A POOLED CROSS SECTIONAL TIME SERIES MODEL OF COUPON PROMOTIONS

by

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Advertising is playing an increasing role in the marketing of agricultural products and the literature on advertising theory is beginning to reflect this importance to agriculture trade [Hochman, Thompson, Ward, 1976]. Surprisingly, a number of specific media for informing and/or stimulating the consumer have received minimal analysis by economist. In particular, the economic theory of coupons and its applications have all but been ignored in the studies of advertising effectiveness. Yet, coupons represent a multi-million dollar industry with the majority of the programs related directly to food consumption [Henderson, Nielson, 1965, Nielson]. Recognizing the deficiency in both conceptual and applied research relating to the use of coupons, this paper will set forth some basic economic principles of coupons and then report the empirical results from a cross-sectional time series study of national coupon programs for promoting frozen concentrated orange juice.

Coupon programs are implemented by first having a major coupon drop through mass distributions via a number of different media [Schwartz].

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Once the drop is made consumers redeem the coupons over the life of the program. Ward and Davis have shown the patterns of coupon redemption given various media and drop levels for coupon programs intended to promote frozen orange concentrate retail sales. Given that C coupons are dropped at some point in time, Ward and Davis' model shows the maximum redemption that will ultimately be realized. The time path and rate of redemption indicates how and when the promotional effort reaches the consumer. Once redemption occurs, then the basic question of its economic effect must be addressed.

Since redemptions represent individual uses of coupons, changes in individual consumption behavior must be related to the coupons redeemed. Further, the effectiveness of coupons may differ depending on the characteristics of the redeemer. Given the importance of the redeemer to understanding the economics of coupons, subsequent analysis are based on cross sections of consumers with their consumption data recorded over time. Procedures for pooling cross sectional and time series data are used in estimating the coupon model.

The first and second sections will be devoted to developing the coupon theory and estimation procedures. The remaining sections provide a direct measure of the effect of coupons on the retail consumption of frozen concentrated orange juice. Monthly panel data from 9,231 households reporting for 48 months are used for the analysis [MRCA].

Coupon Theory

Consider the simple demand model where for consumer i, $q_i = f(p_i, z_i, c_i)$ letting q = consumption of the commodity by consumer i, p = retail container price (¢/unit), z = exogenous demand variables, and c = coupons redeemed ($) by consumer i. Coupons are expected to have two independent effects on the consumer. First, the redemptive value represents reductions in the purchasing price to the consumer, i.e., the <u>price effect</u>. Second, the coupon has some informative and/or stimulating value to the consumer in which case the redemption should yield greater consumption than that realized from a simple price cut, i.e., the <u>advertising effect</u>. The sum of these two effects yields the <u>total coupon effect</u>. If the price effect equals the total effect the use of coupons are no different from an on-package price cut to the consumer.

While the container price is independent of the level of c, the purchase price is not. The purchase price will change depending on the total value of the coupons redeemed and the total purchases. Define:[1]

(1) $p^* = p - (\frac{c}{q})\ (100),$

where p^* = purchase price (¢/unit). The effect of a small change in the coupons on purchases and the purchase price then follows where

(2) $dq = f_c\ dc,$

(3) $dp^* = \{\frac{c}{q^2} f_c - \frac{1}{q}\}\ (100)\ dc,$ where the limits are

$$\lim_{q \to q'} dp^* = -\frac{(100)\ dc}{q} \quad \text{and} \quad \lim_{q \to \infty} dp^* = 0.$$

For small levels of consumption (q), the level of c yields a relative large reduction in the purchase price. For larger purchases (q´), the per unit price reduction over the total purchases becomes small. The per unit coupon value applied to the purchases of very large levels of q then approaches zero.

The redemption of coupons should lead to the total coupon effect as shown in (2). This represents a simple shift in the demand curve and includes both the price and advertising effects . Assuming that a change in the purchase price generates the same response as a change in the container price (i.e., the effect of dp^* and dp on q are equivalent), the components of the total coupon effect holding the container price fixed follows as:

(4) $dq = f_{p^*} dp^* + A_c, \; f_{p^*} < 0$

letting A_c = advertising effect from coupons. Equating (2) and (4), and substituting from (3) the advertising effect can be measured as in (5).

(5) $\{f_c - f_{p^*} \{\frac{c}{q^2} f_c - \frac{1}{q}\} (100)\} \, dc = A_c$

Note from (3) that the level of A_c changes with q as shown in (6) since the per unit value of the coupons depends on the level of consumption.

