were incorporated into the model: (a) the influence of weather; (b) uncertainty about the coefficient of the response function; and (c) product price variability. The model is applied to grain sorghum in the Texas Blackland Prairie.

Paper presented at AAEA annual meeting, Blacksburg, Va., Aug. 6-9, 1978.

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INTRODUCTION

In a recent WJAE article, Talpaz and Taylor presented a theoretical framework for determining optimal fertilization rates under risk aversion. Two sources of risk were incorporated into the model: (a) the influence of weather and other stochastic factors on the marginal product of fertilizer; and (b) uncertainty about the coefficients of the response function. Another important source of risk -- that due to product price variability -- was not considered in this framework. This source of risk is of obvious importance unless the decision maker has a forward contract for the crop. This paper extends the Talpaz-Taylor framework to include price risk. It is assumed that a price forecasting model is available to the decision maker and from this model the relevant characteristics of the price probability density function are obtained. The extended model is applied to the fertilization of dryland grain sorghum in the Texas Blackland Prairie.

THE THEORETICAL MODEL

The decision criterion under consideration can be mathematically stated as:

(1) $\text{MAX } E \ [P \cdot Y(N,W) - mN]$

SUBJECT TO:

(2) $\Pr[P \cdot D(N,W) \geq mN] \geq \delta$

where: E = expected value operator; P = unit price of the product; Y = per-acre yield; N = fertilization rate; W = vector of stochastic weather variables; m = fertilizer price; Pr = probability; D(N,W) = Y(N,W) - Y(N=0,W); and δ = subjective probability threshold (maximum risk).

As with the two sources of risk case, it is necessary to find the probability distribution given by expression (2) in order to find the fertilization rate that satisfies the decision criterion. The stochastic variables in the above model are P, W, and D, about which it is assumed that the following conditional probability density functions are known:

(3) $f_1(D|N,W)$

(4) $f_2(W|A)$

(5) $f_3(P|Z)$

where:

A = vector of variables that can be measured prior to fertilization and that affects the probability distribution of W. In this study, A is available soil moisture at fertilization time and W is the number of stress days after fertilization.

Z = a forecast of the price P.

Distributions (3) and (4) were considered in the earlier paper, while (5) is the additional source of risk emphasized here. For generality, the distribution of P, f_3, is made conditional on a forecast Z. The forecast can be dropped from the framework if a reliable forecast of P is not available.

A critical assumption that must be made at this point in the derivation is that the three probability distributions, (3), (4) and (5) are independent. If one is dealing with a very small part of the total production of a crop and a small geographical area, as one is when making recommendations to individual farmers, the independence assumption appears reasonable. 1/

Following the logic developed in the previous paper the weather variable can be integrated to obtain:

$$(6) \quad f_4(D|N,A) = \int_0^{\infty} f_1(D|N,W) \; f_2(W|A)dW$$

and the problem now becomes one of obtaining the distribution of the return to fertilizer, $R = P \cdot D$.

Since $f_3(P|Z)$ and $f_4(D|N,A)$ are distribution functions of two random and independent variables, we know from the definition of conditional probability that the joint distribution of P and D is:

$$(7) \quad f_5(P,D|N,Z,A) = f_4(D|N,A) \cdot f_3(P|Z)$$

The problem now becomes one of finding the distribution of the random variable R from the joint distribution, f_5, of the random variables P and D. To obtain the distribution of R it is necessary to call on a theorem from mathematical statistics about the probability distribution of a function of two random variables (see, for example, Meyer, p. 107). The basic idea behind this theorem is that it is often simplest to introduce another variable, X, and first obtain the joint distribution of R and X from the joint distribution of P and D. Then the distribution of R can be obtained by integrating the joint distribution of R and X with respect to X.

Letting R = P·D and X = P, the joint distribution of R and X is:

$$(8)\quad f_6(R,X|N,Z,A) = f_5(X,\tfrac{R}{X}|N,Z,A)\,|J(X,R)|$$

where J(X,R) is the Jacobian of the transformation (P,D) → (X,R) which is the determinant:

$$(9)\quad J(X,R) = \begin{vmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial x} \\ \frac{\partial D}{\partial X} & \frac{\partial D}{\partial R} \end{vmatrix}$$

which, for this problem, becomes:

$$(10)\quad J(X,R) = \frac{1}{X}$$

The distribution (8) thus becomes:

$$(11)\quad f_6(R,X|N,Z,A) = f_5(X,\tfrac{R}{X}|N,Z,A)/X$$

And integrating X over all values, we obtain the distribution of R

$$(12)\quad f_7(R|N,Z,A) = \int_X \left[\frac{f_5(X,\frac{R}{X}|N,Z,A)}{X}\right]dX$$

