A MODEL OF DYNAMIC PLANNING UNDER UNCERTAINTY FOR THE COMPETITIVE FIRM*

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I. INTRODUCTION

The problem of decision making under uncertainty has, for obvious reasons, attracted increased attention from agricultural economists in recent years. The decision problem facing the firm when the planning horizon is one period long has been discussed extensively and from a variety of perspectives [3,5,6,7, and 13]. The problem of multiperiod planning under uncertainty has also been discussed by a number of authors [2,8, and 12]. The aforementioned studies have advanced significantly our insights into decision making under uncertainty. Yet, from the perspective of an individual for whom the ultimate purpose of economic activity is consumption, either for oneself or one's descendants, and who thinks it is rational to maximize expected utility when operating under uncertainty, the current literature is deficient in two important respects. The first is that it is not clear that the objective functions employed are consistent with the above view on the purpose of economic activity. The second deficiency in the current literature is that the models do not take proper account of the fact that the firm or its owner is a financial entity. The purpose of the present paper is to present a model which remedies the deficiencies mentioned above and, in addition, to indicate how the optimal decision rules can be computed in such a model.

The remainder of the paper is organized as follows. In the next section, a model is presented which considers simultaneously decisions regarding consumption, finance, production, and investment or capital accumulation for the risk averse owner of a firm which operates in an environment in which there is uncertainty regarding prices and output. The third section formulates the decision problem facing the entrepreneur and indicates how it can be solved. The final section of the paper contains some concluding comments.

II. THE MODEL[1]

We consider an entrepreneur who owns a firm. His wealth consists of liquid wealth and the capital inputs of the firm. The entrepreneur wishes to plan the allocation of his wealth over a finite planning horizon of T+1 time periods. Decisions are made at the start of each of the first T periods.[2] Liquid wealth can be allocated to consumption expenditures, a riskless single-period asset the holdings of which may be positive or negative,[3] and both single-period inputs and capital inputs for the firm's production activities. The firm's capital inputs can be allocated among M production activities. Capital inputs cannot be sold during the planning horizon. They can be disposed of for salvage at the end of the horizon.

The model used in the present paper consists of the following relations: i) a utility function, ii) a revenue generating equation, iii) relations representing the technological possibilities, and iv) a budget equation. The ultimate purpose of the entrepreneur's economic activity is consumption. Alternative decision strategies are evaluated according to their effect on the desirability of a stream of comsumption expenditures $y_1, y_2, \ldots, y_T$ and terminal wealth k_{T+1}. The desirability of such a stream is measured by a utility function

[1] Equations are numbered within sections. When referring to an equation in a different section we use both the equation number and the section number. For example, when referring to Equation 4) of Section II in a discussion in Section III, we use "equation II.4."

[2] No decisions are made in period T + 1.

[3] Borrowing takes the form of negative holdings of the riskless asset in the present model.

[4] Terminal wealth can be viewed as that which is left over in order to facilitate consumption after the end of the planning horizon.

of the form

$$1) \qquad \phi(y,k_{T+1}) = -\sum_{t=1}^{T} \rho^{t-1}\exp\{-\gamma y_t\} - \rho^T\exp\{-\gamma k_{T+1}\}$$

where ρ, a positive fraction, is a known subjective discount factor. The utility function in 1) is the discounted sum of single period utility functions of the form

$$2) \qquad \phi(y_t) = -\exp\{-\gamma y_t\},$$

a function which displays constant absolute risk aversion. Pratt's index of absolute risk aversion is $-\phi''/\phi'$ which is γ for the function in 2) [11]. Thus, γ in 1) is an index of absolute risk aversion. A person whose utility function displays constant absolute risk aversion is one for whom the odds at which he is willing to accept a bet of a given absolute amount remain unchanged as his wealth varies [1, pp. 33-35]. The utility function in 2) displays increasing relative risk aversion. A person having such a utility function will require more favorable odds as his wealth increases if he is to be induced to continue to accept a bet the amount of which is a constant proportion of his wealth [1, pp. 34-35]. The recent paper by Hammond broadens the scope of application of the above utility function by showing that it can under certain conditions be used to reflect properly the attitudes towards risk of people who do not necessarily display constant absolute risk aversion. [4].

