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EXCHANGE RATES IN INTERNATIONAL COMMODITY MODELS: THE COMMON CURRENCY QUESTION REVISITED

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ABSTRACT

This paper clarifies the difference between estimating supply and demand equations for individual countries in national currencies or converting to common currency prior to estimation. The appropriate procedure for a pooled cross-country time-series estimation model depends upon the cross-sectional constraints and whether nominal or real values are used.

EXCHANGE RATES IN INTERNATIONAL COMMODITY MODELS:
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In a recent review of problems with international commodity models, one area of concern raised by Labys was the omission of monetary factors such as inflation and exchange rates. These factors have been more volatile in recent years and consequently their effects have become more important. Two communications in recent years have dealt briefly with methods of incorporating exchange rates into international commodity models. Bjarnason, McGarry and Schmitz advocated converting to a common currency prior to the estimation of supply and demand equations. They "found it best to use a base exchange rate for conversion purposes rather than yearly rates" regardless of whether or not a price deflator was used. But when prices are deflated the base years of the deflator and the exchange rate must coincide. They did not indicate whether or not differing exchange rate assumptions could be parameterized in the model for simulation or projection purposes.

Elliott implied that the Bjarnason-McGarry-Schmitz (BMS) method did not have the flexibility to consider different exchange rates from those used in estimation. Elliott preferred to estimate the equations in national currencies and then convert to a common currency by multiplying the price coefficients by the appropriate exchange rates. This "permits consideration of different exchange rate situations in successive runs of the model." While this method is quite satisfactory for estimating separate country regressions, it is not an option if one desires to combine observations from two or more countries in a pooled cross-section time-series model.

The purpose of this paper is, first, to show that identical results can be obtained from the BMS and Elliott methods; second, to show that converting to common currency with annual exchange rates prior to estimation results in biased estimates; and third, to investigate the common currency question in the context of a pooled cross-section time-series model. It is important at the outset to distinguish between the estimation of equations and the operational use of these equations in an equilibrium model. Elliott's concern was that the estimation procedure could limit the flexibility of the model, so these aspects are both considered in the discussion that follows.

Currency Conversion in a Country Model

Multiplying any independent variable in a linear regression by a constant factor k merely changes the estimated coefficient by the multiple 1/k (Kmenta, pp 376-78). This is all that is involved in the BMS method, which used a fixed base exchange rate for conversion. Consider a time-series model with K independent variables. All variables are assumed to be in units of national currency.[1/] Let the estimation model be defined as:

(1) $$Y = X\hat{B} + u$$

where

Y = vector of T observations on the dependent variable

X = matrix of T observations on the K independent variables

$\hat{B}$ = vector of K + 1 ordinary least squares estimates of true parameters

u = vector of ordinary least squares residuals.

Let r be the base exchange rate in dollars per unit national currency.[2/] Thus r is scalar and the estimation model in dollars is:

$$(2) \qquad Y = Z\hat{C} + v$$

where Z = rX.

The ordinary least-squares estimator of C is,

$$(3) \qquad \hat{C} = (Z'Z)^{-1} Z'Y$$

$$(4) \qquad = (r^2X'X)^{-1} rX'Y \qquad \text{since r is scalar}$$

$$(5) \qquad \hat{C} = \frac{1}{r}(\hat{B}) \qquad \text{since } \hat{B} = (X'X)^{-1} X'Y .$$

It is easily shown that the intercept, the residuals and the significance tests are identical to those obtained in model (1).

From the relationship between the estimated coefficients in (5) it is clear that having done the regression in dollars using a fixed exchange rate, one can always find the national currency parameters $\hat{B}$; and one can always convert to a different exchange rate r* if necessary--say for projection or simulation purposes--by calculating new coefficients C*, such that:

$$(6) \qquad C^* = \frac{r}{r^*}(\hat{C}) = \frac{1}{r^*}(\hat{B}) .$$

These changes can be made directly in the equations as in Elliott's method.

This is precisely what Elliott implied was not possible with the BMS method, and it demonstrates that no flexibility is lost by converting with a fixed base exchange rate.

Bjarnason, McGarry, and Schmitz did not make explicit an important reason why it is preferable to convert with a base exchange rate rather than annual rates. The former merely changes the estimated coefficients by a known constant. However, using annual exchange rates would, in general, give estimates that are biased and not merely "less than favorable."

