# A RENEWAL THEORETIC APPROACH TO ENVIRONMENTAL

# STANDARD SETTING

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## ABSTRACT

The process of environmental regulation is usually a two-step one. In the first step, a standard for environmental quality is set. Then, in the second step, a regulatory mechanism is put in place to achieve the standard. In this paper I show how renewal theory can be used to set the quality standard optimally.

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## 1. Introduction

It is well known that first best solutions to problems of environmental regulation typically

do not exist. As a result, research and policy discussions have typically focussed on the design

of second best regulatory policies with certain desirable properties. In this setting, the design of

environmental policy is usually a two-step process. As Cropper and Oates (1992, p. 685) noted,

". . . first, standards or targets for environmental quality are set, and, second, a regulatory system

is designed and put in place to achieve these standards."

While a significant amount of research effort has gone into analyzing the second step of

the above-described two-step process,[1] economists have paid considerably less attention to the

first step. Indeed, researchers typically *assume* that an environmental standard has been provided

exogenously;[2] they then proceed to analyze the task of achieving this standard optimally. Oates,

Portney, and McGartland (1989, p. 1234) are representative. In a study of the effects of

command and control versus incentive-based regulatory policies, these authors say that "[l]et us

suppose that some standard for environmental (or workplace) quality has been set—we take it

as predetermined." This is not to say that the standard setting task is either trivial or insignificant.

A poorly set standard can lead to inefficiencies and significant losses from regulation.[3] Given

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[1]See Batabyal (1995) or Cropper and Oates (1992) for recent surveys.

[2]Typically on the basis of scientific dose-response relationship studies.

[3]For more on this in a practical setting, see Fri (1995).

this scenario, the purpose of this paper is to show how renewal theory can be used to approach the standard setting task effectively in an explicit cost/benefit framework.[4]

There are two main reasons as to why the use of renewal theory is useful. First, the regulatory environment involves decision making in a dynamic setting marked with—in the language of Batabyal and Yoo (1994)—systemic uncertainty. Systemic uncertainty refers to the fact that many of the underlying processes that a regulatory authority (RA) would like to affect have a tendency to deteriorate probabilistically over time, if left unregulated. Second, the renewal/reward theorem—on which more is in the next section—provides us with a useful way of: (a) modeling the cyclical nature of the RA's actions, and (b) characterizing the RA's objectives. I now turn to a discussion of how to optimally set the environmental standard.

## 2. The Renewal Theoretic Framework

I shall first describe the renewal/reward theorem, which will form the centerpiece for all my subsequent analyses.[5] A stochastic process $\{Q(t): t \geq 0\}$ is said to be a counting process if $Q(t)$ denotes the total number of events that have occurred by time $t$. Now, since $Q(t-2)$, $Q(t-1)$, $Q(t)$, etc., are random, the time between any two counts $Q(t-1)$ and $Q(t-2)$ is also random. This time between any two counts is called the interarrival time. A counting process in which the interarrival times have an arbitrary distribution is called a renewal process.

Consider a renewal process $\{Q(t): t \geq 0\}$ with interarrival times $X_q$, $q \geq 1$, which have a distribution function $F(\bullet)$. Furthermore, assume that a monetary reward $R_q$ is earned when the

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[4]For more on renewal theory, see Karlin and Taylor (1975, pp. 167-227) or Wolff (1989, pp. 52-79).

[5]This discussion of the renewal/reward theorem is taken from Batabyal and Yoo (1994).

*qth* renewal is completed. Let *R(t)*, the total reward earned by time *t*, be given by $\sum_{q=1}^{Q(t)} R_q$ . Let

$E[R_q]= E[R]$, and let $E[X_q] = E[X]$. The renewal/reward theorem tells us that if $E[R]$ and $E[X]$ are

finite, then with probability one,

$$\lim_{t \to \infty} \frac{E[R(t)]}{t} \quad \frac{E[R]}{E[X]} \quad .$$ **(1)**

In other words, if we think of a cycle being completed every time a renewal occurs, then the

long-run expected reward is simply the expected reward in a cycle divided by the expected time

it takes to complete that cycle.

