Division of Agricultural Sciences
UNIVERSITY OF CALIFORNIA

A MODEL FOR ANALYZING LENDERS' PERCEIVED DEFAULT RISK

Gershon Feder and Richard E. Just

California Agricultural Experiment Station
Giannini Foundation of Agricultural Economics
March 1978

Working Paper No. 49

A MODEL FOR ANALYZING LENDERS' PERCEIVED DEFAULT RISK

by

Gershon Feder and Richard E. Just

GERSHON FEDER and
RICHARD E. JUST*

A MODEL FOR ANALYZING LENDERS' PERCEIVED DEFAULT RISK

Bank lending involves, in most cases, the risk that the borrower will not be able or willing to honor his obligations. The existence of default risk is an important factor in explaining the observed behavior of lenders as demonstrated in the works of Jaffee and Modigliani [6], Smith [9], Azzi and Cox [2], Jaffee and Russel [7], and others. Lenders' behavior in this case depends crucially on their subjective evaluation of the probability of default. Thus, to explain lending behavior, knowledge of lenders' subjective probabilities is critical. But, of course, this subjective information is generally unobservable; and empirical analysis of lending behavior is therefore difficult if not impossible. Furthermore, there is often reason to believe that subjective information may vary considerably from lender to lender or from transaction to transaction because of previous experience, personal relationships, etc.; and, hence, the role of subjective perceptions cannot be ignored.

The purpose of this paper is to develop a model which facilitates inference about lenders' subjective default probabilities. Several alternative approaches of constructing both point and interval estimates of subjective default probabilities are proposed under various assumptions about competition, loss rate distribution, risk aversion, and relative loan size.

The assumption which makes these approaches possible is that lending takes place with a fixed probability of default. That is, lending transactions are assumed to be of sufficiently negligible size relative to the borrower's scale of operations (e.g., lending to sovereign borrowers) so that the probability of

---

Gershon Feder is economist, Development Research Center, World Bank, and Richard E. Just is associate professor of agricultural and resource economics, University of California, Berkeley.

default is not influenced by the lender's current decision, i.e., the interest rate on the loan does not affect default probability. This assumption differs from that employed in the theoretical papers cited above and, hence, removes complications associated with endogenizing the probability of default. Nevertheless, the simplification which leads to the empirical possibilities developed in this paper is plausible in many situations--particularly in international lending. For example, suppose the state of São Paulo, Brazil, borrows $100 million in the Eurodollar market with the guarantee of the federal government of Brazil. The likelihood of default in this case would surely be affected only slightly by the lender's decision since the overall debt of Brazil is $32 billion dollars [8].

1. THE MODEL

The lender is assumed to have a given amount W of loanable funds, all or part of which can be lent to the particular borrower under consideration at interest rate $\hat{r}$. The interest rate is exogenously given if the lender operates within a competitive capital market, or it depends on the volume of loans granted (according to the borrower's demand schedule) if the lender has some degree of monopoly power. Alternatively, part or all of the lender's funds can be lent at interest ρ which is the opportunity cost of capital and which is considered risk free.[1] The borrower's demand schedule is given by the relation,

$$\hat{r} = \hat{r}(L), \qquad \begin{array}{l} \hat{r}' < 0 \text{ in the monopolist's case} \\ \hat{r}' = 0 \text{ for the competitive lender,} \end{array} \tag{1}$$

where L denotes the volume of loans granted. It will be useful to define the risk premium r which is the difference between the lender's opportunity cost of capital and the interest rate actually paid by the borrower, namely,

$$\hat{r} \equiv r + \rho. \tag{2}$$

Using equation (1) and considering ρ as a constant, the demand schedule can be defined for the risk premium r,

$$r = r(L), \qquad r' = \hat{r}' \leq 0, \tag{3}$$

where, as before, $r' = 0$ applies to the case of the competitive lender.

