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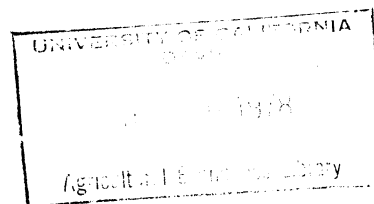
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A METHODOLOGY FOR DURABLE ASSET  
REPLACEMENT DECISIONMAKING

by

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## ABSTRACT

### A Methodology for Durable Asset Replacement Decisionmaking

Durable asset replacement theory typically assumes (1) a constant conversion rate between the stock of the asset and its flow of services and (2) perfectly substitutable services from either asset. A methodology is presented which relaxes both assumptions. The variable usage rate for the asset becomes an important determinant of replacement.

# A METHODOLOGY FOR DURABLE ASSET REPLACEMENT DECISIONMAKING

Alan E. Baquet

## Introduction

Managerial decisions concerning the replacement of durable assets have received considerable attention by the Agricultural Economics profession. (Faris, Winder and Trant, Perrin). The general economics profession has also devoted resources to the study of durable asset replacement. (Jorgenson, Smith, Feldstein and Rothschild). Two simplifying assumptions are typically made in the analysis of asset replacement. The first is that the original asset and the proposed replacement both yield services which are perfect substitutes for each other. The second assumption deals with the conversion of the stock of the durable asset to the flow of services generated by the asset. Most analyses assume a one-to-one correspondence between the stock and the flow. They further assume that the same quantity of services is generated during each year of the durable's lifetime.

In this paper a model which does not rely on either assumption is presented. Relaxing the first assumption requires us to treat the replacement decision as being composed of both a disinvestment decision and an investment decision. Thus, we treat each durable as an independent asset capable of generating services which are close but not perfect substitutes in the production process.

Relaxing the second assumption permits us to specify an asset's value in use as a function of the quantity of services generated in each time period, as well as the number of time periods in which the asset is used. This introduces an important new variable in the investment/disinvestment decision - the quantity of services generated.

In the following section a methodology for investment/disinvestment decisions is developed. It incorporates the notion of acquisition/salvage price divergence.<sup>1/</sup>

The paper concludes with a statement of the potential benefits to be derived from the application of our methodology.

#### A Model for Investment/Disinvestment Decisionmaking

As indicated above, we break the replacement decision into an investment and disinvestment decision. It is generally accepted that the appropriate investment decision rule is to acquire an asset when its expected value in use exceeds its acquisition price. Less generally recognized is the corresponding disinvestment decision rule: dispose of an asset when its future value in use is less than its salvage price.<sup>2/</sup>

It is in the calculations of an asset's value in use that the method presented in this paper diverges from previous methods. An asset's value in use is derived from the services used in the firm's production process.

When we allow the flow of services from a durable asset to vary over time, we must alter our traditional concept of the production process.

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<sup>1/</sup>The notion of acquisition/salvage price divergence is well documented in the literature and will not be presented here. (Johnson)

<sup>2/</sup>Glenn L. Johnson and Clark Edwards have both written on the appropriateness of this disinvestment decision rule.

In our model, we conceive the production process to be a vertically integrated process with the flow of services being generated from the durable at one level and subsequently fed into the production process which determines final output. A diagrammatic representation for a production process which uses the services of one durable is presented in Figure 1.

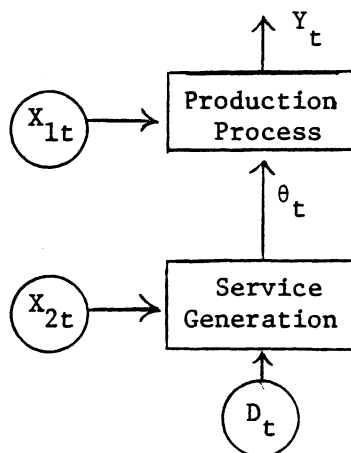


Figure 1. Vertically Integrated Production Process.

The mathematical characterization of this production process is contained in equations (1) through (3).

$$(1) \quad Y_t = f(X_{1t}, \theta_t)$$

$$(2) \quad \theta_t = g(X_{2t}/D_t)$$

where  $Y_t$  = quantity produced and sold in time period 2.

$X_{1t}$  = quantity of nondurable  $X_1$  used in production of  $Y_t$  in time period  $t$ .

$\theta_t$  = quantity of services generated from durable,  $D_t$ , in time period  $t$ .

$X_{2t}$  = quantity of nondurable  $X_2$  used in production of services in time period  $t$ .