(6) $\lim_{q \to q'} A_c = \{f_c + f_{p^*} (\frac{1}{q}) (100)\} \, dc$

and $\lim_{q \to \infty} A_c = f_c \, dc.$

Considering the coupon effect c as illustrated in Figure 1 assuming $f_{cc} = 0$ and $f_{pp} = 0$, consumption increases from q to q^c for the same container price p. The purchase price declines to p^* as a result of the per unit coupon value and the resulting price effect is $q^* - q$. The residual $q^c - q^*$ then represents the increase in consumption resulting from the advertising effect on the consumer. If $q^c - q^*$ is near zero coupons are being used only as a price discounting tool by the consumer. Whereas, when $(q^c - q^*)/(q^c - q)$ is near one the advertising effect is predominant. The value of $q^c - q$ and the above ratio become major inputs to the planning of coupon programs and for the evaluation of the performance of such programs from both the couponer and consumer perspective.

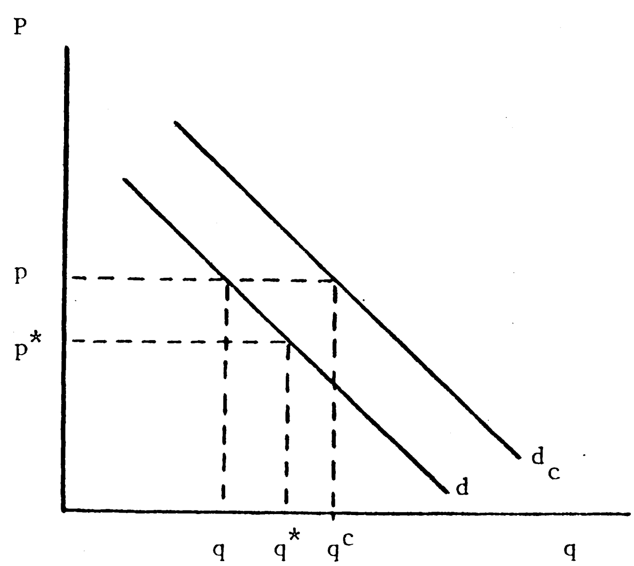

Figure 1. Coupon effect on consumption.

Variance Component Model

The coupon estimation model illustrated above becomes quite complex when the consumption data is based on both cross sections and time series information. Three procedures have been proposed for dealing with pooled information as available for this study. First, the model can be estimated using all the information assuming that all parameters are fixed, i.e., homogeneity assumption [Bass and Wittink]. In this case, OLS procedures for all data are used. Second, the intercept may change over time and/or over consumers and dummy variables are used to capture this effect. Third, the intercept may be assumed to be random with variations occurring between cross sections (specific ignorance) or over time and with variation occurring within cross sections (general ignorance) [Maddala].

Intuitively, the first procedure is very restrictive since no weighting of differences in cross sections are considered. The use of dummy variables is appealing when a small number of cross sectional characteristics can be identified. These characteristics should reduce the specific ignorance resulting from the uniqueness of cross sections. As the number of cross sections increase, the dummy variable procedure becomes computationally unmanageable and the dummy coefficients lose much of their meaning. Also, the use of dummy variables still do not assure that all the variation over time or cross sections have been considered. Recognizing these limits, the variance component pooling procedure has considerable appeal.

To estimate the coupon model for this study, two restrictive assumptions have been made. First, only the intercept of the model will be assumed random. That is, the slope coefficients for coupons and other variables are assumed to be non-random. Interaction terms and dummy variables can be used to account for non-random differences in slopes. Second, the model is restricted to specific ignorance over cross sections assuming that a change in the intercept over time can be measured[2]. Given these limiting assumptions, the variance component model follows where the total error is:

$$(7) \quad \varepsilon_{it} = \mu_i + \nu_{it}$$

letting μ_i = error due to different cross sections (specific ignorance), ν_{it} = error within cross sections (general ignorance), and assuming that $E(\mu_i) = E(\nu_{it}) = E(\mu_i \nu_{it}) = E(\nu_{it}\nu_{is}) = 0$. Following the notation developed by Maddala, the variance component model is shown assuming e_i to be a matrix of errors for the i^{th} cross section:

$$(8) \quad E(e_i\, e_i') = \begin{bmatrix} \sigma_\mu^2 + \sigma_\nu^2 & \sigma_\mu^2 & \cdots & \sigma_\mu^2 \\ \sigma_\mu^2 & \sigma_\mu^2 + \sigma_\nu^2 & & \vdots \\ \sigma_\mu^2 & & \ddots & \sigma_\mu^2 \\ \sigma_\mu^2 & \cdots & \cdots & \sigma_\mu^2 + \sigma_\nu^2 \end{bmatrix}$$

Letting $\sigma^2 = \sigma_\mu^2 + \sigma_\nu^2$ and defining $\rho = \sigma_\mu^2/\sigma^2$, then

$$(9) \quad E(e_i\, e_i') = \sigma^2 \begin{bmatrix} 1 & \rho & \rho & \rho \\ \rho & 1 & \ddots & \rho \\ \vdots & & \ddots & \vdots \\ \rho & \cdots & \cdots & 1 \end{bmatrix}$$

$$= \sigma^2 W_i$$

where if cross section i has T_i observations over time, then W_i is a $T_i \times T_i$ matrix. For all cross sections in the pooled model, the variance-covariance matrix follows as:

$$(10) \quad E(ee') = \sigma^2 \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & W_n \end{bmatrix} = \sigma^2 W$$

Since W is a diagonal matrix of W_i's, the inverse will also be diagonal matrix of W_i^{-1}. If ρ were known, then the coupon model could be estimated allowing for appropriate weighting according to the variation between and within cross sections.