When assessing a loan request, the lender considers the possibility that the borrower will default on part of the interest and principal due. Earlier models have implied that credit-worthiness analysis on the part of the lender produces a subjective distribution which specifies the probability of various degrees of default as a function of the volume of loan and interest charged in the particular deal considered. The present model, however, assumes that the distribution of potential outcomes is determined by existing economic attributes of the borrower and that the particular deal at hand has a negligible impact on the distribution.[2] Thus, the general formulation of default probability may be specified as

$$P = \int_{\underline{h}}^{1} p(h)\, dh, \tag{4}$$

where h denotes the rate of loss (evaluated in present value terms) suffered in case of default; $\underline{h}$ is the smallest possible rate of loss (which can be negative); P is the probability that a default will occur; and $p(h)/P$ is the probability density function of the loss rate h, given the case of default.

The duration of loan (denoted by N) is assumed to be an exogenous factor since it is dictated to a large extent by the lender's and borrower's overall liquidity projections [3, p. 723]. For simplicity, the loans considered here are assumed to be of a "balloon" type; i.e., interest is paid annually, and the principal is repaid at year N when the loan is terminated. Thus, if no default takes place (an event with probability 1 - P), the following discounted value of funds (say, y_1) will accrue to the lender:

$$y_1 = W - L + (r + \rho) L \sum_{i=1}^{N} (1 + \rho)^{-i} + L (1 + \rho)^{-N} = W + r\Theta L, \qquad (5)$$

where

$$\Theta \equiv \frac{1 - (1 + \rho)^{-N}}{\rho}. \qquad (6)$$

If a default causing a rate of loss h takes place, the lender's present value of funds is

$$y_h = W - hL. \qquad (7)$$

Suppose that the lender has a utility function defined over the present value of funds,[3] given by

$$U = U(y), \qquad U' > 0, \qquad U'' \leq 0, \qquad (8)$$

where $U'' < 0$ implies risk aversion and $U'' = 0$ reflects risk neutrality. It is further assumed that, in the case $U'' < 0$, relative risk aversion is no greater than one, i.e., $-U''/U' \cdot y \leq 1$. This is a plausible assumption as argued by Arrow [1, p. 98]. The lender's objective is to maximize expected utility by optimal choice of loan size, i.e.,

$$\max_{L} \Pi \equiv (1 - P) \cdot U(W + r\Theta L) + \int_{\underline{h}}^{1} U(W - hL)\, p(h)\, dh. \qquad (9)$$

The first-order condition for an optimum (assuming an internal solution) is given by

$$\frac{\partial \Pi}{\partial L} = (1 - P) \cdot \Theta \cdot U'(y_1) \cdot (r + Lr') - \int_{\underline{h}}^{1} U'(y_h) \cdot h \cdot p(h)\, dh = 0. \quad (10)$$

The second-order condition requires

$$\frac{\partial^2 \Pi}{\partial L^2} = (1 - P) \cdot U''(y_1)\, \Theta^2\, (r + Lr')^2 + \int_{\underline{h}}^{1} U''(y_h)\, h^2\, p(h)\, dh$$
$$+ (1 - P) \cdot \Theta \cdot U'(y_1) \cdot (2r' + Lr'') < 0. \quad (11)$$

From equation (11), it is obvious that risk aversion is a necessary and sufficient condition for $\partial^2\Pi/\partial L^2 < 0$ in the case of a competitive lender ($r' = r'' = 0$). In the case of a risk neutral monopolistic lender, $(2r' + Lr'') < 0$ is necessary and sufficient to guarantee concavity of the objective function. With risk aversion and monopoly, the latter condition is not necessary but remains sufficient. Henceforth, it will be assumed that second-order conditions hold for the lender under consideration.

2. COMPARATIVE STATIC RESULTS

It is first useful to examine the various comparative static properties of the model for the purpose of showing that the model is indeed plausible and yields results consistent with the literature.

Demand Elasticity. Considering the monopolistic lender, equation (10) implies that the lender operates on the nonelastic portion of the borrower's demand schedule. To see this, note that $r + Lr' = r \cdot (1 - \eta)$, where η is the absolute value of demand elasticity at the optimal point. Obviously, if $\eta > 1$, equation (10) cannot hold since the left-hand side is negative.