The revenue or wealth generating relation gives liquid wealth at the start of next period as a function of decisions taken in the current period and random events that take place at the end of the current period. Liquid wealth at time t+1, k_{t+1}, is the sum of revenues from investment in the riskless asset in period t and revenues from the production activities in period t. That is,

3) $k_{t+1} = b_t v_t + p_t' q_t \qquad t = 1,2,\ldots T-1$

where $b_t - 1$ is the rate of return on the riskless asset in period t,

v_t is the amount expended on the riskless asset at time t,

p_t is the vector of output prices in period t,

q_t is the vector of output levels in period t.

At the end of the planning horizon, i.e., at time T+1, liquid wealth includes also the revenues from the sale of used capital inputs. Thus, the revenue generating relationship for time T+1 is

4) $k_{T+1} = b_T v_T + p_T' q_T + P' Z_{T+1}$

where P is the vector of salvage prices for capital inputs at time T+1,

Z_{T+1} is the vector of stocks of capital inputs at time T+1.

The firm has S capital inputs which can depreciate. The rate of depreciation for capital input s is constant and equal to $1-\delta_s$ where $0 < \delta_s < 1$ and s=1,2,...,S. We define the matrix $\delta = \text{diag}\{\delta_1, \delta_2, \ldots, \delta_S\}$. Then, the firm's stocks of capital inputs at time t is

5) $Z_t = \delta(Z_{t-1} + I_{t-1}) \qquad t = 2,3,\ldots,T+1$

where Z_t is the vector of stocks of capital inputs at time t

I_t is the vector of purchases of new capital inputs at time t.

The vector of capital inputs available for production in period t is $Z_t + I_t$, that is, the stock of old capital at the start of the period plus the newly acquired capital.[5]

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[5] Two assumptions implicit in this formulation are that newly acquired capital inputs are immediately available for production and that depreciated capital and new capital are perfect substitutes. Both of these assumptions can be modified without much difficulty.

There are M production activities which the firm can operate. Each of these activities may require both single period inputs and capital inputs. Production takes one time period to accomplish. The capital input requirements and the implied production possibilities are summarized in the following inequalities:

6) $A_t\bar{q}_t \leq Z_t + I_t$ $t = 1,2,\ldots,T$

$\bar{q}_t \geq 0$

where A_t is the technology matrix, the j-th column of which is the capital input requirements vector for the j-th activity in period t,

$\bar{q}_t$ is the vector of activity levels in period t.

It is assumed that activities can be operated at non-negative levels only.

The output vector in period t is equal to the activity vector plus a random vector ε_t. That is,

7) $q_t = \bar{q}_t + \varepsilon_t$ $t = 1,2,\ldots,T.$

The cost of acquiring the single period inputs to operate the j-th production activity at the unit level in period t is denoted by c_{jt}. The single period inputs required to operate the j-th activity at a positive level λ in period t will cost λc_{jt}. We define

8) $c_t = (c_{1t}, c_{2t},\ldots, c_{Mt})'$.

In each of the first T time periods, the entrepreneur allocates his wealth between consumption y_t, the riskless asset v_t, single period inputs, and capital inputs. Thus, the budget equation for time t is

9) $k_t = y_t + v_t + c_t'\bar{q}_t + W_t'I_t$ $t = 1,2,\ldots,T$

where W_t is the vector of acquisition prices of capital at time t.

Of the decision variables, only the level of expenditure on the riskless asset, v_t, can be non-positive.

For the subsequent discussion, it is convenient to combine the budget equation and the revenue generating relation to obtain the difference equation which governs the development of liquid wealth over time. When the budget equation is solved for holdings of the riskless asset v_t and the value of v_t substituted into the appropriate revenue generating relation, one obtains the following difference relations:

$$10) \quad k_{t+1} = b_t(k_t - y_t) + (p_t - b_t c_t)' q_t - b_t W_t' I_t \qquad t = 1,2,\ldots,T-1$$

$$k_{T+1} = b_T(k_T - y_T) + (p_T - b_T c_T)' q_T - b_T W_T' I_T + P' Z_{T+1}.$$

The first equation is obtained from equations 3 and 9, while the second is obtained from equations 4 and 9. We define

$$11) \quad \pi_t = (p_t - b_t c_t) \qquad t = 1,2,\ldots,T$$

which is the vector of revenues minus the costs of single period inputs per unit of activity.

