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The Use of Shadow Prices in Determining Marginal Values for Agricultural Land

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ABSTRACT

The problem of evaluating the need for land development projects for agricultural uses is addressed in a marginal analysis utilizing shadow prices. A linear programming framework was used with physical and historical characteristics of the land resource being defined and constrained. A procedure for assigning marginal values to an entire package of land resource characteristics is presented.

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INTRODUCTION

Comprehensive planning of future land and water resource use requires some knowledge of the value of these resources in all existing and potential uses. Limited market activity and market imperfections hamper the accessability and collection of these data. Difficulties arise in simulating marginal values when all inputs and outputs are not priced through effective markets. This paper is limited to the development of marginal values for land resources used in agricultural production. Such values are useful indicators of the benefits of additional units of the land resources in agriculture or costs of removing units for non-agriculture uses. Major attention, however, is directed at valuing additions to the resource base through specific development activities.

Land has both quantitative and qualitative characteristics which reflect on its appropriate use and effective land use planning models need to incorporate these characteristics. Description of the land resource may include a "package" of characteristics for each of several land resource groups. These characteristics could include the absolute quantity of land in each group, historical cropping patterns, and physical properties which influence the type of cultivation practices recommended.

A broad land and water resource study setting can be used for the specific application of marginal resource values. Such values can assist in analysis of the general needs for future agricultural land development programs such as erosion control, drainage, and flood protection. In such studies, development needs are generally predicated upon an assumed level of future agricultural production. In this paper a method of estimating marginal values is pursued for land resources which have a "package" of characteristics that influence their value. The marginal value of existing agricultural land resource groups can be used to evaluate the desirability of various land development programs by comparing program costs with the marginal value for the entire "package" of characteristics that make up the land resource. Though not discussed in detail, these values could also be used to reflect the opportunity cost of removing land resources from agriculture to other uses such as recreation or industrial development.

Marginal values specifically address the value of an additional unit of resource, and the primary application of information presented here is limited to analysis of development projects resulting in additions to the agricultural land base. Development projects designed to improve production efficiency of agricultural land already in production would have to be analyzed in a slightly different manner, allowing for consideration of original value in use, and will not be discussed here.

SHADOW PRICES

Linear programming models that maximize profits or minimize costs for a specific level of agricultural production subject to a series of constraints will produce a dual activity value (shadow price) whenever one or more of these constraints are effective. The shadow price reflects the value, at the margin, for the last unit of the limiting resource. If a resource is defined to have several characteristics, a shadow price will exist for each limiting resource characteristic.

With profit maximization, the shadow price indicates the loss in profits if the limiting resource were decreased by one unit. With cost minimization the shadow price indicates the increase in costs from a reduction in the limiting resource of one unit.

Although the interpretation of the shadow price is slightly different depending on whether maximization or minimization is used, the absolute value of the shadow price will be the same under either case when a given level of agricultural production is specified. Thus what follows will be applicable for either optimization procedure when the production level is given.

A problem arises when a land resource has a number of joint characteristics, several of which may be limiting in the problem context, and thus each having a shadow price. Because these are joint characteristics and cannot be provided separately, individual shadow prices have no real meaning. It is necessary to have a single value for all shadow prices associated with the "package" of characteristics to have an estimate of the value of an additional unit of the land resource.

APPLICATION

Resource "Package"

A sample problem with three crops and three agricultural land resource groups was developed to examine shadow prices for joint characteristics (Table 1). The "package" of characteristics for land resource 2 was initially constrained to a total of 100 acres cropland with a maximum of 40 percent in corn and 30 percent in wheat and a minimum of 30 percent in pasture. In the optimal solution output, the constraints

