RISK AND DECISION MAKING IN AGRICULTURAL PRODUCTION*

by
Jean-Paul Chavas and Gustavo Arcia**

Introduction

Risk considerations in producer and consumer decision problems have become a subject of increased interest in agricultural economics. Choices, implying judgments about alternative subjective probability distribution of outcomes, are common for agricultural production units. Factors affected by this aspect of the decision problem include the choice of technique and/or the adoption of new technologies. For decision makers with concave utility functions and low levels of income, the implications of uncertainty for farm decisions may be especially important. In fact, there is evidence from several empirical studies indicating that risk may have been an important factor in slowing down the adoption of Green Revolution technologies in LDC's [Roumasset, Anderson].

A variety of approaches to handling risk in the firm decision model have been advocated. Of these, the expected utility approach appears most plausible and tractable for applied work. Furthermore, decision theory based on the maximization of expected utility is normatively coherent and represents a logical and complete basis for choice in uncertain

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**Graduate students, Department of Agricultural Economics, University of Missouri. We are indebted to S.R. Johnson and Mel Blase for their invaluable help.

circumstances, a factor which makes the approach especially suited for policy analysis.

The purpose of this paper is to provide for a clearer link between maximization of expected utility and a functional form consistent with the characteristics of crop response under risky conditions. Attention is given to the practical relevance of the mean-variance space in connection with the functional forms of utility, and to the econometric properties of the estimators in the production function.

Sources of Risk

The two principal sources of uncertainty for the agricultural entrepreneur are the physical production relationship and prices. Uncertainty in agricultural production functions arises because there are some inputs into the process which are neither known nor controlled or known and uncontrolled by the farm operator at the time of the decision. This idea can be incorporated into the production and decision theory by specifying the production function as:

$$Y = f(X_1,\ldots X_n;\ X_{n+1},\ldots X_m)$$

where $X_1,\ldots X_n$ are inputs controlled at the time of the decision. In crop production these inputs might be fertilizer, herbicides, insecticides, seeds, etc. Inputs $X_{n+1},\ldots X_m$ are uncontrolled (perhaps unknown) at the time of the decision. This class would include climatic variables, rainfall, temperature, growing season, etc.

Generally, there is an interaction effect between the decision inputs $(X_1,\ldots X_n)$ and the uncontrolled inputs $(X_{n+1},\ldots X_m)$. A typical example is the interaction between fertilizer and rainfall. Thus, uncertainty is an important factor in the appraisal of best operating conditions.

Assuming that at the time of the decision, the factors, $X_1, \ldots X_n$ are known with certainty while $X_{n+1}, \ldots X_m$ are random variables described by a multivariate probability distribution, the subjective production function for decision making becomes:

$$Y = f(X_1, \ldots X_n, \varepsilon)$$

where ε is a random disturbance representing the uncertainty associated with $X_{n+1}, \ldots X_m$.

Price uncertainty also may be influential either through uncertainty about the product price, P_y, input prices, p_i, or both. Generally, because of production lag, the input prices are known, but the product price is uncertain at the time of the production decision. Accordingly, it is assumed that price uncertainty occurs only with respect to product price.

Optimality Decisions With Price and Yield Uncertainty

The present analysis of the farmer's decision making process is based on the Expected Utility Theorem. The theorem states that a utility function exists for any decision-maker whose preferences are consistent with a prespecified set of axioms of ordering, continuity and independence. Each decision maker's preference is based on his/her subjective probability distribution of each risky alternative. Therefore, a utility function is simply a device which assigns an index of values to a set of consequences in order to help a decision maker to maximize his/her subjective expected utility.[1] Such a utility function U associates a single real number with

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[1]For a review of the Expected Utility Theorem and the axioms see Dillon; Anderson, Dillon, and Hardaker, and Roumasset. For a good

a risky choice and has the following properties:

Let A and B be two random probability distributions; then

1) If A is preferred to B then $U(A) > U(B)$.

2) The utility of a risky choice A is its expected utility value:

$U(A) = E[u(A)]$

3) Utility measure is completely arbitrary.