The incorporation of the budget relations into the revenue generating relations has eliminated holdings of the riskless asset v_t from explicit consideration as a decision variable. Its optimal values can be obtained from the budget equation if one knows the optimal values of the other decision variables.

III. THE DECISION PROBLEM

In order to formulate the decision problem facing the entrepreneur, it is necessary to specify the information available to him. The acquisition prices of inputs,[6] the rates of return on the riskless asset, the technology

[6] The assumption that future acquisition prices of inputs are known can be relaxed with little difficulty.

matrices, and depreciation rates are known with certainty. The uncertain quantitites are the revenues from the production activities, the end-of-horizon salvage prices of capital inputs, and output levels. The decision maker is assumed to have beliefs regarding the uncertain quantities and it is further assumed that these beliefs can be expressed in the form of probability distributions.

The random quantities in the model are normally distributed. The revenue vector in period t, π_t, has mean vector μ_t and variance-covariance matrix Ω_t, written compactly as $\pi_t \sim N(\mu_t, \Omega_t)$. Similarly, for the vector of salvage prices of capital inputs, $P \sim N(EP, \Sigma)$. The output vector q_t has mean vector $\bar{q}_t$ and variance-covariance matrix Θ_t, $q_t \sim N(\bar{q}_t, \Theta)$. The output and price vectors are both independent of each other and serially independent.[7]

The decision problem facing the entrepreneur at the beginning of the planning horizon is: given his stocks of liquid wealth and physical capital and given his beliefs, choose the sequence of decision rules for i) consumption expenditures, ii) capital acquisitions, and iii) activity vectors which maximize expected utility. Given the definitions in the previous section the decision problem can be stated formally as

$$\text{12)} \quad \underset{\substack{(y_t, q_t', I_t') \geq 0 \\ t=1,2,\ldots,T}}{\text{maximize}} \quad E\left[-\sum_{t=1}^{T} \rho^{t-1} \exp\{-\gamma y_t\} - \rho^T \exp\{-\gamma k_{T+1}\}\right]$$

subject to:

(k_1, Z_1) is given

$$k_{t+1} = b_t(k_t - y_t) + \pi_t' q_t - b_t W_t' I_t \qquad t=1,2,\ldots,T-1$$

[7] All variance matrices are assumed to be positive definite. Thus, exact linear relationships between random variables are excluded.

$$k_{T+1} = b_T(k_T - y_T) + \pi_T' q_T - b_T W_T' I_T + P' Z_{T+1}$$

$$A_t \bar{q}_t \leq Z_t + I_t \qquad t = 1,2,\ldots,T$$

$$Z_t = \delta(Z_{t-1} + I_{t-1})$$

$$\pi_t \sim N(\mu_t, \Omega_t)$$

$$P \sim N(EP, \Sigma)$$

$$q_t \sim N(\bar{q}_t, \Theta_t)$$

As is well known, decision problems of the above type are amenable to analysis by means of stochastic dynamic programing. In general, however, the method does not lead to a computational procedure which can be executed on existing equipment at non-prohibitive cost. The important point about the above model is that when the dynamic programming argument is applied in a modified way, it is found that the optimal strategies in the decision problem in 12) can be found by a combination of quadratic programming and algebra.

The modified way in which the dynamic programming method is applied is as follows. A sequence of non-negative vectors of purchases of capital inputs and a sequence of feasible activity vectors are chosen. Then, treating these as given, the dynamic programming method is applied to obtain conditionally optimal consumption strategies and to obtain the function which gives the expected utility of such a conditionally optimal program. The expected utility of a conditionally optimal program is found to be monotonically related to a function quadratic in the capital input purchases and activity levels. Then, the optimal production and investment decisions can be obtained by maximizing said quadratic function subject to the technology constraints. The conditionally optimal consumption strategies are then evaluated at the optimal values of the production and investment variables.