Default Probability. A borrower who is more risky (i.e., who carries a higher probability of default) will be granted less credit whether the lender is competitive or not. This can be confirmed by assuming that, for any given rate of loss h, the relative likelihood is at least as great as before, i.e., $d\ p(h) \geq 0$ for $\underline{h} \leq h \leq 1$. Hence, using equation (4),

$$\int_{\underline{h}}^{1} d\ p(h) > 0$$

if $dP > 0$. By differentiating equation (10), one then obtains:

$$\frac{dL}{dP} = \left[\frac{\partial^2 \Pi}{\partial L^2} \right]^{-1} \cdot \int_{\underline{h}}^{1} [U'(y_1) \cdot \Theta \cdot (r + Lr') + U'(y_h) \cdot h \cdot] \frac{d\ p(h)}{dP} < 0, \quad (12)$$

where the sign is established using (11) and the earlier result that $r + Lr' > 0$ for both a monopolist and a competitive lender. The result in (12) implies, in the case of a monopolist, that riskier borrowers are charged a higher risk premium. This is also the case in a competitive market since, with a higher probability of default, a smaller amount of loans will be offered by any individual lender in the market at any given interest rate. The aggregate supply of loans to the borrower under consideration will thus decline. With a negatively sloped demand for loans and a positively sloped aggregate supply, a decline in supply must yield a higher equilibrium level of risk premium (with a lower amount of loans contracted). It thus remains to show that the aggregate supply of loans in a competitive market is indeed positively sloped (i.e., that supply is increasing with higher interest rates). This is done by differentiating equation (10):

$$\frac{dL}{dr} = -\left(\frac{\partial^2 \Pi}{\partial L^2}\right)^{-1} (1 - P) \cdot \Theta \cdot U'(y_1) \cdot \left[1 + \frac{U''(y_1)}{U'(y_1)} r\Theta L\right]. \quad (13)$$

Under the assumption that relative risk aversion is no greater than one, the term in square brackets on the right-hand side of (13) is positive; and it is thus concluded that $dL/dr > 0$. It should be noted in the case where the lender considers the loan to have an impact on the probability of default, however, that the offer curve by each individual lender is backward bending irrespective of attitudes toward risk as shown in Jaffee and Modigliani [6], Smith [9], and Azzi and Cox [2]. The different results in the present analysis are due to the fact that a higher interest rate increases the marginal expected utility while, in the earlier models, a higher interest rate may reduce expected utility by causing a higher probability of default.

Cost of Capital. A common result in the earlier models of lending is that, with risk neutrality, a higher opportunity cost of capital reduces the size of the optimal loan. It is easy to show that this result holds in the present model for the case of a risk-neutral monopolist. Differentiation of equation (10), recalling equation (2), yields

$$\frac{dL}{d\rho} = -\left[\frac{\partial^2 \Pi}{\partial L^2}\right]^{-1} \left[(1 - P)\ (r + Lr')\ \frac{\partial \Theta}{\partial \rho} - (1 - P)\ \Theta\right]. \quad (14)$$

One can show that $\partial\Theta/\partial\rho < 0$; thus, $dL/d\rho < 0$.

As mentioned earlier, in the present model risk neutrality is consistent with an internal solution for a noncompetitive lender only. Nevertheless, with relative risk aversion less than one, it can be shown that for the competitive lender an increase in the cost of capital reduces the volume of lending.

Risk Aversion. Considering the role of risk aversion in the model, it is intuitively expected that, when lenders are more risk averse, the volume of loans will be lower and risk premiums will be higher. This can be shown simply by assuming a specific form of the utility function such that risk aversion is reflected in a single parameter. The two most common such utility functions are the constant relative risk-aversion family of functions and the constant absolute risk-aversion family of functions. These are given, respectively, by

$$U(y) = ay^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{15a}$$

$$U(y) = a - e^{-\alpha y}, \qquad \alpha > 0 \tag{15b}$$

where α is a parameter of risk aversion such that the higher α is associated with higher risk aversion. Assuming form (15a), differentiation of equation (10) obtains

$$\frac{dL}{d\alpha} = \left[\frac{\partial^2 \Pi}{\partial L^2}\right]^{-1} \cdot (W + r\Theta L)^{-\alpha} \cdot \Bigg[(1 - P) \cdot \Theta \cdot (r + Lr') \cdot \int_{\underline{h}}^{1} h \cdot (W - hL)^{-\alpha} \cdot p(h) \ln\left(\frac{W - hL}{W + r\Theta L}\right) \cdot dh\Bigg]. \tag{16a}$$

Since $\ln [(W - hL)/(W + r\Theta L)] < 0$, it follows that $dL/d\alpha < 0$.