Bass and Wittink have outlined the matrix procedures for calculating the GLS estimates assuming various weighting according to the value of ρ. If $\rho = 0$ the homogeneity assumption is valid since $\sigma_\mu^2 = 0$ and $W_i = I_i$ and the simple pooled model can be estimated using OLS. If $\rho > 0$ then $\sigma_\mu^2 > 0$ and the effect of cross sections must be weighted in the calculation of the parameters. The value of ρ in essence determines the weight to be given to the between cross section variation in the model.

The ratio ρ is generally unknown and must be estimated. Nerlove has suggested a two ban estimating procedure. However, it becomes quite cumbersome with a large number of cross sections. Alternatively, maximum likelihood procedures can be used where the generalized least squares estimates of the model are calculated over values of ρ.

The ρ is selected that minimizes the error sums of squares.

The variance component procedures searching over ρ have been used to estimate the coupon model discussed earlier. This procedure proved to be desirable given the large number of observations included in the study.

Empirical Coupon Model

Preliminary estimation indicated that the coupon model for frozen concentrated orange juice is nonlinear and a number of demographic variables proved useful for explaining differences in households. The final structure of the coupon model estimated follows in (11).

$$\text{(11)} \quad q_{it} = p_{it}^{\xi_1} c_{it}^{\xi_2} \exp^{(\xi_0 + \mu_i + \nu_{it})}$$

letting $$\xi_1 = \sum_{j=1}^{5} \gamma_j z_{jit} + \sum_{k=1}^{4} \beta_k r_{kit},$$

$$\xi_2 = \sum_{j=1}^{5} \lambda_j z_{jit} + \theta \log(p_{it}),$$

$$\xi_0 = \sum_{j=0}^{7} \alpha_j z_{jit} + \omega \log(I_{it}),$$

and q_{it} = ounces of single strength equivalent FCOJ purchased by consumer i in month t,

p_{it} = container price reported by consumer i for month t, (cents per ounce),

c_{it} = coupons redeemed by consumer i in month t, (dollars),

I_{it} = income level (1, high; 2, med.-high; 3, mod.-low; and 4, low),

z_0 = intercept,

z_1 = race (0, caucasian; 1, other),

able 1.--A pooled cross sectional-time series model for measuring the effect of coupon redemption

$$\log(q_{it}) = \underset{(139.45)}{4.954} + \underset{(0.84)}{.0832}\, z_1 + \underset{(4.20)}{.1750}\, z_2 + \underset{(3.05)}{.1011}\, z_3 + \underset{(0.17)}{.0057}\, z_4 + \underset{(1.55)}{.0630}\, z_5 + \underset{(28.01)}{.0036}\, z_6 + \underset{(4.77)}{.0120}\, z_7 - \underset{(-17.70)}{.105}\, \log$$

$$+ \{ \underset{(-21.42)}{-1.636} + \underset{(21.24)}{.5675}\, r_1 + \underset{(4.57)}{.1318}\, r_2 + \underset{(6.05)}{.2263}\, r_3 + \underset{(8.25)}{.2903}\, r_4 \} \log(p_{it})$$

$$+ \{ \underset{(-2.15)}{-.0873}\, z_1 + \underset{(9.84)}{.2266}\, z_2 + \underset{(1.11)}{.0205}\, z_3 - \underset{(-4.97)}{.1131}\, z_4 + \underset{(2.98)}{.0731}\, z_5 \} \log(p_{it})$$

$$+ \{ \underset{(4.13)}{.0317} - \underset{(-12.07)}{.1888} \log(p_{it}) \} \log(c_{it})$$

$$+ \{ \underset{(.05)}{.0011}\, z_1 - \underset{(-1.16)}{.0106}\, z_2 + \underset{(1.39}{.0101}\, z_3 + \underset{(3.29)}{.0283}\, z_4 + \underset{(3.88)}{.0384}\, z_5 \} \log(c_{it}) + \mu_i + \nu_{it}$$

$\bar{R}^2 = .9412$ $\sigma_u^2/(\sigma_u^2 + \sigma_\nu^2) = .13$ No. of Households[a] = 9231 Total Observations = 127,3