If the initial liquid wealth is k_1 and the initial vector of capital inputs is Z_1 and if we choose a given set of capital input purchases $I_1, I_2, \ldots, I_T$ and a given set of feasible activity vectors $\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_T$, then when the dynamic programming procedure is applied the following results are obtained [9]. The conditionally optimal consumption strategy in period t is

$$13) \qquad y_t = h_t k_t - \frac{1}{\gamma} \log \bar{m}_t \qquad t = 1,2,\ldots,T$$

while the expected utility of such a conditionally optimal program is

$$14) \qquad J_T(k_1, Z_1, {}_1\bar{q}_T, {}_1I_T) = -\bar{m}_1(Z_1, {}_1\bar{q}_T, {}_1I_T) h_1^{-1} \exp\{-\gamma h_1 k_1\}$$

where $\quad {}_1\bar{q}_T = (\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_T),$

$${}_1I_T = (I_1, I_2, \ldots, I_T).$$

We postpone for a moment giving the precise expression for the quantities h_t and $\bar{m}_t$ where $t = 1,2,\ldots,T$ in terms of quantities already defined. The important point to note at this stage is that in order to find the optimal activity levels and capital input purchases, the function J_T in 14) is maximized with respect to $\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_T, I_1, I_2, \ldots, I_T$ subject to the appropriate constraints. But maximizing J_T is equivalent to maximizing $-\bar{m}_1(Z_1, {}_1\bar{q}_T, {}_1I_T)$. Futhermore, since the uncertain quantities in the model are normally distributed, the quantity $\bar{m}_1$ in 14) can be written

15) $$\bar{m}_1(Z_1, {}_1\bar{q}_T, {}_1I_T) = R_1 \exp\{-\gamma h_1 Q_1(Z_1, {}_1\bar{q}_T, {}_1I_T)\}$$

where R_1 is a positive constant and Q_1 is quadratic in $({}_1\bar{q}_T, {}_1I_T)$. Now, maximizing Q_1 is equivalent to maximizing $-\bar{m}_1$.

Given the foregoing, the optimal production and investment decisions can be found by solving the following quadratic programming problem.

16) $$\underset{{}_1\bar{q}_T, {}_1I_T \geq 0}{\text{maximize}} \quad Q_1(Z_1, {}_1\bar{q}_T, {}_1I_T).$$

subject to:

(k_1, Z_1) given

$$A_t\bar{q}_t \leq Z_t + I_t$$

$$Z_{t+1} = \delta(Z_t + I_t) \qquad t=1,2,\ldots,T.$$

The quantity h_t depends only on the rate of return on the riskless asset and is defined recursively by the formulae, $h_{T+1} = 1$ and

17) $$h_t = b_t h_{t+1}(1+b_t h_{t+1})^{-1} \qquad t=1,2,\ldots,T.$$

Thus, h_t which is the marginal propensity to consume is a positive fraction. In view of 15) and 17), giving expressions for Q_1 and R_1 is equivalent to giving one for $\bar{m}_1$. In order to simplify the exposition, three cases will be discussed, namely, i) price uncertainty, ii) production uncertainty, and iii) price and production uncertainty.

When there is price uncertainty together with technological certainty, $q_t = \bar{q}_t$, the quadratic function Q_1 in 15) is shown in [9, pp. 31-33] to be[8]

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[8]The expression given in [9] is for h_1Q_1.

18) $$Q_1(Z_1, {}_1q_T, {}_1I_T) = \sum_{t=1}^{T} v_t [r_t' q_t - \frac{1}{2}\gamma h_{t+1} q_t' \Omega_t q_t - b_t W_t' I_t]$$

$$+ r_T [EP' Z_{T+1} - \frac{1}{2}\gamma Z_{T+1}' \ddagger Z_{T+1}]$$

where $r_t = \prod_{j=1}^{t} b_j^{-1}$ and

$$Z_{T+1} = \delta^T Z_1 + \sum_{t=1}^{T} \delta^{T+1-t} I_t$$

In the present case, the quantity R_τ is

19) $$R_\tau = \prod_{t=\tau}^{T} (\rho b_t)^{\prod_{j=\tau}^{t} (1-h_j)} \qquad \tau=1,2,\ldots,T$$