Similarly, assuming the formulation (15b) and differentiating equation (10) yields

$$\frac{dL}{d\alpha} = \left[\frac{\partial^2 \Pi}{\partial L^2}\right]^{-1} \cdot \left[\int_{\underline{h}}^{1} h \cdot p(h) \cdot e^{-\alpha(W-hL)} \cdot (h + r\Theta) \cdot L \cdot dh\right] < 0. \tag{16b}$$

The results in (16a) and (16b) imply that a higher degree of risk aversion causes a smaller volume of loans and higher risk premiums in both competitive and noncompetitive markets.

3. SUGGESTED APPLICATIONS

The results in the previous section are plausible and intuitive and imply that the equilibrium relation in equation (10) can be usefully considered for further expositional and empirical purposes. In this section several such empirical applications are demonstrated. For this purpose, suppose that lenders' information and evaluation distinguish between the overall probability that a default will take place and the (conditional) probability that a given rate of loss (h) will be incurred. Hence, for conceptual purposes, and so that subjective default probability can be meaningfully discussed, suppose that the probability density function of loss rate h is proportional to the overall default probability such that the conditional probability of h depends on h alone and not on the economic factors which explain the overall probability. This implies

$$\frac{p(h)}{P} = \psi(h), \qquad \int_{\underline{h}}^{1} \psi(h)\, dh = 1, \tag{17}$$

where ψ depends only on h irrespective of the economic factors which determine P. Note that, with this assumption, the (unconditional) probability of any given loss rate increases when the overall default probability increases, as one would expect, since equation (17) implies $p(h) = \psi(h) \cdot P$; thus, $d\, p(h)/dP = \psi(h) > 0$. Applying equation (17) in equation (10) and rearranging yields

$$r = \frac{1}{(1 - \eta)} \frac{P}{(1 - P)} \frac{\bar{h}}{\Theta} R \tag{18}$$

where

$$\bar{h} \equiv \int_{\underline{h}}^{1} \psi(h)\ h\ dh$$

and

$$R = \frac{\int_{\underline{h}}^{1} h \cdot \psi(h) \cdot U'(y_h)\ dh}{\bar{h} \cdot U'(y_1)} .$$

For a monopolistic lender, equation (18) demonstrates that the risk premium includes a demand elasticity factor, $(1 - \eta)^{-1}$; the odds of default, $P/(1 - P)$; a time-effect factor, Θ^{-1}; an average loss-rate factor $\bar{h}$; and a risk-aversion factor, R. The latter factor is identically equal to one under risk neutrality in which case $\bar{U'(y_h)} = U'(y_1)$. With risk aversion, it is easy to show that $R > 1$ and $\partial R/\partial L > 0$.

Equation (18) offers a great deal of flexibility in empirical inference of subjective default probabilities because of its simplicity. This flexibility is demonstrated in the following three suggested applications.

A. The Threshold Default Probability

A simple application of the model is related to the concept of "threshold probability" for competitive lenders. The threshold probability is the highest value of default probability (say, P*) for which a loan will be granted to any borrower and depends on r, Θ, and $\bar{h}$. Borrowers with probability lower than P* will be given credit with the amount of credit increasing in the ratio $[P^*/(1 - P^*)]/[P/(1 - P)]$.