$D_t$  = stock of the durable asset in time period  $t$ .

Equation (1) is self-explanatory. Equation (2) is a production relationship which indicates that service flows from durable assets are

generated or produced according to the function  $g(\cdot)$  by using one non-durable input with a given stock of the durable asset.<sup>3/</sup> Thus, at this level of integration we need both stocks and flows in the production of services.

The exact function of maintenance in production processes is not clear. There are situations in which services from durable assets can be generated without maintenance being performed. For example, an engine may operate for a certain number of hours without maintenance being performed. However, if the engine is to operate over a longer time interval, some maintenance may be required. It is clear that performing maintenance can extend the physical life of a durable. For example, changing the oil in an engine may extend the life of the engine.

For this paper, we assume the time period is such that current services,  $\theta_t$ , can be generated without the use of maintenance inputs. This assumption is reflected in equation (2) preceding. The role of maintenance will be to extend the physical life of the asset.

The physical life of a durable asset is related to both the services extracted and the maintenance performed during each year of its life. In our model we specify this physical relationship as equation (3).

$$(3) \quad T_D = h(\theta_1, \dots, \theta_t, \dots, \theta_{T_H}, X_{31}, \dots, X_{3t}, \dots, X_{3T_H})$$

where  $T_D$  = physical life of durable D.

$\theta_t$  = services extracted from durable in time period t.

$X_{3t}$  = aggregated maintenance input  $X_3$ .

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<sup>3/</sup> The variable  $D_t$  is an integer quantity, e.g. one tractor, one combine. However, to make the model definitive, we require a description of the physical characteristics of the durable asset.

$T_H$  = planning horizon for the firm.  $T_H$  is chosen such that costs and returns beyond  $T_H$  would be discounted essentially to zero for any positive discount rate.

$$T_H \geq T_D$$

$h(\cdot)$  = functional relationship among services, maintenance and the physical life of the durable. It has the following properties:

$$\frac{\partial h}{\partial \theta_t} < 0, \quad \frac{\partial h}{\partial X_{3t}} > 0.$$

Specification of the production process in this manner allows us to vary the rate of use for durable assets. It also permits us to determine investment and disinvestment in durables simultaneously with the production activities associated with the durable. The optimal length of life for the durable is also determined internally.

Since the focus of this paper is on the investment/disinvestment activities of the firm, we will not derive the optimizing conditions for the firm's production activities.<sup>4/</sup>

For a given durable,  $D_t$ , the decision variables for the firm's production activities are  $X_{1t}$ ,  $X_{2t}$ ,  $X_{3t}$ , and  $\theta_t$ , for  $t=1, \dots, T_H$ . Optimal levels for these variables will maximize the firm's profit from its production activities. These optimal levels are also used to determine the durable asset's value in use. It is this later purpose which we will focus on in this paper.

With our conception of variable usage rates for durable assets, the durable's value in use depends on both the services generated in each time period and the number of time periods the durable is used.

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<sup>4/</sup> The interested reader is referred to Baquet, Alan E. A Theory of Investment and Disinvestment, Including Optimal Lives, Maintenance and Usage Rates for Durable Assets, unpublished Ph.D. dissertation, Department of Agricultural Economics, Michigan State University, 1978.



We will denote the durable's value in use<sup>5/</sup> as  $NRD(\theta^*, T_D)$ . The optimal quantity of services to generate is derived from the firm's production activities. The optimal length of time to use the durable,  $T_D^*$ , is derived internally as part of the firm's disinvestment activities.

Specifying the durable's value in use as a function of both the services generated in each time period and the number of time periods,  $T_D$ , is a significant advancement over previous specifications. The typical procedure has been to specify the durable's value in use as depending only upon the length of time the durable is used. Our more accurate specification is possible because of our assumption concerning the variable usage rates for the durable.

The factors which comprise  $NRD(\theta^*, T_D)$  are specified in equation (4).

$$(4) \quad NRD(\theta^*, T_D) = \sum_{t=1}^{T_D} \left[ \int_{\theta_t=0}^{\theta_t^*} MVP_{\theta_t} d\theta_t - \int_{\theta_t=0}^{\theta_t^*} MFC_{\theta_t} d\theta_t \right] \frac{1}{(1+r)^t} + S(T_D) \frac{1}{(1+r)^{T_D}}$$

where  $NRD(\theta^*, T_D)$  = net return to durable.

$MVP_{\theta_t}$  = marginal value product of services generated from the durable used to produce product Y.