Therefore a person who combines the axioms of utility theory with the properties of the utility function in his/her decision making process has both a unique utility function and a subjective probability list. Hence, it is obvious that in choosing among risky choices such person will maximize expected utility. Perhaps the most difficult task facing a researcher of risk problems is the choice of the functional form of the utility functions. It involves finding the preference function of the decision makers and then fitting an algebraic form to it. However, finding an utility function free of interview bias is not a simple task [Roumasset] and fitting a good functional form requires full knowledge of its limitations. Consider, for instance, the decision concerning the

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exposition on the Bernoulli Principle see Borch. The Expected Utility Theorem is proved in Von Neumann and Morgenstern. Such a proof is not within the scope of this paper. The axioms of utility theory (listed here to preserve the flow of the paper) are:

i) Ordering and Transitivity. Let f_1 and f_2 be the probability distributions of two mutually exclusive events. Either a person prefers one distribution over the other or is indifferent between them. Let f_3 be a third probability distribution of another event. If f_1 is preferred (or indifferent) to f_2 and f_2 is preferred (or indifferent) to f_3, then f_1 is preferred (or indifferent) to f_3.

ii) Continuity. If f_1 is preferred to f_2 there exists a unique probability p such that f_2 is indifferent to a lottery that yields f_1 with probability p and f_3 with probability (1-p).

iii) Independence. If f_1 is preferred to f_2, then a lottery with f_1 and f_3 as prizes will be preferred to a lottery with f_2 and f_3 as prizes provided the probability of occurrence for f_1 and f_2 is the same.

assumption of normality in the distribution of the stochastic variables. Such assumption greatly simplifies the analysis since it implies that only a two-parameter functional form is needed in order to portray the entire density function: the sample mean and sample variance. The use of functional forms operating only in the (μ, σ^2) space has created substantial controversy [see Roumasset; Hanoch and Levy; Anderson]. The prime objection is that the mean-variance space is relevant for analysis only if one or both of the following conditions hold [see Feldstein; Tobin]:

i) The utility function is quadratic.

ii) The random variables are normally distributed.

So far, is is not yet entirely clear whether or not such conditions hold in the empirical world. The use of quadratic utility functions such as $U(X) = X + bX^2$ is very appealing due to its convenient properties. It is monotomically increasing over the range of interest and has positive marginal utility since it's also nondecreasing. However, such function is consistent with utility theory only if

$X > -1/2b$ if $b < 0$

$X < -1/2b$ if $b > 0$.

There are also other theoretical limitations involved [see Roumasset, p. 23; Dillon (a and b)] with quadratic functional forms; nevertheless, empirical findings have not yet shown evidence that the inclusion of moments beyond the second (that is, beyond the mean-variance space) significantly alter the results. Hence, if quadratic utility functions cannot be *a priori* eliminated from consideration, the question of normality of the random variables becomes irrelevant. In summary, as Dillon puts it [Dillon (a), p. 34]:

> Overall, despite the theoretical criticisms that have been made of quadratic and cubic polynomials as utility functions, they must still be regarded as satisfactory first-steps to practical application of utility analysis.

The quadratic form is one of several functional forms operating in the mean-variance space. Moreover, non-normal distributions with easily obtainable mean and variance are also available. What is left is more work in exploring such avenues in order to find satisfactory alternatives within the simple framework of the (E, V) space. Under the framework established in the preceding discussion our whole-farm model is based on the assumption that farmers maximize the expected utility of profits. Therefore, let

$$\pi = \sum_j P_{yj} Y_j - \sum_{ij}\sum P_i X_{ij} - F \qquad (1)$$

where $i = 1,2,\ldots n$ inputs,

$j = 1,2,\ldots k$ outputs,

π = total profit,

Y_j = output $j = f_j(X_{ij},\ldots X_{nj}, \varepsilon_j)$,

P_{yj} = price of output j,

X_{ij} = input i used to produce output j,

p_i = price of input i,

F = fixed cost.