Using equation (18), the threshold probability is calculated by noting that, at $P = P^*$, it must hold that $L = 0$ which implies that $U'(y_1) = U'(y_h) = U'(W)$. This obtains

$$P^* = \frac{r}{(\bar{h}/\Theta) + r} . \tag{19}$$

By differentiating equation (19), one can verify that the threshold probability is positively related to the risk premium and the loan duration (i.e., $\partial P^*/\partial r > 0$, $\partial P^*/\partial N > 0$) while being negatively related to the average loss rate and to the opportunity cost of capital (i.e., $\partial P^*/\partial \bar{h} < 0$, $\partial P^*/\partial \rho < 0$). Using equation (19) in (18), with $\eta = 0$, further yields

$$R(L) = \frac{P^*/(1 - P^*)}{P/(1 - P)} . \tag{20}$$

The right-hand side of (20) is the ratio of odds evaluated at the threshold and actual values of probability. From equation (20), one can derive the result that loans (and the risk aversion premium) increase as the right-hand side of equation (20) increases.

A useful property of the threshold probability is that it is independent of the degree of risk-aversion as is apparent from equation (19). Since the values of ρ, N, and $\bar{h}$ are either known or can be estimated without major difficulty, one can calculate the threshold probability which applies to different risk premiums. This approach is demonstrated in Table 1 for $\rho = .065$ and $\bar{h} = .15, .2$. The results are presented in terms of q^* (the short-run threshold probability) so as to allow comparison between cases with different loan durations. That is, if at most, one default can take place within the duration of the loan, then the probability P of default over the entire period of the loan is $P = 1 - (1 - q)^N$, provided that q, the probability of default in

TABLE 1

Threshold Short-Run Probabilities and Risk Premiums for 5- and 10-Year Loans[a]

Loan duration	r	q* ($\bar{h}$ = .15)	q* ($\bar{h}$ = .2)
years	percent		
N = 5	.25	.013	.010
N = 10	.25	.011	.009
N = 5	.50	.026	.020
N = 10	.50	.021	.016
N = 5	.75	.037	.029
N = 10	.75	.03	.024
N = 5	1.00	.048	.037
N = 10	1.00	.038	.03
N = 5	1.25	.058	.045
N = 10	1.25	.046	.036
N = 5	1.50	.067	.053
N = 10	1.50	.053	.042
N = 5	1.75	.076	.060
N = 10	1.75	.059	.048
N = 5	2.00	.085	.067
N = 10	2.00	.065	.053

[a]Assuming the opportunity cost of capital ρ is 6.5 percent and N = 5, the value of Θ is 4.156; with N = 10, the value of Θ is 7.1888.

any given year, is constant.[4] It is reasonable to expect that, given current data, lenders assume q to remain essentially constant since no data beyond the period in which the loan is granted are available for projection. The q* in Table 1 is a threshold or upper bound on q and is related to P* just as q is related to P.

Using this approach, one can very easily confine the subjective probability q to the interval [Q, q*] under competition by simply using data on r, ρ, N, and $\bar{h}$ (note that Θ is determined by ρ and N); moreover, the interval [0, P*] is fairly narrow even for small $\bar{h}$. Furthermore, if $\bar{h}$ is unknown but can be bounded from below by $\bar{h}^*$, then P can still be confined to the interval [0, P**] where

$$P^{**} = \frac{r}{(\bar{h}^*/\Theta) + r}$$

(since P* is decreasing in $\bar{h}$) or q can still be confined to an associated interval [0, q**] where $P^{**} = \bar{1}\ (1 - q^{**})^N$. Hence, the simple transaction data on r, ρ, and N is sufficient to develop some information about the magnitudes of subjective default probabilities which might exist.