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<sup>5/</sup>The "\*" notation is used to denote optimal levels. Thus,  $\theta_t^*$  is the optimal quantity of services to generate from the durable in time period t.

$MFC_{\theta t}$  = marginal factor cost of services generated from the durable. As derived in Baquet,  $MFC_{\theta t}$  is composed of elements which reflect the cost of maintenance, acquiring the nondurable input  $X_{2t}$ , and the marginal user cost.<sup>6/</sup>

$S(T_D)$  = salvage value of durable asset.

$r$  = discount rate.

Equation (4) indicates that the durable's value in use is calculated as the summation over time of the discounted net value of the optimal quantity of services generated in each time period plus the discounted salvage value.

Determining the optimal lifetime for a durable, in essence, determines the point in time when the firm should disinvest in the durable. To apply our disinvestment criteria to determine the optimal length of life, we rewrite our expression for the durable's value in use as:

$$(5) \text{NRD}(\theta^*, T_D) = \text{PVS}(T_D) + S(T_D)$$

The variable  $\text{PVS}(T_D)$  in (5) is the summation portion of  $\text{NRD}(\theta^*, T_D)$  from (4) expressed as a function of the final time only, since we have determined  $\theta^*$  already.  $T_D^*$  is determined so as to maximize  $\text{NRD}(\theta^*, T_D)$ . If we treated time as a continuous variable, we would differentiate (5) with respect to  $t$  and equate with zero (this procedure was used by Perrin). However, our model treats time as a discrete variable; thus, we cannot take derivatives. We can only state approximate marginal rules for determining  $T_D^*$ . Our marginal rule is to equate the additions to  $\text{PVS}(T_D)$  with the reductions in  $S(T_D)$ ,  $T_D^*$  is the point in time when the additions

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<sup>6/</sup>User cost was conceived by Keynes, and extended by Neal and Lewis. As used in Baquet, marginal user cost is the change in the value of the asset as a result of generating services during the time period as opposed to not generating services during that time period.

to  $PVS(T_D^*+1)$  are less than the reductions in  $S(T_D^*+1)$ . In other words,  $PVS(T_D^*) > S(T_D^*)$ , but  $PVS(T_D^*+1) < S(T_D^*+1)$ . This procedure determines when to disinvest in a durable. It is based on comparing the durable's value in use with its salvage value.

The above procedure will determine a durable's value in use. This value in use depends on both the services generated in each time period and the number of time periods in which the durable is used. Our procedure for calculating the optimal length of time to use the durable was based on our disinvestment criteria and hence yielded the optimal point in time to disinvest in the durable.

For the investment decision, the firm compares  $NRD(\theta^*, T_D^*)$  with the durable's acquisition price,  $P_a$ . If  $P_a < NRD(\theta^*, T_D^*)$  the firm will acquire the asset; if  $P_a > NRD(\theta^*, T_D^*)$  the firm will not invest in the asset.

In this section we have developed a model which considers variable usage rates for durable assets. We have incorporated this variable usage rate into the calculation of a durable's value in use. We then stated the optimizing conditions for a firm's investment and disinvestment activities.

### Summary

The analysis of asset replacement is typically based on two assumptions. The first assumption is that assets generate services which are perfect substitutes for each other. The second is that the same quantity of services is generated from the asset in each production period. A methodology for analyzing the replacement decision when both of these assumptions are dropped was presented above. Relaxing the first assumption

required us to treat the replacement decision as being composed of both an investment decision and a disinvestment decision. Relaxing the second assumption permitted us to treat an asset's value in use as depending explicitly upon the amount of services generated in each time period as well as the number of time periods.

The more detailed specification of the production process, upon which our calculations of an asset's value in use are derived, permits us to develop a more realistic specification of an asset's value in use.

By using the methodology presented in this paper we can analyze the potential affects on investment and disinvestment decisions of changes in a firm's economic environment. The variable usage rate for durable assets permits us to link the firm's investment and disinvestment decisions directly and explicitly to the firm's production activities.

As an example of the potential use for this methodology, consider the affects of adopting minimum tillage practices on the investment and disinvestment decisions relating to farm tractors and complementary machinery implements.

The affect on investment/disinvestment decisions of changing the usage rate of tractors as implied by minimum tillage practices cannot be analyzed with traditional replacement methodologies, whereas the variable usage rate is an explicit part of the methodology presented in this paper. Thus the analysis of varying cultural practices can be handled directly and explicitly with our methodology.

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Biographical Sketch

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