This bias can be demonstrated by considering the case of conversion with annual exchange rates (r_t) that vary across time. Let the estimation model in dollars be redefined as:

(7) $$Y = Z\hat{D} + v$$

where $Z = PX$ and $P = \begin{bmatrix} r_1 & 0 & \cdot & \cdot & 0 \\ 0 & r_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & r_T \end{bmatrix}$

The OLS estimator of D is:

(8) $$\hat{D} = (X'P'PX)^{-1}X'P'Y \; .$$

Let the true model be

(9) $$Y = X\beta + e$$

where

β = vector of K + 1 unknown true parameters

e = vector of stochastic disturbances.

Substituting (9) into expression (8) and taking the expected value of $\hat{D}$

$$E(\hat{D}) = (X'P'PX)^{-1}X'P'X\beta \quad (10)$$

The expected value of $\hat{D}$ is not equal to a constant transformation of the true coefficient vector β except in the trivial and uninteresting case where X is nonsingular and there are zero degrees of freedom.

The conclusions are clear. For estimation purposes it makes no difference whether one uses national currencies or transforms the data to a common currency with fixed exchange rates. The choice of method would be a question of convenience to the researcher. The use of annual exchange rates, however, would introduce bias into the regression estimates.

Currency Conversion in a Pooled Model

An important instance in which it may be necessary to convert all values to a common currency is in a pooled cross-section time-series model. If there are no cross-sectional constraints on the estimated coefficients, either national currencies or fixed exchange rates can be used. This would be the case in estimating the coefficients of "seemingly unrelated regression" equations. However, if a cross-sectional constraint is imposed on a price or income variable in a pooled model it is imperative to convert that data to a common currency prior to estimation. A simple example of this case is the estimation of an Engel curve using cross-country time-series data.

Unconstrained model. To demonstrate the case of "seemingly unrelated regression" equations, consider two countries with estimation models of the form specified in (1) above:

(11) $$Y_i = X_i B_i + u_i \qquad i = 1, 2$$

The unconstrained pooled model is:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

or

(12) $$\underline{Y} = \underline{X}\,\hat{\underline{B}} + \underline{u} \,.$$

Let the currency in country 1 be dollars and that in country 2 be francs, and let r_2 be the base exchange rate for country 2 in dollars per franc. Then the pooled estimation model in dollars is:

(13) $$\underline{Y} = \underline{Z}\,\hat{\underline{C}} + \underline{u}$$

where

$$\underline{Z} = \underline{X}\, R \qquad \text{and} \qquad R = \begin{bmatrix} I & 0 \\ 0 & r_2 I \end{bmatrix} .$$

$$I = (K + 1)x(K + 1) \text{ identity matrix}$$

The relationship between the OLS estimators of $\underline{B}$ and $\underline{C}$ is:

(14) $$\hat{\underline{C}} = (R'\underline{X}'\underline{X}\, R)^{-1} R'\underline{X}'\underline{Y}$$

(15) $$\hat{\underline{C}} = R^{-1}\, \hat{\underline{B}} \qquad \text{since R is nonsingular.}$$

The OLS estimator is consistent ~~but not efficient~~ in the case of "seemingly unrelated regressions." An efficient estimator is Aitken's generalized least-squares (GLS) formula, and the relationship between the Aitken GLS estimates of $\underline{B}$ and $\underline{C}$ is also:

$$\tilde{\underline{C}} = R^{-1}\,\tilde{\underline{B}} \;. \tag{16}$$

In both the OLS and GLS results, the coefficients in country 2 are all changed by the multiple $\frac{1}{r_2}$ when the variables are converted into dollar units. The coefficients in country 1 remain the same because the units of measurement were not changed. These results do not differ from those for a single country model.

Constrained model. To demonstrate the case of a constrained estimation model, consider an Engel curve to be estimated with data from two countries. Estimation with nominal and real values will be compared, because the results of these two cases differ.