Substituting f_4 from (6) into (7) and the result into (12) for f_5 gives:

$$(13)\quad f_7(R|N,Z,A) = \int_X \left[\frac{f_3(X|Z)\int_o^\infty f_1(\frac{R}{X}|N,W)f_2(W|A)dW}{X}\right] dX$$

Finally, the probability of recovering the cost of fertilizer, equation (2), can be obtained by integrating (13) from the cost of the fertilizer, mN, to infinity:

$$(14)\quad Pr(R \geq mN) = \int_{R=mN}^{\infty} f_7(R|N,Z,A)dR$$

which is

$$(15)\quad Pr(R \geq mN) = \int_{R=mN}^{\infty} \int_x \left[\frac{f_3(X|Z)\int_o^\infty f_1(\frac{R}{X}|N,W)f_2(W|A)dW}{X}\right]dXdR$$

The solution to this decision framework for three sources of risk is characterized by two regions, as is the solution to the two-source risk model: (a) if the risk constraint is not binding, namely, $\Pr(R>mN|N,Z,A) \geq \delta$ then $N^* = N_e$, where N_e is that N at which expected profit is maximum and N^* is the optimal N; (b) if $\Pr(R \geq mN|N,Z,A) < \delta$, then N^* is found by setting expression (15) equal to δ and solving for N.

AN APPLICATION

The extended theoretical framework was applied to grain sorghum fertilization problem used in the earlier paper. A discussion of how weather and response uncertainty are evaluated can be found there and thus need not be reiterated here. To evaluate price uncertainty a simple price forecasting model was estimated. This model bases the price forecast on ending stocks for the previous crop and a time trend. This forecasting model, while certainly not the definitive forecasting model, may provide a reasonable "first approximation" forecast and illustrate how such a model can be used with the theoretical decision framework developed in this paper. The forecast equation, which was estimated with ordinary least squares regression with data for the 1958-76 period, is: 2/

$$(16) \quad P = \underset{(0.14)}{2.1557} - \underset{(0.02)}{0.1143T} + \underset{(0.0006)}{0.00319T^2} - \underset{(0.001)}{0.00168S}$$

$$R^2 = .62 \quad \hat{\sigma}^2 = 0.043 ,$$

where

P = price of grain sorghum; T = time trend variable (year-1900); and S = ending stocks of feed grains.

Assuming that the distribution of P is normal, for a finite sample, $f_3(P|Z)$ will be distributed as student's -t: [3/]

(17) $$f_3(P|Z) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)\sqrt{\Pi k}} \left(1 + \frac{t^2}{k}\right)^{-(k+1)/2} \qquad -\infty<t<\infty$$

where

$Z = [T,S]$; $t = \frac{(P-\mu)\sqrt{k}}{\sigma}$; k = degrees of freedom; $\mu = \hat{P}$ (estimated P given Z); and σ = standard error of the estimate $\hat{P}$.

Substituting this distribution into (15) along with the distribution for weather and response uncertainty (Talpaz and Taylor, pp.) gives:

(18) $$Pr(R \leq mN|N,Z,A) =$$

$$\int_{R=-\infty}^{mN} \left[\int_{x=o+}^{\infty} \left[\frac{\left[\int_{w=o}^{\infty} \frac{w^{(a-1)} e^{-w/b} \Gamma[(k_1+1)/2]}{\Gamma(a) b^a \Gamma(k_1/2) \sqrt{\Pi k_1}} \left(1+ \frac{t_1^2}{k_1}\right)^{-(k_1+1)/2} dW \right]}{X} \right] \frac{\Gamma[k_2+1)/2]}{\Gamma(k_2+2)\sqrt{\Pi k_2}} \left(1 + \frac{t_2^2}{k_2}\right)^{-(k_2+1)/2} dX \right] dR$$

where

a,b = parameters of the weather distribution function; $t_1 = \frac{(\frac{R}{X} - \mu_1)\sqrt{k_1}}{\sigma_1}$; $t_2 = \frac{(X - \mu_2)\sqrt{k_2}}{\sigma_2}$; μ_1 = expected value of D; k_1 = degrees of freedom for the response function; σ_1 = standard error of D; μ_2 = predicted mean of P; k_2 = degrees of freedom for the price forecasting model; and σ_2 = standard error of P.