[a]Some households did not report for all 48 months of the study, therefore the estimating procedures allow T_i to vary with each household. Since the estimated model was expressed in logs, the coupon variable was set equal to .001 when no coupons were redeemed by a household.

z_2 = children (1, children present; 0, no children),

z_3 = education (1, college; 0, less than college),

z_4 = age (1, under forty; 0, otherwise),

z_5 = age (1, greater than sixty; 0, otherwise),

z_6 = monthly time periods (1, 2, 3, . . . beginning with January 1972 through December 1975 for 48 months),

z_7 = seasonality variable,

r_1 = region (1, Northeast; 0, otherwise),

r_2 = region (1, North Central; 0, otherwise),

r_3 = region (1, Mountain-Southwest; 0, otherwise),

r_4 = region (1, Pacific; 0 otherwise),

r_5 = region (Southern region deleted for estimation of dummies).

Of particular importance to the analysis is the value of ξ_2 in equation (11) since ξ_2 represents the total elasticity of demand with respect to coupons. ξ_2 in turn depends on both demographic characteristics as well as the container price. A priori, the effects of demographics cannot be hypothesized, i.e., $\beta_j, \lambda_j, \gamma_j, \alpha_j \gtrless 0$. In contrast, as prices increase the stimulant from a given level of coupons may decrease since the consumer must pay more for the increased consumption. If so, the effectiveness of coupons will likely decline with higher container prices, i.e., $\theta < 0$ and the demands illustrated with Figure 1 would not be parallel.

The empirical counterpart of equation (11) is given in Table 1 where the estimates are based on the variance component pooling procedures. The model is estimated from panel data of 9,231 households reporting intermitantly over a 48 month period with a total of 127,329 observations [MRCA]. Maximum likelihood estimates yield a ρ value of

.13 suggesting that only a small weight be given to the between group variation. The relative low value of ρ is due in part to the fact that the dummy variables included in the model accounted for much of the differences in consumers and hence reduced the between group variation. The estimates, while weighted, will not differ greatly from that of a simple pooled procedure assuming homogeneity of the intercept.

The last part of the equation shown in Table 1 gives the complete effect of demographic characteristics on concentrate consumption. Race, presence of children, educational level, and age of the housewife were included as interacting variables with coupons. A slight numerical increase in the coupon elasticity was noted for the non-caucasian consumer however statistically there was little difference among races and their responsiveness to coupons. Likewise, presence of children in the household and the educational level of the housewife showed low statistical interaction with coupons. In contrast, the age of the housewife proved to be an important factor influencing the effectiveness of coupons.

The housewife's age was identified as under forty, over sixty, or in-between these years and coupon elasticity increased with both the younger and older housewives. The greater elasticity occurred with the older housewives where the elasticity increased by a factor of .038. Such results should be greatly beneficial to those planning the media and/or copy for distributing coupons since generally coupon drops can be distributed in media reaching consumers having specific characteristics.

Coupon Effectiveness

As indicated above, the total effect of the coupons redeemed and the ratio of the advertising effect to the total are of primary interest. The model estimated further shows that both effects are dependent on the specific demographic characteristics of the consumer.

Using equation (11) where $f_c = \xi_2(q/c)$, $f_{p^*} = \{\xi_1 + \theta \log(c)\}(q/p)$, and $A_c = q^c - q^*$ in Figure 1, then the advertising effect relative to the total is calculated (see Figure 1 and equations (3) and (5)) using equation (5):

$$(12) \quad \frac{q^c - q^*}{q^c - q} = 1 - \frac{\{\xi_1 + \theta \log(c)\}}{\xi_2} \{(\frac{c}{p}) \left(\frac{\xi_2 - 1}{q}\right) (100)\}$$

Equation (12) is nonlinear in all variables and it is somewhat cumbersome to derive the effect of changes in these variables on the advertising and total effects. Alternatively, it is relatively easy to simulate the effects under varying levels of the independent variables. The empirical value of (12) should directly show the extent of the informational effect coupons have on consumers.

The total and informational effects of coupons can be analyzed for specific demographic characteristics and for the average consumer. Calculating the average from the nonlinear model yields a geometric rather than a simple mean. Hence, the average must be converted to an arithmetic mean in order to reference the average consumer[3]. Using the average proves most useful when aggregating over households is desired. Also, household responses to coupons and/or prices are more readily illustrated when the possible combinations of demographic effects are averaged.