B. An Application in Inferring Lenders' Subjective Probabilities: The Case of the Eurodollar Market

Another application of the model in this paper can be made by inferring interval estimates of lenders' subjective probabilities of default from a minimal amount of observed market data. To demonstrate this possibility, 20 observations on loans granted in the Eurodollar market during the third quarter of 1973 have been selected. These observations (presented in the Appendix) represent all the loans made during that period to public (or publicly quaranteed) entities in developing countries for which data on risk premium and loan duration were available.[5]

The procedure employed here uses the assumptions that the market operates competitively and that the subjective loss-rate distribution is singular in order to establish bounds on the lenders' perceived default probabilities. The first assumption implies that $\eta = 0$, and the second implies that h is a constant, say, $\bar{h}$. Assuming that the utility function can be specified by the constant elasticity formulation [equation (15b)], the risk-aversion premium R can then be written

$$R = \left(\frac{1 + r\Theta\ell}{1 - \bar{h}\ell} \right)^{\alpha}, \qquad (21)$$

where $\ell \equiv L/W$ is the share of the loan in total loanable funds and α is the relative risk-aversion parameter ($0 < \alpha < 1$).

While the 20 observations on Eurodollar loans range in loan duration between 7 and 15 years, the probability concept which is of interest is again one that is free of the loan duration effect. Thus, combining equations (18) and (21) and the relation between q and P, the following formula is obtained:

$$q = 1 - (1 - P)^{1/N} = 1 - \left[1 + \frac{r\Theta}{\bar{h}} \left(\frac{1 - \bar{h}\ell}{1 + r\Theta\ell} \right)^{\alpha} \right]^{-1/N} . \qquad (22)$$

To make use of equation (22) in the Eurocurrency market, it can be observed that the average rate of loss in international banking is low; in almost all cases, loans are renegotiated and rescheduled.[6] Hence, for exemplary purposes, a value of $\bar{h} = .1$ is used.[7] As for ℓ, ρ, and α, it can be noted from (22) that $\partial q/\partial \ell$, $\partial q/\partial \rho$, and $\partial q/\partial \alpha < 0$. Thus, specifying sets of upper and lower bounds on these coefficients leads to lower and upper bounds, respectively, for the short-run probability q. For example, consider upper bounds of $\ell = .3$, $\rho = .08$, and $\alpha = .8$ and for lower bounds of $\ell = 0$, $\rho = .06$, and $\alpha = 0$. These values seem to span the range of reality. That is, $\ell = 0$ and $\alpha = 0$ are the lowest possible values (assuming no risk-loving behavior), while $\ell = .3$ and $\alpha = .8$ seem to be above and beyond all likely possibilities. No bank seems to loan anything near 30 percent of its funds to one borrower; also, α close to 1, in the case of the constant elasticity utility function, implies an extremely high degree of risk aversion. Finally, the opportunity cost of capital seems to be very close to 7 percent. Using these bounds, the associated interval estimates for subjective default probability (of the lender) in Table 2 are possible for the Eurodollar market transactions considered above.

Several interesting observations can be made on the basis of these results. First, the upper and lower bounds span a range for the probability of each case within 7/10 of 1 percent or less. This is a small interval relative to the overall variation in Table 2. Second, the calculated q's for those countries with more than one observation (e.g., Algeria) are fairly close (theoretically, they should be identical). This may serve as an indication of the creditability of the assumptions. In summary, it appears that fairly precise estimates of subjective default probability are possible under competition using only the terms of the loan if loss rate for the event of default can be determined (estimated).

C. Possible Econometric Applications

In addition to the above examples, equation (18) also has interesting econometric possibilities. For the purpose of econometric work, it is useful to consider the common logistic specification for the probability, namely,

TABLE 2

Perceived Short-Run Default Probabilities in the Euromarket
With Probability q, Third Quarter, 1973

Observation number	Country	Upper bound[a]	Lower bound[b]
1	Algeria (N = 15)	.044	.039
2	Algeria (N = 12)	.047	.042
3	Algeria (N = 10)	.048	.044
4	Brazil (N = 10)	.054	.049
5	Brazil (N = 12)	.049	.044
6	Colombia	.043	.039
7	Gabon	.079	.072
8	Greece	.040	.036
9	Iran (N = 12)	.037	.033
10	Iran (N = 10)	.037	.033
11	Jamaica	.063	.057
12	Korea	.059	.052
13	Mexico (N = 12)	.036	.032
14	Mexico (N = 10)	.035	.031
15	Nicaragua	.066	.060
16	Peru	.075	.068
17	Senegal	.083	.075
18	Sudan	.093	.086
19	Zaire	.079	.072
20	Zambia	.069	.063

[a]Calculated with $\alpha = 0$, $\ell = 0$, $\rho = .06$, and $\bar{h} = .1$.