I am now in a position to discuss the renewal theoretic approach. Let the *state* of the

resource system—for example, air or water—that is sought to be regulated be represented by a

Brownian motion process $\{S(t): t \geq 0\}$, with mean $\mu t$ and variance $\sigma^2 t$, where $\mu > 0$. Further, I

assume that at time $t_0$, the state of the resource system is $s_0$. The reader should think of the state

at any time *t*, as the level of pollutants in the resource system at that time. In other words, as the

level of pollutants increases, the quality of the resource system declines. The goal of the RA is

to set a standard so as to cap the level of pollutants at some maximum level. Let this maximum

level, i.e., the standard, be denoted by $\bar{s}$ .[6] Whenever the state of the resource system, *s*, reaches

or exceeds $\bar{s}$ , where $\bar{s} > s_0$, the RA will take action so as to bring the resource system back to

some acceptable state, say state $s_a$. Note that because $\mu > 0$, and $\bar{s} > s_0$, if left unregulated, the

state of the resource system will hit $\bar{s}$ with probability one (Cox and Miller, 1965, p. 212).

Corrective action by the RA involves social costs and benefits. Let the monetary social costs and

benefits *from regulation* in state $s = \bar{s}$ be given by $\bar{C}(s)$ and $\bar{B}(s)$, respectively, where

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[6]More formally, we can think of the standard as an absorbing barrier.

$C'(\bar{s}) \geq 0$ , and $B'(\bar{s}) \geq 0$ . Let $N(\bar{s}) \equiv B(\bar{s}) - C(\bar{s})$ be the social net benefit from regulation

in state $\bar{s}$ . Because $C(\bullet)$ and $B(\bullet)$ are both increasing functions in deciding where to set the

standard, the RA will have to take into account the fact that these two variables pull in opposite

directions.

I assume that the goal of regulatory policy is to set the standard $\bar{s}$ so as to maximize the

long-run expected net social benefit from regulation. Note that whenever $S(t) \geq \bar{s}$ , the RA will

take action; this action successfully brings the resource system to state $s_a$ . Furthermore, this

action also involves the completion of a renewal. As a result, I can appeal to the renewal/reward

theorem to characterize the long-run expected net social benefit from such regulatory action.

From (1), the numerator is $N(\bar{s}) \bullet Pr\{S(t) \geq \bar{s} / S(t_0) = s_0\}$. Further, let the denominator be given

by $g(\bar{s})$, where $g(\bar{s})$ is the expected time it takes to get to state $s = \bar{s}$ . The RA's objective is to

maximize over $\bar{s}$ ,

$$\frac{N(\bar{s}) \bullet Pr\{S(t) \geq \bar{s} / S(t_0) - s_0\}}{g(\bar{s})} \ . \tag{2}$$

By the properties of Brownian motion processes (see Karlin and Taylor, 1975, p. 356),

the    numerator    of    (2)    can    be    simplified    to

$N(\bar{s}) \bullet \int\limits_{\bar{s}-s_0}^{\infty} \left\{\sigma\sqrt{2\pi(t-t_0)}\right\}^{-1} \exp\left[-\{x-\mu(t-t_0)\}^2/2\sigma^2(t-t_0)\right]dx$ . To simplify the denominator of

(2), it will be necessary to derive a differential equation satisfied by $g(\bar{s})$. I shall proceed as in

Batabyal and Yoo (1994, pp. 239-40). Conditioning on the random variable $H = S(\Delta) - S(0)$,

where $H$ denotes a change in the resource system in a small time increment $\Delta$, I get

$$g(\bar{s}) - \Delta - E[g(\bar{s} - H)] - o(\Delta) , \tag{3}$$

where $o(\Delta)$ denotes the probability that the resource system will already have reached state $\bar{s}$ in

time $\Delta$.  Note that $H$ is distributed normally with mean $\mu\Delta$ and variance $\sigma^2\Delta$.  Now expanding (3) in a Taylor series, I get

$$g(\bar{s}) \quad \Delta \quad E\left[g(\bar{s}) \quad \frac{Hg'(\bar{s})}{1!} \quad \frac{H^2g''(\bar{s})}{2!} \quad h.o.t\right] \quad o(\Delta) . \tag{4}$$