Average Coupon Effect

The empirical counterpart to Figure 1 is shown in Figure 2 where the demand represents the average consumer using the mean values for all variables other than prices and coupons. Both the nonlinear nature of demand and the interaction of prices and coupons are evident. Consumers facing higher prices respond less to the use of coupons in terms of additional consumption. This decrease in effectiveness is illustrated best with Figure 3 in conjunction with Figure 2. At the lowest (.5 cents) price monthly consumption increases by 73 ounces with five cents worth of coupons. As the price increases to .6 cent, coupons are approximately 73 percent as effective as at the lower price. The remaining percentages in Figure 3 show the decline in coupon effectiveness as prices increase. Note that once prices reach 1.2 cents per ounce coupons are nearly ineffective for the average relationship.

The price interacting phenomenon usually works to the advantage of the concentrate industry since during periods of higher prices there is generally less incentive to promote sales. Whereas, lower prices reflect greater supplies and a greater need for promotions and at these lower prices coupons tend to be more effective.

Coupons also show a declining marginal rate of return. Using a price of .5 cents in Figure 2, coupon values totalling twenty cents lead to 156.5 ounces increase in consumption. Approximately 47 percent of this gain could have been achieved with five cents worth of coupons; 72 percent would have resulted with ten cents; and nearly 88 percent of the gain could have achieved with coupons equalling fifteen cents.

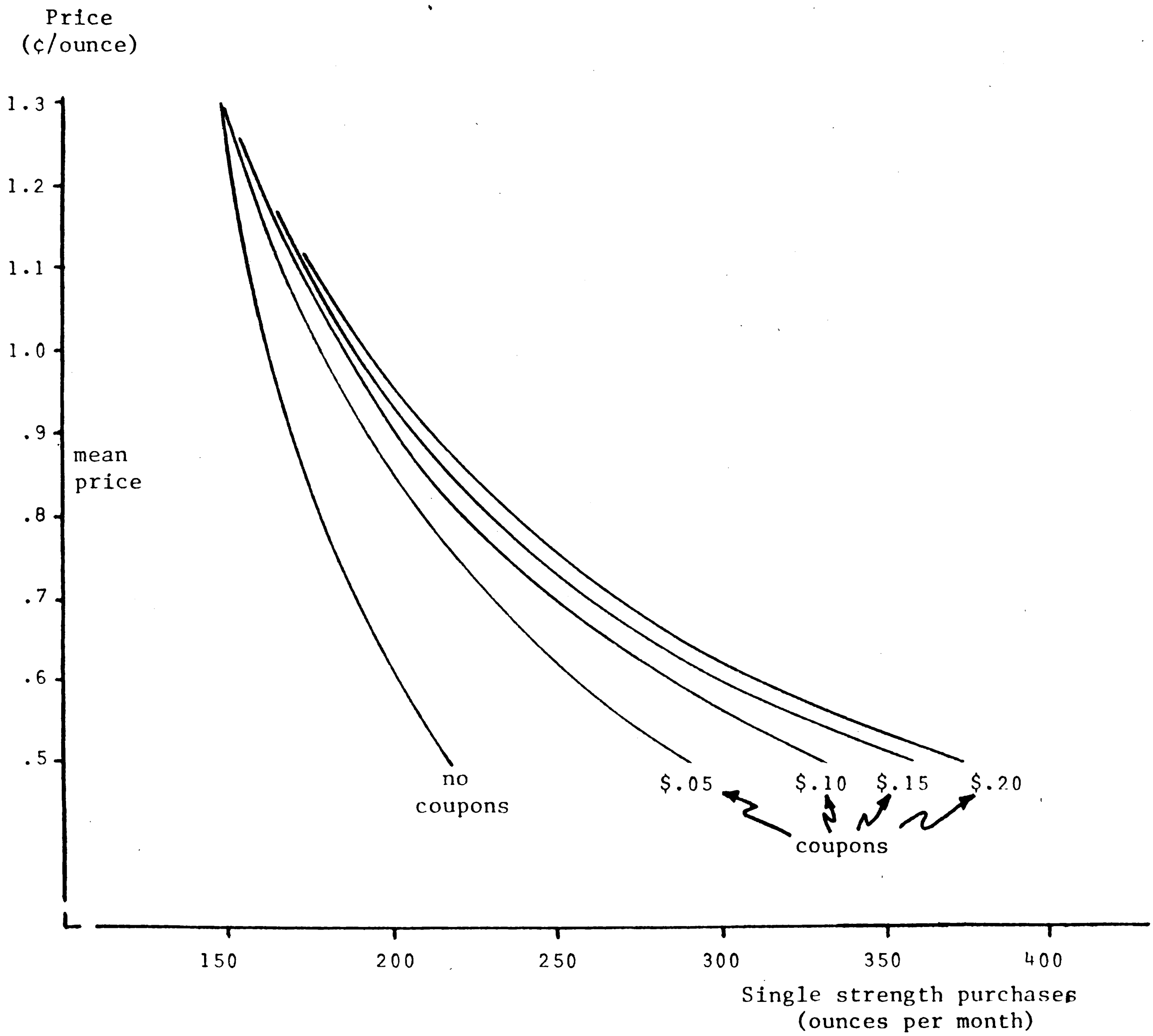

Figure 2. Coupon effect on the average household demand relationship per month.