[b]Calculated with $\alpha = .8$, $\ell = .3$, $\rho = .08$, and $\bar{h} = .1$.

$$P = \frac{e^{\beta' X}}{1 + e^{\beta' X}} \tag{23}$$

where β is a vector of coefficients and X is a vector of economic indicators, including loan duration, which are considered by lenders as relevant risk indicators. With this specification, the odds are log linear, i.e., $\ln [P/(1 - P)] = \beta X$. Hence, equation (18) becomes

$$\ln r = -\ln (1 - \eta) + \ln \bar{h} - \ln \Theta + \beta' X + \varepsilon, \tag{24}$$

where

$$\varepsilon \equiv \ln h \equiv \ln \left[\int_{\underline{h}}^{1} h\psi(h)\, U'(y_h)\, dh \right] - \ln [\bar{h} U'(y_1)].$$

The model in (24) has several interesting potential simplifications. First, for risk-neutral lenders it is clearly the case from above that $\varepsilon = \ln R = 0$; hence,

$$\ln r = -\ln (1 - \eta) + \ln \bar{h} - \ln \Theta + \beta' X. \tag{25}$$

With the presence of a disturbance term, equation (24) can be estimated in a variety of situations to determine which factors are considered important by lenders in assessing default probabilities. For instance, η, $\bar{h}$, and Θ can be calculated (Θ depends only on ρ and N which are often recorded, and $\bar{h}$ is also occasionally recorded for different classes of borrowers) and then included in a regression estimating β where a coefficient of 1 or -1 is imposed for $\ln (1 - \eta)$, $\ln \bar{h}$, and $\ln \Theta$ as needed according to (25). Alternatively, one may find data where combinations of η, $\bar{h}$, and Θ are held constant and can thus be included in a constant term. Another approach [4] is to consider some of the first right-hand terms as randomly distributed among borrowers and then use a variance-components approach in estimating β from a time series of cross-section data.

In the case of a competitive market, equation (25) can be further simplified obtaining $\ln r = \ln \bar{h} - \ln \Theta + \beta' X$ since $\eta = 0$ under competition. Hence, estimation can be further simplified; or one can, in fact, test the underlying assumption regarding the degree of competition in the market by testing whether $\ln (1 - \eta) = 0$ in equation (25).

Finally, an interesting observation can be made for equation (24) in the general case of risk preferences. As it stands, the ε term in equation (24) would be very difficult to treat econometrically; besides, it would imply inclusion of the dependent variable r on the right-hand side. But in a broad range of cases, it turns out that ε is negligible relative to both ln r and the usual variations in ln r among observations. This can be demonstrated assuming a constant elasticity utility function in which case equation (21) applies. Using the same plausible limits on parameters used to generate Table 2, together with additional limits on r of $0 \leq r \leq 0.025$ and on N of $0 \leq N \leq 10$, it can be shown through simple calculations that $1 \leq R < 1.07$ and hence $0 < \ln R < 0.068$. By comparison, as r varies from 0.00575 to 0.01875, as in the Eurocurrency market data used in generating Table 2, the dependent variable ln r varies from -3.977 to -5.159. Hence, even with most conceivable limits on ℓ, ρ, and α, it is thus clear that ε is negligible and can, for most practical purposes, be included with any disturbance in (24).

In summary, then, it appears that point estimates of default probability (based on β estimates) are possible in a wide variety of cases if one is able to obtain data for economic or other indicators considered by the lender in granting loans as well as the terms of loans.

4. CONCLUSION

This paper presents a model of lending under default risk with the underlying assumption that the probability of default is determined by factors not controlled by the lender. The effects of risk, risk aversion, cost of capital, and expected loss rate on the equilibrium levels of loan supply and risk premiums are analyzed for both competitive and noncompetitive markets. The model is shown to be useful for assessing subjective default probabilities from market data using the Eurodollar market. The model also enables the determination of upper bound threshold probabilities for acceptable customers, given the relevant duration of loan and interest rate. Further utilization of the model for various econometric purposes is discussed in detail, and it is shown that in many cases the complicated term reflecting risk aversion can be ignored since its magnitude and range of variation are negligible. This simplifies greatly the procedures for econometric application of the model in obtaining point estimates of subjective default probabilities.