The quadratic objective function in 18) is the present value of a sequence of terms. The term for period t is the expected net cash intake by the firm in period t, $\mu_t' q_t - b_t W_t' I_t$, less a multiple of the variance of revenues, $q_t' \Omega_t q_t$, where the multiple is one half the product of the coefficient of risk aversion γ and the quantity $h_{t+1}=1$, which is a positive fraction and is a function of future rates of return on the riskless asset. If there were no capital input acquisition and if $h_{T+1}=1$, then the term for period t is of the same form as that in Freund [3] and in Scott and Baker [13]. The term for time T+1 is the expected scrap value of the stock of capital inputs at the end of the horizon less $(1/2)\gamma$ times the variance of the scrap value. If the decision maker is risk neutral, that is, $\gamma=0$, then the objective function in 18) becomes the present value of expected net revenues plus the present value of the capital stock at the end of the horizon.

Under conditions of price certainty and production uncertainty, the expression for Q_1 in 16) is

$$20)\qquad Q_1(Z_1,{}_1\bar{q}_T,{}_1I_T) = \sum_{t=1}^{T} r_t[\pi_t'\bar{q}_t - \frac{1}{2}\gamma h_{t+1}\pi_t'\Theta\pi_t - b_t W_t' I_t] + r_T P' Z_{T+1}$$

The expression for R_τ is the same as in 19). The objective function in 20) is linear in both the activity and the capital investment levels. This is a familiar certainty equivalent result which stems from specification in 7) that the output vector is equal to the activity vector plus a random vector.[9]

When there is both price and production uncertainty it is necessary, in order to obtain the following result, that the matrix $\Theta_t^{-1}-\gamma^2 h_{t+1}^2\Omega_t$ be positive definite for t=1,2,...,T. Then, the function Q_1 in 16) is shown in [10] to be

$$21)\qquad Q_1(Z_1,{}_1\bar{q}_T,{}_1I_T) = \sum_{t=1}^{T} r_t[\mu_t' B_t\bar{q}_t - b_t W_t' I_t - \frac{1}{2}\gamma h_{t+1}\bar{q}_t' B_t\Omega_t\bar{q}_t$$

$$- \frac{1}{2}\gamma h_{t+1}\mu_t' B_t\Theta_t\mu_t] + r_T[EP'Z_{T+1} - \frac{1}{2}\gamma Z_{T+1}'\ddagger Z_{T+1}]$$

where $B_t = (\Theta_t^{-1} - \gamma^2 h_{t+1}^2\Omega_t)^{-1}\Theta_t^{-1}$. In the present case, the expression for R_τ is

$$22)\qquad R_\tau = \prod_{t=\tau}^{T} ((\rho b_t)|I-\gamma^2 h_{t+1}^2\Omega_t\Theta_t|^{-1/2})^{\prod_{j=\tau}^{t}(1-h_j)} \qquad \tau=1,2,\ldots,T$$

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[9]It is important to note, however, that while uncertainty does not affect the optimal activity level it does affect both borrowing and comsumption in this case.

The function Q_1 in 21) is a quadratic function of the activity vectors and of the expected revenue vectors. The quantities appearing in the expression do not lend themselves to a straight forward interpretation as was the case when there was only one source of uncertainty. It is, however, true that when the variance-covariance matrix for output in period t, Θ_t, is null for t=1,2,...,T then the expression for Q_1 in 21) reduces to that in 18). Similarly, if the covariance matrix of prices is null in all cases, then the expression for Q_1 in 21) reduces to that for the production uncertainty case in 20). Before running the quadratic programming problem when there is both price and production uncertainty, it is necessary to check that the matrices $\Theta_t^{-1}-\gamma^2 h_{t+1}^2 \Omega_t$, t=1,2,...,T are postive definite. This can be done by means of a standard program for computing characteristic roots, all of which should be positive.

An expression for $\bar{m}_\tau$, τ=2,3,...,T, can be obtained easily. In the expressions in 15), 18), 20), and 21), if the subscript one is replaced by τ everywhere, then one obtains the formula for $\bar{m}_\tau$, τ=2,3,...,T.