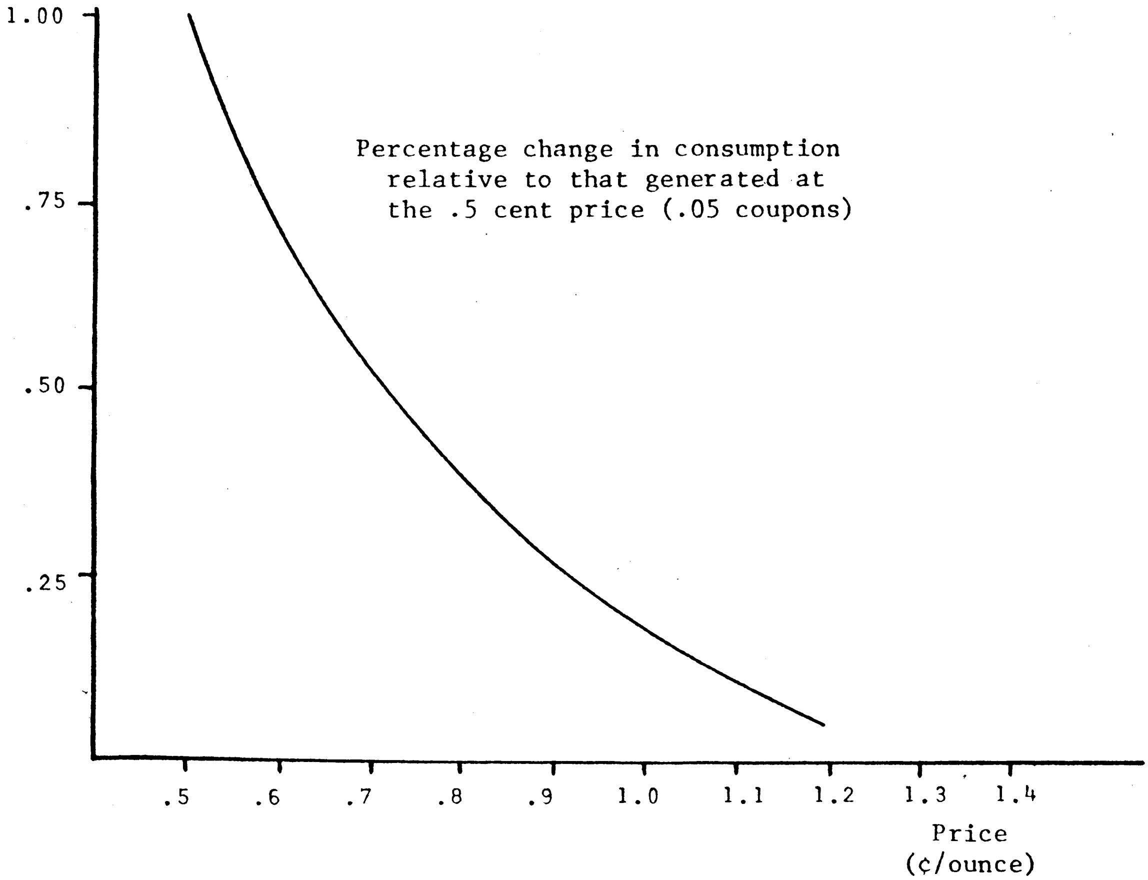

Figure 3. Decline in coupon effectiveness as the retail container price increases.

Advertising Effect

The results from Figure 2 clearly establish that consumers can be stimulated to increase their consumption via the use of coupons. This increase results from a change in the purchasing price as well as from the addition of new information. Using equation (12) the informational component of the total response can be measured.

Figure 4 reports the ratio of the informational or advertising effect to the total coupon response shown in Figure 2. First considering the consumption gains realized from five cents worth of coupons at a container price of .6 cents, 79 percent of the gain in consumption was directly due to the informational stimulant. For higher prices, the total response declined and the informational impact decreased relative to the total effect. As consumers face higher prices, the model clearly indicates that it is more difficult to stimulate the consumer using new information. This phenomenon is especially important to couponers since during periods of rising prices the coupon program generally cost more to implement and the program becomes a less effective marketing tool.