be the profit equation for the whole farm unit. The expected utility function is

$$U = f[E(\pi), V(\pi)] \qquad (2)$$

where $E(\pi)$ is the expected value of profits and $V(\pi)$ is the variance. The first order condition for utility maximization is:

$$\frac{du}{dx_i} = \frac{\partial u}{\partial E(\pi)} \cdot \frac{\partial E(\pi)}{\partial X_1} + \frac{\partial u}{\partial V(\pi)} \cdot \frac{\partial V(\pi)}{\partial X_1} = 0 \qquad (3)$$

$$\text{or} \quad \frac{\frac{\partial E(\pi)}{\partial X_i}}{\frac{\partial V(\pi)}{\partial X_i}} = - \frac{\frac{\partial u}{\partial V(\pi)}}{\frac{\partial u}{\partial V(\pi)}} = \text{RSU (rate of substitution in utility)} \qquad (4)$$

With risk aversion, RSU is positive and an increase in variance must be compensated by an increase in expected profit to stay at the same utility level. If RSU = 0, the decision maker is indifferent to risk, and his utility is maximized when his profit is maximized. Since farmers are generally risk averters we would expect RSU $\geq$ 0.

Since generally P_{yj} is correlated with Y_j the expected value of π is:

$$E(\pi) = E(\sum_j^k P_{yj}Y_j - \sum_{ji}^{kn}\sum P_iX_{ij} - F)$$

$$= \sum_j^k E(P_{yj})E(Y_j) + \sum_j^k Cov(P_{yj}, Y_j) - \sum_{ji}^{kn}\sum P_iX_{ij} - F \qquad (5)$$

Differentiating with respect to X_{ij} we have:

$$\frac{\partial E(\pi)}{\partial X_{ij}} = \frac{\partial[Cov(P_{yj}, Y_j)]}{\partial X_{ij}} + \frac{E(P_{yj})\partial E(Y_j)}{\partial X_{ij}} + \frac{E(Y_j)\partial E(P_{yj})}{\partial X_{ij}} - P_i \qquad (6)$$

Similarly, our variance for profits is:

$$V(\pi) = V(\sum_j^k P_{yj}Y_j - \sum_{ji}^{kn}\sum P_iX_{ij} - F) \qquad (7)$$

$$= V(\sum_j^k P_{yj}Y_j)$$

$$= \sum_j^k [V(P_{yj}Y_j)] + 2\sum_{j<g}^{k}\sum^{k} [Cov(P_{yj}Y_j, P_{yg}Y_g)]$$

$$= \sum_j^k \{E(P_{yj}^2Y_j^2) - [E(P_{yj}Y_j)]^2\}$$

$$+ 2\sum_{j<g}^{k}\sum^{k} [E(P_{yj}Y_jP_{yg}Y_g) - E(P_{yj}Y_j)E(P_{yg}Y_g)]$$

which, after several manipulations becomes:

$$= \Big[\text{Cov}\ \{V(P_{yj})V(Y_j) + V(P_{yj})[E(Y_j)]^2 + V(Y_j)[E(P_{yj})]^2 + [E(P_{yj})E(Y_j)]^2$$

$$+ \text{Cov}\ (P_{yj}{}^2, Y_j{}^2) - [E(P_{yj})E(Y_j)]^2 + 2[E(P_{yj})E(Y_j)\text{Cov}(P_{yj}, Y_j)]$$

$$- [\text{Cov}\ (P_{yj}, Y_j)]^2\} + 2\sum_{j<g}^{k}\sum^{k} \{\text{Cov}\ (P_{yj}P_{yg}, Y_jY_g) + \text{Cov}(P_{yj}, P_{yg})\ \text{Cov}$$

$$(Y_j, Y_g) + E(Y_j)E(Y_g)\ \text{Cov}\ (P_{yj}, P_{yg}) + E(P_{yj})E(P_{yg})\ \text{Cov}\ (Y_j, Y_g)$$

$$- E(P_{yj})E(Y_j)\ \text{Cov}(P_{yg}, Y_g) - E(P_{yg})E(Y_g)\ \text{Cov}\ (P_{yj}, Y_j)$$

$$- \text{Cov}\ (P_{yj}, Y_j)\ \text{Cov}\ (P_{yg}, Y_g)\}\Big]^{1/2} \qquad (8)$$