Table 1. Linear Programming Model

	Corn 1	Wheat 1	Pasture 1	Corn 2	Wheat 2	Pasture 2	Transfer 2	Corn 3	Wheat 3	Pasture 3		RHS
Cost (Objective function)	66.40	96.40	80.14	151.20	93.80	84.16	.01	102.60	61.50	75.48		
Corn Requirement	80			60				40			=	7500
Wheat Requirement		40			35				25		=	2500
Pasture			2.2			2.0				1.8	=	170
Land 1 (Cropland)	1	1	1								≤	100
Corn 1 L (Corn Restriction on Land 1)	1										≤	60
Wheat 1 L (Wheat Restriction on Land 1)		1									≤	20
Pasture 1 L (Pasture Restriction on Land 1)			1								≥	20
(Resource 2)							(1)				(≤	1)
Land 2 (Cropland)				1	1	1	-1				≤	100
Corn 2 L (Corn Restriction on Land 2)				1			-.4				≤	40
Wheat 2 L (Wheat Restriction on Land 2)					1		-.3				≤	30
Pasture 2 L (Pasture Restriction on Land 2)						1	-.3				≥	30
Land 3 (Cropland)								1	1	1	≤	100
Corn 3 L (Corn Restriction on Land 3)								1			≤	20
Wheat 3 L (Wheat Restriction on Land 3)									1		≤	30
Pasture 3 L (Pasture Restriction on Land 3)										1	≥	30

on land, corn and wheat were effective giving shadow prices as follows:

Row	At	Activity	Dual Activity
Land 20	UL	100	-1.15694
Corn 21	UL	40	-1.54306
Wheat 22	BS	30	
Pasture 23	LL	30	1.44583

It might appear that the marginal value of the land and its corn, wheat and pasture restrictions, which together make up a "package" of characteristics, would equal the sum of all their shadow prices. However, it is the weighted sum of their shadow prices that is actually equal to the marginal value of the cropland resource "package". 1/ The weights to be used in this process are the proportions of the cropland resource that act as constraints for each specific crop, i.e., .4 for corn, .3 for wheat and .3 for pasture in this example.

Stability of the Marginal Value

When a particular agricultural land resource is at the upper limit in the solution, it is of policy interest to know over what range of additional acres the marginal value will remain stable. The interval over which the shadow prices for individual characteristics remain constant is not the same as the interval over which the shadow price for the entire "package" of resources remains constant. This can be seen from the range analysis information in Table 2. 2/ The intervals for stable shadow prices for each of the components of the agricultural resource "package" were exceeded from Run 1 to Run 2, but the shadow prices remained exactly the same. This lack of accurate interval information led to another approach. A transfer activity, TRANSFER 2, ble 2. Range Analysis for Row Entries [a/]

mber	Row	At	Activity	Slack Activity	Lower Limit Upper Limit	Lower Activity Upper Activity	Unit Cost	Limiting Process	AT
n 1	Land 2 (Cropland 2)	UL	105.00000	.	None 105.00000	100.64279 105.00000	1.14917 1.14917-	Pasture 1L Wheat 2L	LL UL
	Corn 2 (Corn Restriction on Cropland 2)	UL	42.00000	.	None 42.00000	42.00000 45.00000	1.55083 1.55083-	Wheat 2L Corn 3	UL LL
	Pasture 2 (Pasture Restriction on Cropland 2)	LL	31.50000	.	31.50000 None	31.50000 35.85712	1.44250- 1.44250	Wheat 2L Pasture 1L	UL LL
n 2	Land 2 (Cropland 2)	UL	91.00000	.	None 91.00000	90.84280 91.00000	1.14917 1.14917-	Pasture 1L Wheat 2L	LL UL
	Corn 2 (Corn Restriction on Cropland 2)	UL	36.40000	.	None 36.40000	36.40000 36.55714	1.55083 1.55083-	Wheat 2L Wheat 1L	UL UL
	Pasture 2 (Pasture Restriction on Cropland 2)	LL	27.30000	.	27.30000 None	27.30000 27.45711	1.44250- 1.44250	Wheat 2L Wheat 1L	UL UL

[a/] The information presented here is in the format of the RANGE post-optimal procedure on the IBM-MPSX linear programming routine.

was added to the model. This activity increased all components of the agricultural resource "package" in the proper proportion for each characteristic. (See TRANSFER 2 column in the LP model, Table 1.) The "reduced cost" reported in the program solution for TRANSFER 2 (Table 3), plus the cost for the activity, was exactly the same as the weighted sum of the shadow prices determined earlier. [3/] In addition to supplying marginal value information, the range analysis in Table 3 gives the precise interval over which the marginal value will remain unchanged. Adding the numbers given as Lower Activity and Upper Activity to the initial level of land resource 2 for Run 3 (100 acres) gives the interval as 90.48 to 105.52 acres.