The optimal consumption strategy is obtained by evaluating the conditionally optimal strategy in 13) at the optimal production and investment levels obtained from solving the maximization problem in 16). One obtains, using 15,

$$23) \qquad y_1^* = h_1[k_1 + Q_1(Z_1, {}_1\bar{q}_T^*, {}_1I_T^*)] - \frac{1}{\gamma} \log R_1$$

where an asterisk indicates the optimal values. The optimal amount to lend, which can be negative, is by the budget equation.[10]

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[10] Expressions for the optimal decision rules for consumption and lending in period t are obtained by replacing the subscript one by the subscript t everywhere in 23) and 24).

24) $v_1^* = k_1 - y_1^* - c_1'\bar{q}_1 - W_1'I_1^*.$

There are a number of features of the foregoing results that are worthy of note. The optimal production and investment decisions are independent of wealth. This result is a direct consequence of assuming that the decision maker displays constant absolute risk aversion. The decision maker's rate of time preference, as reflected in ρ, does not affect the production and investment decisions.[11]

When the objective function Q_1 in 16) is evaluated at the optimal decisions for production and investment $({}_1\bar{q}_T^*, {}_1I_T^*)$ it gives the decision maker's personal valuation of the firm. This quantity $Q_1(Z_1, {}_1\bar{q}_T^*, {}_1I_T^*)$ is also the decision maker's valuation of his non-liquid wealth as can be seen from the consumption function in 23). The effect of various factors on the decision maker's valuation of his firm can be seen most clearly in the price uncertainty case. From 18) it follows that an increase in expected revenue, a reduction in the rate of return on the riskless asset, a reduction in risk aversion, a reduction in input prices, and a reduction in the variance of revenues will each result in an increase in the individual's valuation of the firm.

The optimal consumption policy in 23) is linear in wealth, which is the sum of liquid and non-liquid wealth. The marginal propensity to consume is a positive fraction and is a function of the rates of return on the riskless asset.

The amount of lending, i.e. negative borrowing, is determined by the

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[11]This contrasts with the assertion in [12, p. 452].

decision maker in the present model. The optimal amount of borrowing, which takes the form of negative holdings of the riskless asset, decreases with current liquid wealth and increases with the value of the firm. The risk averseness of the decision maker ensures that the amount borrowed is finite when price uncertainty is present. When there is only production uncertainty then borrowing is finite when the production and investment levels are finite. The assumption that there is a riskless asset in the model is crucial to the derivation of the results. It is possible, however, to introduce additional financial assets into the model without much difficulty.

CONCLUDING COMMENTS

Much of the information needed to implement the present model is the same as that provided in the existing models. The two important exceptions are the subjective discount factor ρ and the coefficient of risk aversion γ. For individuals whose time preferences and attitude towards risk are correctly represented by the utility function in 1), both ρ and γ can be calculated with little difficulty. The subjective discount factor can be calculated by finding out the amount by which current consumption would have to be increased in order to compensate for a reduction by $1 of consumption in the next period when both current and next period consumption are equal initially. The coefficient of risk aversion γ can be estimated by finding the individual's certainty equivalent for the 50-50 gamble between k^o-A and k^o+A and then solving the following equation for γ:

$$\frac{1}{2}\exp\{-\gamma(k^o-A)\} + \frac{1}{2}\exp\{-\gamma(k^o+A)\} = \exp\{-\gamma c\}$$

where c is the certainty equivalent and A is a finite number of dollars of consumption expenditure.

Even if the decision maker does not display constant absolute risk aversion, Hammond has shown in a recent paper [4] that it is still possible under some circumstances to use the constant absolute risk aversion type utility function to represent adequately the decision maker's attitude toward risk. About all that can be said at this stage is that not much is known about the extent to which this approximation will prove useful in practice. However, the situation seems promising enough to merit thorough investigation.

There are a number of modifications and extensions of the model which can be introduced without destroying the desirable property that the optimal production and investment decisions can be obtained by a quadratic programming routine. Future prices of inputs can be treated as uncertain. Capital inputs which have a productive lifetime which is less than the length of the planning horizon can be introduced into the model. Storage of output can be allowed for. Finally, if prices are allowed to be serially dependent, the optimal production and investment strategies can still be obtained by quadratic programming, although the problem is much more complicated than in the present model.

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