Differentiating with respect to X_{ij} we have:

$$\frac{\partial V(\pi)}{\partial X_{ij}} = \frac{\partial[\text{Cov}(P_{yj}^2, Y_j^2)]}{\partial X_{ij}} + \frac{V(P_{yj})\partial V(Y_j)}{\partial X_{ij}} + \frac{V(Y_j)\partial V(P_{yj})}{\partial X_{ij}}$$

$$+ \frac{[E(Y_j)]^2\partial V(P_{yj})}{\partial X_{ij}} + \frac{V(P_{yj})\partial V(Y_j)}{\partial X_{ij}} + \frac{V(Y_j)\partial V(P_{yj})}{\partial X_{ij}}$$

$$+ \frac{[E(P_{yj})]^2\partial V(Y_j)}{\partial X_{ij}} + \frac{2V(Y_jE(P_{yj})\partial E(P_{yj})}{\partial X_{ij}}$$

$$- \frac{2\text{Cov}(P_{yj}, Y_j)\partial\text{Cov}(P_{yj}, Y_j)}{\partial X_{ij}} - \frac{2\text{Cov}(P_{yj}, Y_j)E(P_{yj})\partial E(Y_j)}{\partial X_{ij}}$$

$$- \frac{2\text{Cov}(P_{yj}, Y_j)E(Y_j)\partial E(P_{yj})}{\partial X_{ij}} - \frac{2E(P_{yj})E(Y_j)\partial\text{Cov}(P_{yj}, Y_j)}{\partial X_{ij}}$$

$$+ 2\sum_{g\neq j} \frac{\partial\text{Cov}(P_{yj}P_{yg}, Y_jY_g)}{\partial X_{ij}} + 2\sum_{g\neq j} \frac{\text{Cov}(P_{yj}, P_{yg})\partial\text{Cov}(Y_j, Y_g)}{\partial X_{ij}}$$

$$+ 2 \sum_{g \neq j} \frac{Cov(Y_j, Y_g)\partial Cov(P_{yj}, P_{yg})}{\partial X_{ij}} + 2 \sum_{g \neq j} \frac{Cov\ (P_{yj}, P_{yg})E(Y_j)\partial E(Y_g)}{\partial X_{ij}}$$

$$+ 2 \sum_{g \neq j} \frac{Cov(P_{yj}, P_{yg})E(Y_h)\partial E(Y_j)}{\partial X_{ij}} + 2 \sum_{g \neq j} \frac{E(Y_j)E(Y_g)\partial Cov(P_{yj}, P_{yg})}{\partial X_{ij}}$$

$$+ 2 \sum_{g \neq j} \frac{Cov(Y_j, Y_g)E(P_{yj})\partial E(P_{yg})}{\partial X_{ij}} + 2 \sum_{g \neq j} \frac{Cov\ (Y_j, Y_g)E(P_{yg})\partial E(P_{yj})}{\partial X_{ij}}$$

$$+ 2 \sum_{g \neq j} \frac{E(P_{yj})E(P_{yg})\partial Cov(Y_j, Y_g)}{\partial X_{ij}} - 2 \sum_{g \neq j} \frac{Cov(P_{yj}, Y_j)\partial Cov(P_{yg}, Y_g)}{\partial X_{ij}}$$

$$- 2 \sum_{g \neq j} \frac{Cov(P_{yg}, Y_g)\partial Cov(P_{yj}, Y_j)}{\partial X_{ij}} - 2 \sum_{g \neq j} \frac{Cov(P_{yj}, Y_j)E(P_{yg})\partial E(Y_g)}{\partial X_{ij}}$$