RESULTS

Optimal fertilization rates for one range of soil moisture at planting time and three expected price levels are given in Figure 1. Figure 2 compares the optimal rates with and without price uncertainty. At high levels of risk aversion, it can be seen that fertilizer recommendations that do not consider price uncertainty will lead to over-fertilization. With a probability of no net loss, σ, of .95, the optimal fertilization rate when price uncertainty is not considered is 63 lbs./acre, and with price uncertainty considered it is 49 lbs./acre. If the decision maker ignores price uncertainty in his decision and applies 63 lbs./acre, the true probability of no loss will be only .91 rather than .95 on which the decision was based.

Figure 3 compares expected profit with and without price uncertainty. This figure shows the bias resulting from relating expected profit to the probability of no loss where price uncertainty is ignored. It is also interesting to note that Figure 3 shows the benefit to a risk averter of having a forward contract for the crop. If the decision maker specifies a probatility of no loss of say .95, expected profit with no price uncertainty is about $37.50. With price uncertainty, the expected profit is about $31.00. In this case, the decision maker could adjust the fertilization rate and increase expected profit by about $6.50 per acre if he could obtain a forward contract for the crop at a price equal to his expectation for it, yet he could still recover the cost of fertilizer 95 percent of the time.

The results of this empirical application for other moisture conditions and other expected prices are similar to those shown in Figure 1 through 3.

FOOTNOTES

* Texas Agricultural Experiment Station Technical Article No.

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1/ For practical use, this assumption should be carefully reviewed.

2/ Standard errors are given in parentheses.

3/ Theoretically, P cannot have a normal distribution since negative values for P are impossible. However, over the relevant range of price variability this may be a reasonable assumption. Other forms of the distribution of P could be used in the model without difficulty.

REFERENCES

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Talpaz, H. and C. Robert Taylor, "Determining Optimal Fertilization Rates Under Variable Weather Conditions," _Western Journal of Agricultural Economics_, 1977, pp. 45-51.