Derived Demand for the Agricultural Land Resource

By increasing the upper limit for TRANSFER 2 beyond 5.52 acres, it is possible to find a new marginal value. Continuing in a similar manner will provide information for a derived demand curve for the agricultural land resource "package". (Figure 1)

Land resource development planning requires future as well as present analyses--preferably by a dynamic charting of expected resource use through time. A further expansion of this procedure to determine derived demand is to vary the specified level of production by some constant factor, and thus produce a range about marginal values and the intervals over which they would remain constant. [4/] The dotted lines on Figure 1 are an example of changes in derived demand resulting from different levels of specified production.

Further Application

The ability to use the weighted sum of shadow prices for all characteristics of a resource "package" makes it possible to determine,

Table 3. Range Analysis for Column Entries [a/]

Number	Row	At	Activity	Input Cost	Lower Limit Upper Limit	Lower Activity Upper Activity	Unit Cost	Upper Cost Lower Cost	Limiting Process	At
Run 3	Transfer 2	UL	1	.01	.	9.52381-	1.32675	1.33675	Pasture 1L	LL
					1.00000	5.52017	1.32675-	Infinity-	Pasture 3L	LL

[a/] The information presented here is in the format of the RANGE post-optimal procedure on the IBM MPSX linear programming routine.

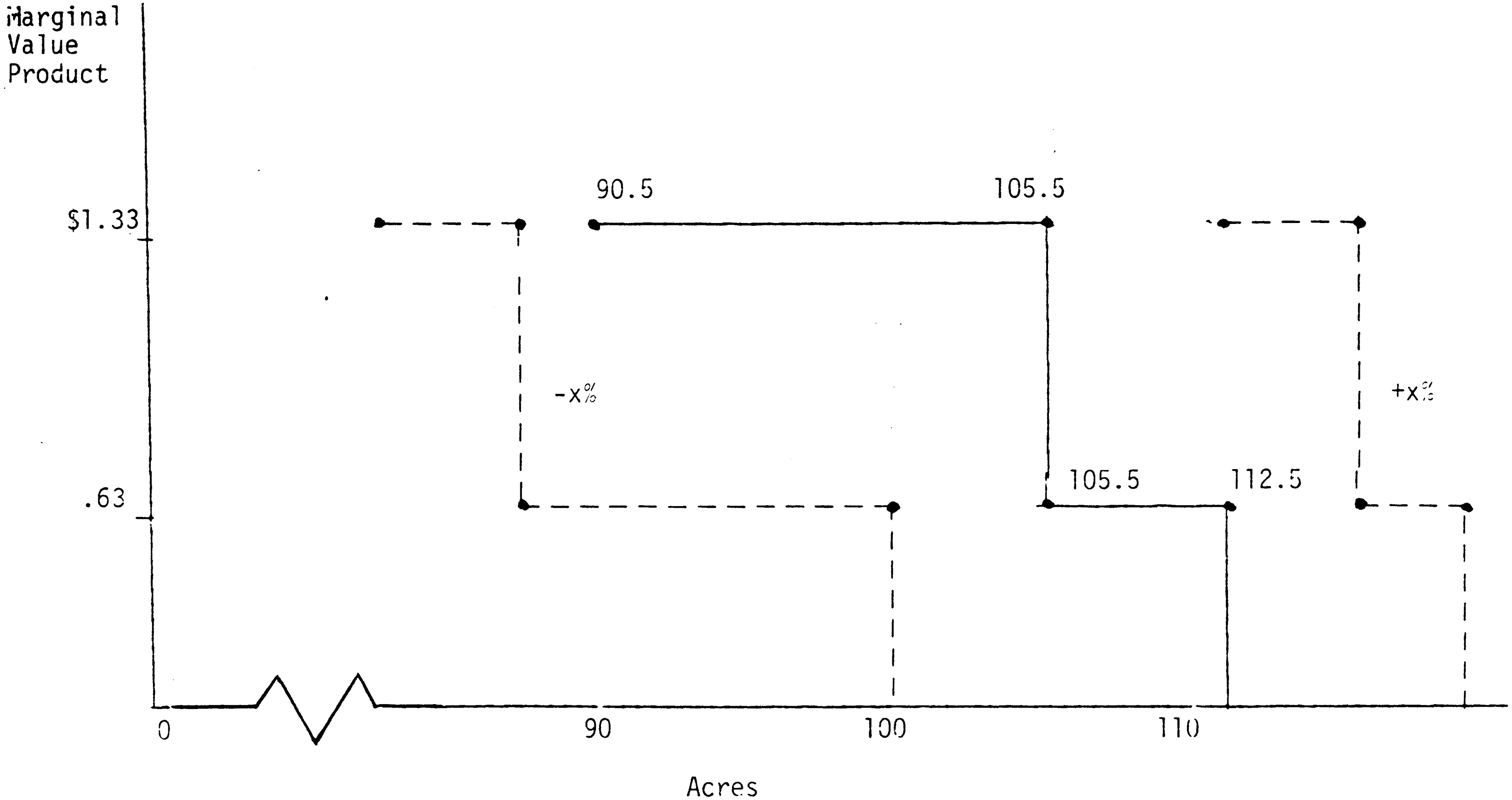

Figure 1. Derived Demand for Land Resource 2

without any changes in an LP model, the marginal value for fully utilized resources. Although the example used in this paper pertains to agricultural land resources, the same procedure could be used to find the marginal value for any resource that was a composite of several restrictive characteristics. Those resources with marginal values exceeding the costs of development programs would normally be considered for further study. The interval over which the marginal value of a resource remains constant can be found by forming a column activity (with an upper limit) that includes all characteristics of the resource being considered. The optimal level of agricultural land treatment activities could be determined by entering the treatment cost for the column activity cost. The same technique could be used to find the optimal level of other resource development programs.

SUMMARY