The lower graph in Figure 4 shows the advertising ratio for coupons totalling twenty cents per month. The declining stimulant with higher prices is still evident. In addition, for the same price, the advertising effect relative to the total has declined. While the total gains increase with more coupons, that portion of the gain resulting from simply giving the consumer a price discount has increased. In this sense, the consumer is using the larger value of coupons as a price cutting device since most of the informational effect could have been

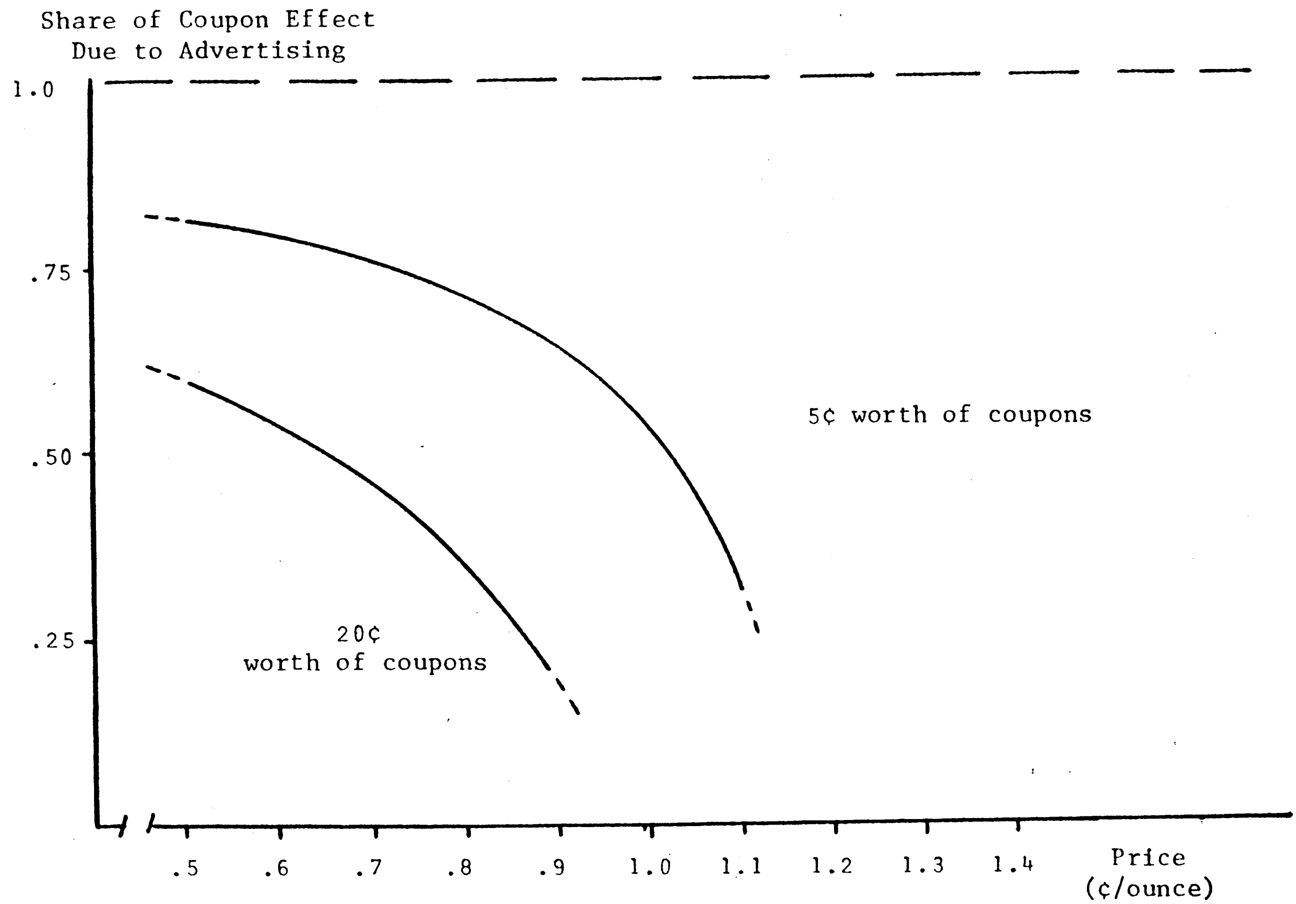

Figure 4. The ratio of the advertising to the total coupon effect (see equation 12).

achieved with a smaller total value of coupons. Again this ratio becomes especially important to the couponer since the coupon value and/or the intensity of the coupon drop can lead to a mechanism for price discounting with secondary informational effects. Likewise, if waste occurs, then both the consumer and couponer are ultimately worse off.

Demographic Effect

As shown in Table 1 a number of demographics were included in the coupon model. Of these, the redeemer's age proved to be significant in having both a direct and indirect effect on coupon effectiveness. Since the over forty and under sixty age group was used as the base, the effectiveness of coupons used by the two remaining age groups relative to this base is readily calculated.