$$- 2 \sum_{g \neq j} \frac{Cov(P_{yj}, Y_j)E(Y_g)\partial E(P_{yg})}{\partial X_{ij}} - 2 \sum_{g \neq j} \frac{E(Y_g)E(P_{yg})\partial Cov(P_{yj}, Y_j)}{\partial X_{ij}}$$

$$- 2 \sum_{g \neq j} \frac{Cov(P_{yg}, Y_g)E(P_{yj})\partial E(Y_j)}{\partial X_{ij}} - 2 \sum_{g \neq j} \frac{Cov(P_{yg}, Y_g)E(Y_j)\partial E(P_{yj})}{\partial X_{ij}}$$

$$- 2 \sum_{g \neq j} \frac{E(P_{yj})E(Y_j)\partial Cov(P_{yg}, Y_g)}{\partial X_{ij}} . \qquad (9)$$

$$, i=1,\ldots,n;\ j=1,\ldots,k;\ j<g.$$

Assuming:

$$\frac{\partial Cov(P_{yg}, Y_g)}{\partial X_{ij}} = 0, \text{ for both } g=j \text{ and } g \neq j.$$

$$\frac{\partial E(P_{yj})}{\partial X_{ij}} = 0^2$$

$$\frac{\partial Cov(P_{yj}P_{yg}, Y_jY_g)}{\partial X_{ij}} = 0, \text{ for both } g=j \text{ and } g \neq j.$$

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[2]This assumption is especially relevant in the case of small farmers.

$$\frac{\partial E(Y_g)}{\partial X_{ij}} = 0, \; g \neq j$$

$$\frac{\partial E(P_{yg})}{\partial X_{ij}} = 0, \; g \neq j$$

and substituting in (4), the first order condition, the optimum input use is given by:

$$\frac{E(P_{yj})\partial E(Y_j)}{\partial X_{ij}} - P_i = RSU \{\frac{\partial E(Y_j)}{\partial X_{ij}} [2V(P_{yj})E(Y_j) - 2Cov(P_{yj}, Y_j)E(P_{yj})$$

$$+ 2 \sum_{g \neq j} Cov(P_{yj}, P_{yg})E(Y_g) - 2 \sum_{g \neq j} Cov(P_{yg}, Y_g)E(P_{yj})]$$

$$+ \frac{2V(Y_j)}{\partial X_{ij}} [V(P_{yj}) + (E(P_{yj}))^2] + \frac{\partial V(P_{yj})}{\partial X_{ij}} [V(Y_j) + (E(Y_j))^2]$$

$$+ \frac{\partial Cov(Y_j, Y_g)}{\partial X_{ij}} [2 \sum_{g \neq j} Cov(P_{yj}, P_{yg}) + 2(P_{yj})E(P_{yg})]$$

$$+ \frac{\partial Cov(P_j, P_g)}{\partial X_{ij}} [2 \sum_{g \neq j} Cov(Y_j, Y_g) + 2 \sum_{g \neq j} E(Y_j)E(Y_g)]\} \quad (10)$$

$i=1,\ldots,n; \; j=1,\ldots,k; \; j<g.$

We assume that the second order conditions are satisfied.

Equation (10) describes the most general case of a multiproduct farm unit where input price is non-stochastic and output and output prices are stochastic and correlated with output levels. Although no a priori quantitative evaluation of the elements in (10) can be given, economic theory along with past empirical findings indicate that RSU>0, that is, farmers are risk averters [Dillon (b)], the covariance of output and its price is negative and the covariance of two different output prices is positive.

The familiar micro relation MR=MC still holds but the components of the equation in (10) show that such equality occurs at a different (lower) output level than in the riskless case. Specifically, optimal input use is reduced if RSU>0 and $V(P_{yj}) \geq 0$ and large. However, the real impact has to be empirically assessed. For small farms with low levels of technology and low speed of adoption of new techniques the aforementioned factors may be particularly significant in reducing input use.