For this purpose a simple 2-variable estimation model is specified for an Engel curve:

$$Q_{it} = \hat{a}_i + \hat{b}_i\, Y_{it} + u_{it} \qquad i = 1, 2;\ t = 1, 2, \ldots, T \tag{17}$$

or

$$Q_{it} = \hat{c}_i + \hat{d}_i\, RY_{it} + v_{it} \tag{18}$$

where

Q_i = per capita food consumption in country i

RY_i = real per capita income in country i

Y_i = nominal per capita income in country i.

If only the income coefficients are constrained to be equal in the two countries the pooled estimation model can be written in matrix notation as

(19) $$Q = YE + u$$

where

$$Q = \begin{bmatrix} Q_{11} \\ Q_{12} \\ \cdot \\ \cdot \\ \cdot \\ Q_{1T} \\ Q_{21} \\ \cdot \\ \cdot \\ \cdot \\ Q_{2T} \end{bmatrix} \quad YE = \begin{bmatrix} 1 & 0 & Y_{11} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & 0 & Y_{1T} \\ 0 & 1 & Y_{21} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & 1 & Y_{2T} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad u = \begin{bmatrix} u_{11} \\ \cdot \\ \cdot \\ \cdot \\ u_{1T} \\ u_{21} \\ \cdot \\ \cdot \\ \cdot \\ u_{2T} \end{bmatrix}$$

In the corresponding model using real income, RY_{it} replaces Y_{it} in the Y matrix.

It is clear that the Y (or RY) variable must have the same units of measurement in the two countries. The question is whether a fixed exchange rate or the annual exchange rates should be used to convert country 2 francs to dollars. In this case "the same units of measurement" means that in the "nominal values" model current francs should be converted to current dollars and in the "real values" model real francs should be converted to real dollars.

Conversion of current francs to current dollars requires the use of annual exchange rates. So

(20) $$DY_{2t} = r_{2t} Y_{2t}$$

where

DY = nominal per capita income in dollars

r_2 = annual exchange rate in dollars per franc

Y = nominal per capita income in francs

But it was shown in (10) above that converting with the annual exchange rates changes the within-country variation in such a way as to bias the estimated coefficients. If a fixed base exchange rate (r_{2b}) is used the result is

(21) $$r_{2b} Y_{2t} = DY_{2t} + (r_{2b} - r_{2t})Y_{2t} .$$

So converting with a base exchange rate will only generate the correct nominal value (DY_{2t}) in years when the annual exchange rate equals the base exchange rate. Thus in the case of a "nominal values" model, neither conversion approach is entirely satisfactory.

For a "real values" model it is necessary for the price deflator to have the same base year in both countries. Suppose the base year is 1975. Then the 1975 exchange rate would convert 1975 francs to 1975 dollars.

(22) $$RDY_{2t} = r_{75} RY_{2t}$$

where

RDY = per capita income in 1975 dollars

r_{75} = 1975 exchange rate in dollars per franc

RY = per capita income in 1975 francs.

In this case there is no conflict. The conversion factor which is preferred for the "real values" pooled model does not have undesirable effects on the country 2 estimates. The country equations can be separated after the pooled estimation, and the country 2 coefficient can be converted to a national currency coefficient if that is desired for simulation or projection purposes.

These results can be extended to price variables and any number of countries. The estimated coefficients from a constrained regression are not in general identical to those of the unconstrained or separate country regressions. But the Fisher F-test of equality of coefficients between regressions can be used to determine whether the constraint causes statistically significant changes in the regressions.

Conclusions

When supply or demand equations are estimated for individual countries, currencies may be converted with a fixed exchange rate prior to estimation or national currencies may be used. In either case there are no constraints on the exchange rate assumptions when the equations are subsequently used in an equilibrium model. The conversion to alternative exchange rates can be handled through price linkages or by making appropriate changes in coefficients of the equations.

If supply or demand equations are estimated with a pooled cross-section time-series model and cross-country constraints are imposed on any coefficients, the corresponding variables must be in common units of measurement. In a model specified in nominal values neither annual exchange rates nor a base exchange rate is fully satisfactory. In a model specified in real values the exchange rate for the base year of the price deflator should be used to convert variables to a common currency.

FOOTNOTES

1/ This is not a necessary assumption but it simplifies the analysis.

2/ If the variables are deflated by a price index, the Bjarnason, McGarry, Schmitz method would use the exchange rate for the year which is the base of the price index.

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