Considering the total effect derived in (1) and illustrated in Figure 2, then the relative differences in a response to coupons redeemed for the age groups are:

$$(13a) \quad \frac{dq|_{<40}}{dq|_{40-60}} = 1.0057\ c^{.0293}\ p^{-.1134}$$

and

$$(13b) \quad \frac{dq|_{>60}}{dq|_{40-60}} = 1.0650\ c^{.0384}\ p^{.0731}$$

Table 2 shows the values of (13a) and (13b) assuming various levels for coupons and prices. For the lower coupon values (i.e., 5¢), both the younger and older consumers respond less to the level of coupons. Given larger values of coupons, both age groups' effectiveness increases relative to the base group. As prices increase, the model further shows that the marginal responses to coupons become substantially less for the younger consumer group. During periods of higher prices, the under 40 group becomes increasingly more difficult to stimulate relative to the older consumers. In contrast, the older consumers become somewhat more responsive relative to the 40-60 group.

While the other demographics included in equation (11) have not been analyzed in detail in this article, it is clear that having an analytical understanding of the consumer profile is essential to the successful implementation of coupon programs.

Table 2.--Coupon effectiveness relative to the 40-60 age group

Price (¢/oz.)	05¢ coupons		20¢ coupons	
	Under 40 yrs.	Over 60 yrs.	Under 40 yrs.	Over 60 yrs.
.6	.976[a]	.914	1.016	.964
.8	.945	.934	.984	.984
1.0	.921	.949	.959	1.001
1.2	.902	.962	.940	1.015

[a]Values correspond to the gain in consumption for that age group relative to that realized by the base group.

Conclusion

The empirical results from the study of coupons for promoting frozen concentrate show coupons to be an effective tool for informing the consumer. While the coupon parameters are for a specific commodity, much of the results are generally expected to be applicable to a broad number of commodities. The demographic differences, the declining marginal returns to coupons, and the advertising effect are likely general to many food products. This particular couponed product is unique in that the product is generic; whereas, a number of similar programs are brand oriented. The analysis does not address the incidence of brand switching resulting from coupon drops.

The demand model in conjunction with the coupon redemption model can be used to evaluate the projected impact of various coupon programs for the concentrate industry. Assessment of such programs prior to implementation can reduce cost and economic waste resulting from excessive drops. Likewise, guidelines for media use are apparent with the analyses. From a theoretical perspective, the coupon model delineates the effect of prices from informational stimulants.

Historical trends indicate that coupons will continue to be used to promote food products and may well be one of the major ways for stimulating consumers to more frequently enter certain areas of food stores. Further, evidence of cooperative couponing between retailers and wholesalers and use of joint ventures to coupon non-competitive products suggest structural changes that result directly from an effort to inform the consumer. Considerable research needs to be completed on relating the success of advertising programs with that of changing market structures in agriculture.

FOOTNOTES

[1]Prices were defined in cents while coupons were measured in dollars. The effect of coupons on price in equation (1) must be adjusted by 100 to compensate of this difference. These variables were expressed in different units for estimation covenience used later in a nonlinear model.

[2]Growth trend and seasonality variables are to be included in the model to account for intercept changes over time. As a general rule it is much easier to account for time series versus cross sectional differences. The gains from simplification of the pooled model is expected to more than offset any small amount of randomness in the intercept over time. The inclusion of these variables imply that the error over time is reduced to zero and equation (7) is the appropriate error specification.

[3]Consider the model $y_i = x^o e^{\Sigma \beta_i D_i}$ where D_i is a dummy variable and $i \leq n$. Suppose the k^{th} group has the probability f_k of occurring, then the average value of y_i over the i groups follows where

$$\Sigma f_i \log(y_i) = \Sigma f_i \log(x^o) + \Sigma \beta_i D_i f_i,$$

or $$\Sigma f_i \log(y_i) = \log(x^o) + \Sigma \beta_i f_i.$$

This average represents the geometric mean of y over the groups donated by D_i where now

$$\text{Geometric Mean } (y_i) = x^o e^{\sum_{i=1}^{n} \beta_i f_i}$$

FOOTNOTES (Cont'd.)

This illustration is analagous to the coupon model where the geometric mean is calculated over a number of independent groups of dummy variables. Ultimately, the geometric mean is calculated using the following values for each group of dummies referenced in (11).

$I = 2.0339$	$z_5 = .253$	$r_3 = .106$
$z_1 = .058$	$z_6 = 24$	$r_4 = .127$
$z_2 = .487$	$z_7 = .014$	
$z_3 = .431$	$r_1 = .309$	
$z_4 = .392$	$r_2 = .305$	

These probabilities then give the geometric average which must be converted to a simple arithmetic mean in order to reference the average consumer. Given the variance of the log normal distribution estimated in Table 1, then the arithmetic mean is approximated over the demographics where

$$A.M = G.M.e^{\sigma^2/2} \quad \text{and } \sigma^2 = .4369$$

then $= G.M\ (1.244)$ [Klein, p. 328].

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