Production Function Under Risk

In order to empirically assess the model set forth in the previous section it is necessary to obtain estimates of the farmer's utility function, along with the parameters of the multivariate distribution of P_y and γ and, at least qualitatively, the derivatives

$$\frac{\partial E(Y_j)}{\partial X_{ij}}, \frac{\partial V(Y_j)}{\partial X_{ij}}, \text{ and } \frac{\partial Cov(Y_j, Y_g)}{\partial X_{ij}}.$$

The main task in this section is to establish a production function which gives a sound estimate of our quantity variables while at the same time incorporating all the restrictions implied by risky prospects. Such restrictions have been outlined by Just and Pope in the form of eight basic postulates. Briefly outlined below, such postulates are:

1) $E(Y_i) > 0$ (Positive expected production)

2) $\frac{\partial E(Y_i)}{\partial X_i} > 0$ (Positive expected marginal product)

3) $\frac{\partial^2 E(Y_i)}{\partial X_i^2} < 0$ (Diminishing marginal productivity)

4) $\frac{\partial E(Y_i)}{\partial V(Y_i)} = 0$ (Changes in output variance should not influence expected output)

5) $\frac{\partial V(Y_i)}{\partial X_i} \gtreqless 0$ (Depending on the input, increased input use may or may not affect output variance)

6) $\frac{\partial \overline{X}_i}{\partial V(\varepsilon)} = 0$, $\overline{X}_i$ = optimum input use. (Changes in risk should not affect factor use among risk-neutral decision makers.)

7) $\frac{\partial V(\partial Y_i/\partial X_i)}{\partial X_j} \gtreqless 0$ (Changes in one factor of production should not a priori constraint the sign of the change in the marginal product.)

8) $F(\theta\chi) = \theta F(\chi)$ (Allow for constant stochastic returns to scale.)

As Just and Pope show, any production function which incorporates stochastic disturbances in a multiplicative fashion violates at least one of the above postulates. However, Just and Pope also proceed to show that a production function with additive disturbances such as

$$Y = f(X) + h(X)\varepsilon \tag{11}$$

where $E(\varepsilon) = 0$

$V(\varepsilon) = \sigma^2$

is consistent with all the risk restrictions set forth previously if f and h are linearly homogenous. Hence, it should be obvious that a Cobb-Douglas production function specified in such fashion would yield attractive estimates. Although Cobb-Douglas production functions can yield parameter estimates free of simultaneous equation bias [see Zellner, Kmenta, and Dreze] which can be applied to risky situations [Blair and Lusky], these results are obtained only if we redefine the Cobb-Douglas production function by assuming that crop response processes are neither instantaneous nor deterministic and that decision makers generally face stochastic production variables and therefore random profits. Under such assumptions

Zellner, Kmenta and Dréze go to show that the estimators obtained are unbiased and consistent. The proof outlined here is set forth by Kelejian, who points out that normality of the disturbances need not be assumed. Let

$$y_t = \alpha_0 X_{1t}^{\alpha_1} \cdots X_{kt}^{\alpha_k} e^{u_t} + v_t \text{ , } t=1,\ldots,n \tag{12}$$

be a Cobb-Douglas production function in which at least a subset of the input mix is stochastic, where X_{it} is the $t^{\underline{th}}$ observation of the i variable (which can be random). Assume

$$E(u_t) = E(v_t) = 0$$

$$V(u_t) = \sigma_u^2$$

$$V(v_t) = \sigma_v^2$$

Although no assumption is made about the distribution of u_t and v_t the proof only requires the first two moments of the distribution. Also, assume

$$E(X_{it}u_t) = E(X_{it}v_t) = E(u_t v_t) = 0$$

and $E(u_s u_t) = E(v_s v_t) = 0$, $s \neq t$.

Let $E(e^{u_t}) = A + \phi_t$ (13)

where $E(\phi_t) = 0$ and $V(\phi_t) = \sigma_\phi^2$.