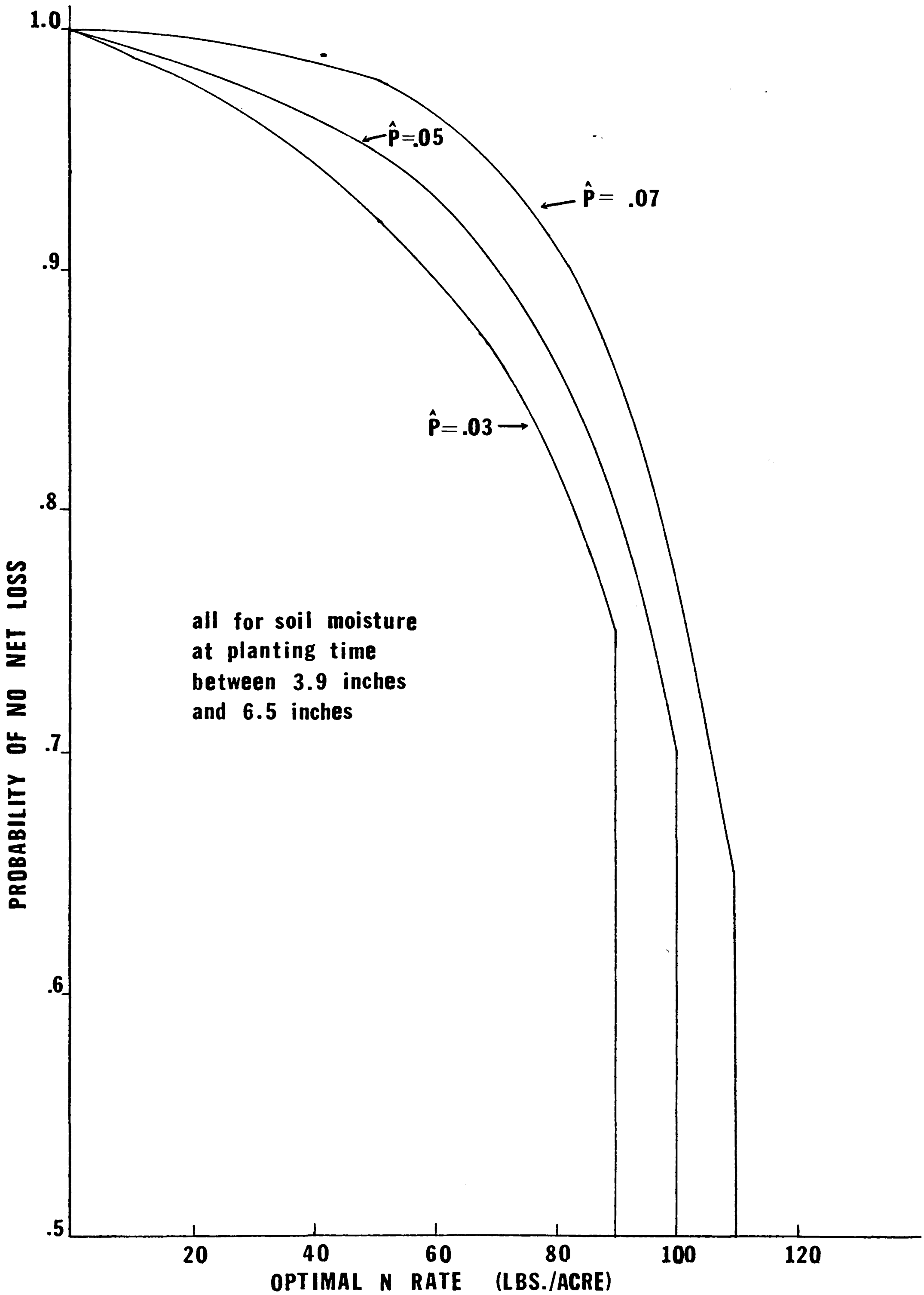

FIGURE 1. OPTIMAL N RATES WHERE THERE IS UNCERTAINTY ABOUT PRICE, WEATHER, AND YIELD RESPONSE

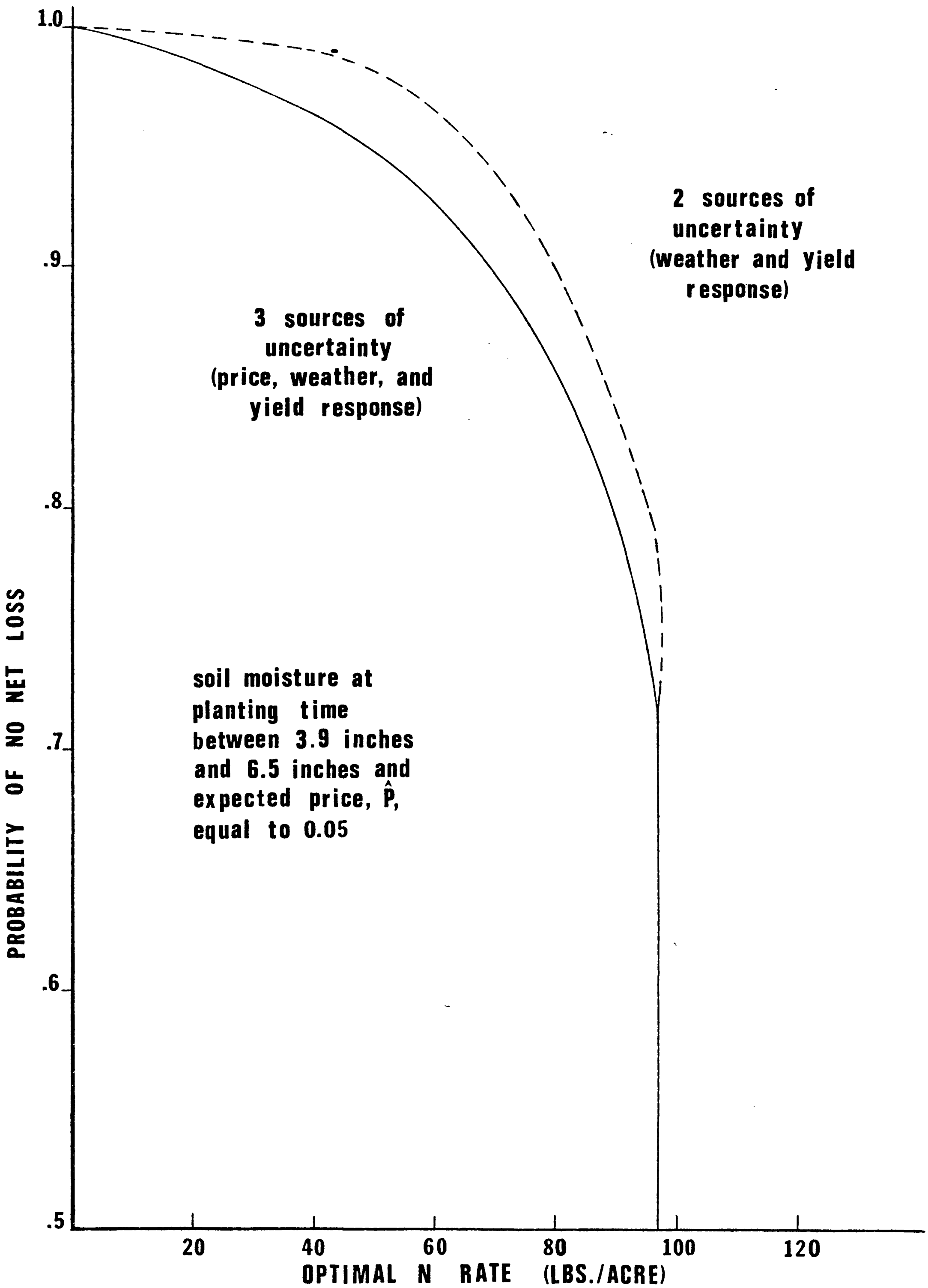