Taking the expectation of the relevant terms in (4), canceling common terms from both sides of (4), dividing both sides of (4) by $\Delta$, and then letting $\Delta \rightarrow 0$ yields the required differential equation.  I get

$$\mu g'(\bar{s}) \quad \frac{g''(\bar{s})}{2} \quad 1 . \tag{5}$$

From (5) and the fact that a Brownian motion process has independent and stationary increments, I can infer that $g(\bar{s}) = k \bullet \bar{s}$ , for some constant $k$.  Using $g(s) = k \bullet s$  in (5), I conclude that $g(\bar{s}) = k \bullet \bar{s}$ .  The RA's objective, as expressed in (2), can now be written as

$$\max_{\bar{s}}\left[\mu \bullet \bar{s}^{-1} \bullet N(\bar{s}) \bullet \int_{\bar{s}-s_0}^{\infty} \left\{\sigma\sqrt{2\pi(t \ t_0)}\right\}^{-1} \exp\left[ \{x \ \mu(t \ t_0)\}^2/2\sigma^2(t \ t_0)\right]dx\right] . \tag{6}$$

The first-order necessary condition is

$$N(\bar{s}^*) \bullet \int_{\bar{s}^*-s_0}^{\infty} \left\{\sigma\sqrt{2\pi(t \ t_0)}\right\}^{-1} \exp\left[ \{x \ \mu(t \ t_0)\}^2/2\sigma^2(t \ t_0)\right]dx$$

$$\bar{s}^*\left[N(\bar{s}^*) \bullet \{d/d\bar{s}^*\int_{\bar{s}^*-s_0}^{\infty} \left\{\sigma\sqrt{2\pi(t \ t_0)}\right\}^{-1} \exp[ \{x \ \mu(t \ t_0)\}^2/2\sigma^2(t \ t_0)]dx\}\right.$$

$$\left. N'(\bar{s}^*) \bullet \int_{\bar{s}^*-s_0}^{\infty} \left\{\sigma\sqrt{2\pi(t \ t_0)}\right\}^{-1} \exp\left[ \{x \ \mu(t \ t_0)\}^2/2\sigma^2(t \ t_0)\right]dx\right], \tag{7}$$

where $\bar{s}^*$ solves (7).  Equation (7) tells us that optimality requires the RA to set the quality standard so that the long-run expected net social benefit from regulation (the LHS) equals a probabilistically weighted sum of the marginal and total net benefit from regulation.  When the

quality standard is chosen in this manner, the long-run expected net social benefit from regulation will have been maximized.

### 3. Conclusions and Extensions

In this paper I have provided a simple framework within which the task of optimal environmental standard setting can be analyzed effectively. This framework does not assume the existence of an exogenous standard; on the contrary, this framework endogenizes the task of optimal standard setting. Further, the cost/benefit aspect of the problem, and the fact that the RA's task involves decision making in a dynamic and stochastic environment are explicitly modeled.

The simple framework presented in this paper can be extended in a number of directions. In what follows, I suggest two possible extensions. First, one could analyze a model in which the step one and step two stages of the regulatory process have been combined. Such an integrated analysis will enable us to have a better understanding of the costs and benefits of alternate forms of environmental regulation, in a model with endogenous standard setting. Second, the RA's objective need not involve expected net benefit maximization. Depending on the context, one could formulate an objective which involves expected cost minimization. Finally, one could introduce learning into the model. This will enable us to have a better understanding of the connections between optimal standard setting and the temporal resolution of uncertainty.

## References

Batabyal, A.A. (1995) "Leading Issues in Domestic Environmental Regulation: A Review Essay." *Ecological Economics* 12: 23-39.

Batabyal, A.A., and S.J. Yoo. (1994) "Renewal Theory and Natural Resource Regulatory Policy Under Uncertainty." *Economics Letters* 46: 237-41.

Cox, D.R., and H.D. Miller. (1965). *The Theory of Stochastic Processes*. Methuen: London, England.

Cropper, M.L., and W.E. Oates. (1992) "Environmental Economics: A Survey." *Journal of Economic Literature* 30: 675-740.

Fri, R.W. (1995) "Using Science Soundly: The Yucca Mountain Standard." *Resources* (120): 15-18.

Karlin, S., and H.M. Taylor. (1975) *A First Course in Stochastic Processes*, 2nd ed. New York: Academic Press.

Oates, W.E., P.R. Portney, and A.M. McGartland. (1989) "The Net Benefits of Incentive Based Regulation: A Case Study of Environmental Standard Setting." *American Economic Review* 79: 1233-42.

Wolff, R.W. (1989) *Stochastic Modeling and the Theory of Queues*. Englewood Cliffs, NJ: Prentice Hall.