Substituting (13) into (12) we get

$$y_t = B\, X_{1t}^{\alpha_1} \cdots X_{kt}^{\alpha_k} + w_t \tag{14}$$

where

$$B = \alpha_0 A \text{ and } w_t = \alpha_0 X_{1t}^{\alpha_1} \cdots X_{kt}^{\alpha k} \phi_t + v_t$$

Hence, $E(w_t|X_t) = 0$ since ϕ_t and v_t are independent of X_t. By using nonlinear least squares on (14) we obtain inefficient but consistent estimates of B and $\alpha_1,\ldots,\alpha_k$ since w_t is heterescedastic. However such estimates are useful in obtaining consistent estimates of σ_w^2. The conditional variance of w_t is

$$E(w_t^2|X_t) = (\alpha_0\phi_t)^2 X_{1t}^{2\alpha_1} \cdots X_{kt}^{2\alpha_k} + \sigma_v^2$$

$$= (\sigma_0^2\sigma_\phi^2) X_{1t}^{2\alpha_1} \cdots X_{kt}^{2\alpha_k} + \sigma_v^2$$

$$= C\ Z_t + \sigma_v^2$$

where $C = \alpha_0^2\ \sigma_\phi^2$ and $Z_t = X_{1t}^{2\alpha_1} \cdots X_{kt}^{2\alpha_k}$. Since

$$\hat{w}_t = y_t - \hat{B}\ X_{1t}^{\hat{\alpha}_1} \cdots X_{kt}^{\hat{\alpha}_k}$$

then, the consistency of $\hat{w}_t$ implies that

$$\hat{w}_t^2 = w_t^2 + \Delta_t$$

where plim $\Delta_t = 0$.

Consequently, a consistent estimate of the conditional variance of w_t can be obtained through the following regression:

$$\hat{w}_t^2 = \hat{\sigma}_v^2 + \hat{C}\hat{Z}_t + \varepsilon_t$$

where $E(\varepsilon_t) = 0$ yielding

$$\hat{\sigma}_w^2 = \hat{\sigma}_v^2 + \hat{C}\hat{Z}_t,$$

a consistent estimate of $V(w_t)$. By dividing (14) by $\hat{\sigma}_w^2$ and applying nonlinear least squares we obtain estimators which are unbiased, consistent and asymptotically efficient.

Kelejian also shows that σ_w^2 can be the smallest variance for that class of estimators, thus proving the estimator's asymptotic efficiency. Furthermore, the proof outlined previously can also be used as a guide for the mechanical procedures needed to obtain the production function's estimates. Once obtained, the results can be used in our equilibrium conditions specified in the preceding section.

The final point in this section relates to input use under risk. Rearranging our equilibrium conditions (10) we obtain

$$P_i = MVP_i - RSU\ \{\cdot\} \tag{15}$$

where MVP_i is the marginal value product of input i. Obviously, for a decision maker who is a risk averter (RSU > 0) a sufficient condition for input use lower than in the riskless case is that the bracketed term $\{\cdot\}$ be positive. As it happens, such case holds generally true in empirical observations [Dillon (a), (b); Anderson, Dillon, and Hardaker]. Nevertheless, the possibility of $\{\cdot\} < 0$ still exists. Terms such as Cov (Y_j, Y_g) present no problem in estimation in cases of uniproduct farms; however, in the case of multiproduct enterprises such terms can be obtained through simultaneous estimation of the production functions. By using procedures such as the one set forth by Zellner, Kmenta and Dreze we are able to obtain results which are free of simultaneous equation bias.

Conclusions

The main conclusion of this paper is that empirical estimation incorporating the procedures established in the preceding sections is a first step in analyzing risky production processes. Only positive economic methodology can be used to determine whether risk response from part of the farmer is of a sufficient degree to merit the tremendous amount of normative risk related work which is currently being undertaken. As it was hopefully indicated, there is still a need for more complete means of obtaining more accurate information if we want to avoid underestimating policy problems or overestimating the farmer's fear [Just and Pope]. Specifically, more research is needed with respect to other functional forms for utility, along with more work on other production functions with better characteristics and less limitations than the Cobb-Douglas function. Empirical estimation of equilibrium conditions in which the third moment is included should be of help in determining the validity of some of our assumptions, along with inclusion of time lags in the response function. Such needed work will certainly improve our perception of risk as a deterrent for development.

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