FIGURE 2. A COMPARISON OF OPTIMAL N RATES WITH AND WITHOUT PRICE UNCERTAINTY

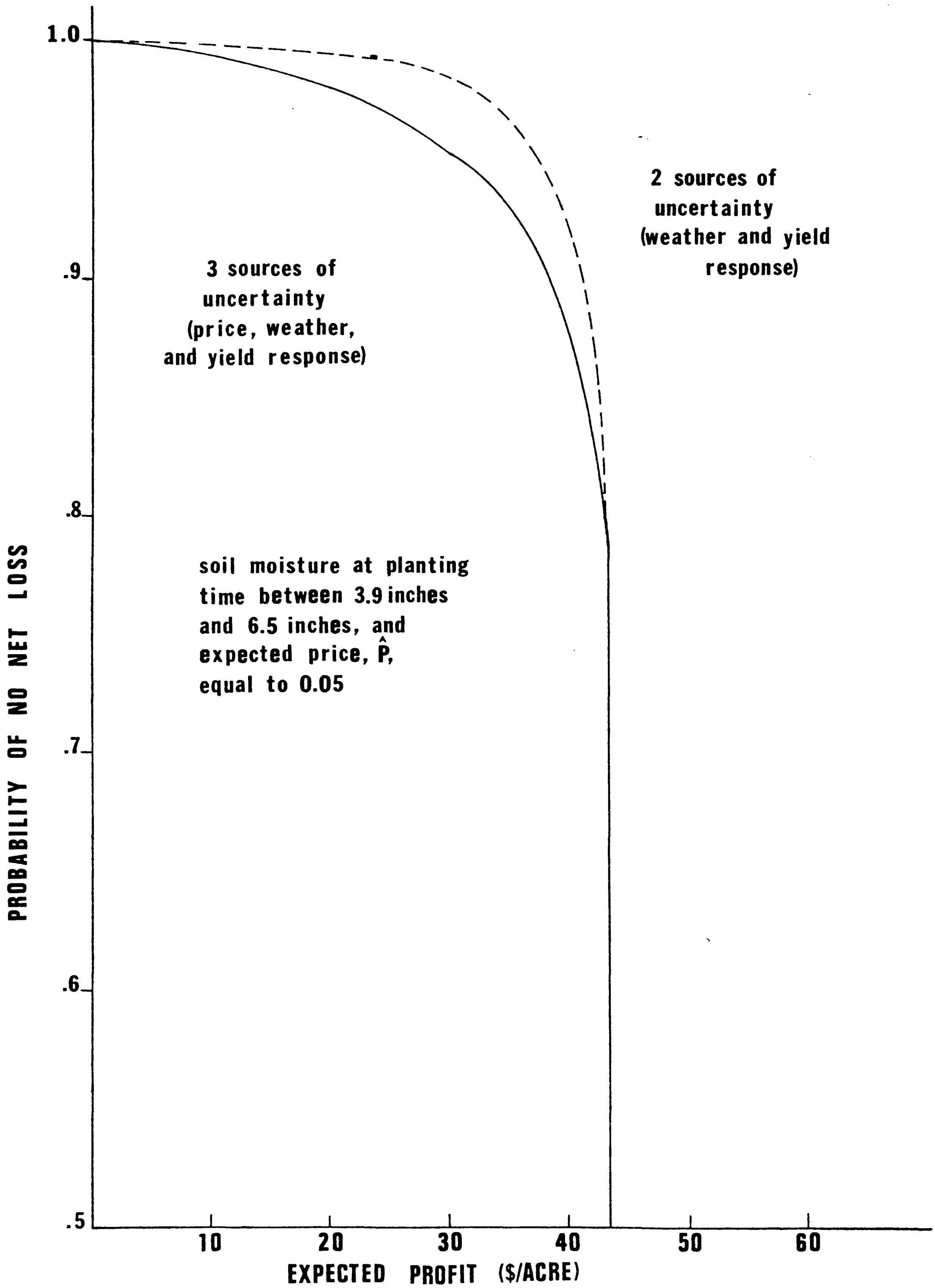

FIGURE 3. EXPECTED PROFIT WITH AND WITHOUT PRICE UNCERTAINTY