The use of a weighted sum of shadow prices, all of which apply to a "package" of land resource characteristics, is considered as a means of determining the marginal value for agricultural land resources. It is shown that this method does give the appropriate marginal value. Determining the interval over which the marginal value for the agricultural land resource "package" remains constant is also possible. This technique of determining intervals can also be used to provide a derived demand curve for the agricultural land resource. The optimal levels for land development programs can be found, given the conditions specified.

FOOTNOTES

The authors would like to thank Ken Clayton, Takashi Takayama, and Glenn Zepp for helpful comments and suggestions on early drafts of this paper.

1/ The reason for using the weighted sum as the marginal value for the combined "package" of characteristics can best be seen by referring to the TRANSFER 2 activity in Table 1. This activity adds one unit of LAND 2, along with the proper proportion of all the characteristics that make up the complete package. The shadow price on RESOURCE 2 (which was used to limit the TRANSFER 2 activity to test this procedure) is the marginal value for the entire package of characteristics and is calculated as follows:

$$\gamma_{\text{RESOURCE 2}} = \gamma_{\text{LAND 2}} + .4\gamma_{\text{CORN 2}} + .3\gamma_{\text{WHEAT 2}} + .3\gamma_{\text{PASTURE 2}} + C$$

Where C is the cost of the TRANSFER 2 activity entered in the objective function.

The RESOURCE 2 row and TRANSFER 2activity were not in the original sample problem, but were added later to check assumptions about shadow prices. It is not necessary to have the RESOURCE 2 row or the TRANSFER 2 activity in the model, the marginal value of the entire "package" of characteristics is still equal the weighted sum of the shadow prices for each characteristic.

2/ An excellent guide to the interpretation of output from the RANGE procedure is available in A User's Guide to the IBM-MPSX Linear Programming Package, by James D. Libbin, Charles A. Moorhead and Neil R.

Martin, Department of Agricultural Economics, University of Illinois at Urbana - Champaign.

3/ This is to be expected since this column activity is the corresponding logical variable of the row activities considered in the "package" of agricultural land resource characteristics.

4/ It was pointed out to us that the derivation of derived demand functions could be simplified somewhat by the addition of an accounting row to monitor the level of the transfer activity and allow parametric programming which would facilitate location of the "steps" in the demand function.