APPENDIX

Transactions in the Eurodollar Market Third Quarter, 1973[a]

Borrowing country	risk premium	Loan duration
	percent	years
Algeria	1.008	15
Algeria	.938	12
Algeria	.875	10
Brazil	1.0	12
Brazil	1.0	10
Colombia	.75	10
Gabon	1.75	10
Greece	.75	12
Iran	.688	12
Iran	.625	10
Jamaica	1.25	10
Korea	1.125	10
Mexico	.659	12
Mexico	.575	10
Nicaragua	1.325	10
Peru	1.6	10
Senegal	1.875	10
Sudan	1.75	7
Zaire	1.75	10
Zambia	1.425	10

[a]When several premiums were charged in different stages of the loan duration period, a weighted average was calculated. Also, when several loans with the same duration were observed, a weighted average was calculated.

Source: World Bank, <u>Borrowing in International Capital Markets (1973-1975)</u>. Report No. EC-181 (Washington, D. C., 1976).

FOOTNOTES

*Giannini Foundation Paper No. . The views expressed in this paper are those of the authors and do not necessarily reflect the views of the institutions with which they are affiliated.

[1]As in the earlier works on lending under uncertainty cited above, portfolio considerations are ignored for the sake of simplicity.

[2]For instance, when the state of São Paulo in Brazil borrows $100 million in the Eurodollar market (with the guarantee of the federal government of Brazil), the likelihood of default is affected only slightly by this specific loan and the terms under which it is granted.

[3]The utility function could be defined over terminal wealth without changing the results.

[4]The reader will note that this definition of P was indeed used in the description of the model.

[5]A period of no more than a quarter was chosen so as to avoid any significant variations in the opportunity cost of capital among observations. Also, it seems that after 1974 some elements of th risk premium in the Euromarket are included in transactions fees which are not reported. Finally, the third quarter of 1973 was selected since it has the largest number of complete and reliable observations as reported by the World Bank [10].

[6]In this respect, Friedman [5, p. 55] noted: "Losses in U. S. banks' overseas operations have been less, both in absolute terms and as a proportion of total risk assets, than in U. S. operations." A similar observation has been made by Beim [3, p. 717].

[7]Normally, one would want to estimate $\bar{h}$ since the estimates are somewhat sensitive to this particular parameter.

LITERATURE CITED

1. Arrow, Kenneth J. Essays in the Theory of Risk-Bearing. Chicago: Markham Publishing Company, 1971, Chapter 3.

2. Azzi, Corry F., and James C. Cox. "A Theory and Test of Credit Rationing: Comment." American Economic Review, 66 (December 1976), 911-917.

3. Beim, David O. "Rescuing the LDCs." Foreign Affairs, 55 (July 1977), 717-731.

4. Feder, Gershon, and Richard E. Just. "An Analysis of Credit Terms in the Eurodollar Market." European Economic Review, 9 (May 1977), 221-243.

5. Friedman, I. S. The Emerging Role of Private Banks in the Developing World. New York: Citicorp, 1977.

6. Jaffee, Dwight M., and Franco Modigliana. "A Theory and Test of Credit Rationing." American Economic Review, 59 (December 1969), 850-872.

7. Jaffee, Dwight M., and T. Russel. "Imperfect Information, Uncertainty, and Credit Rationing." Quarterly Journal of Economics, 90 (November 1976), 651-666.

8. "LDC Debit: The Load Eases." Newsweek, February 27, 1978, 70.

9. Smith, Vernon L. "A Theory and Test of Credit Rationing: Some Generalizations." American Economic Review, 62 (June 1972), 477-483.

10. World Bank. Borrowing in International Capital Markets (1973-1975). Report No. EC-181 (Washington, D. C., 1976).