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THE QUEUING THEORETIC APPROACH TO GROUNDWATER MANAGEMENT

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ABSTRACT

In this paper I propose and develop a new framework for modeling groundwater management issues. Specifically, I apply the methods of queuing theory—for the first time, to the best of my knowledge—to model a groundwater management problem from a long-run perspective. I characterize two simple management regimes as two different kinds of queues and then show how to pose a manager's decision problem as an optimization problem using queuing theoretic techniques. I solve for certain fundamental quantities, such as the expected system size, and then discuss the economic meaning and relevance of the queuing concepts being used. I close by discussing possible extensions to my basic models.

Key words: groundwater, management, stochastic, queuing, theory

THE QUEUING THEORETIC APPROACH TO GROUNDWATER MANAGEMENT

1. Introduction

When economists have studied the question of groundwater management, they have typically cast the problem in a deterministic, control theoretic framework. Brown and Mc Guire (1967), Burt (1967), Burt and Cummings (1977), and Provencher and Burt (1993) have all analyzed different aspects of the groundwater management problem within this kind of control theoretic framework. While this framework has yielded many valuable insights, in this schema, analysts have not been able to satisfactorily model the twin phenomena of *uncertain dynamic demand and uncertain dynamic supply*. Given this situation, in this paper I propose and analyze a new framework for modeling the question of groundwater management. This framework uses the techniques of queuing theory. There are two main advantages to this method as compared to the deterministic, control theoretic framework. First, I am able to model the twin phenomena of uncertain dynamic demand and uncertain dynamic supply effectively. Second, I am able to treat the inherently stochastic nature of the problem explicitly and not as something that is relevant but incidental to the problem owing to modeling difficulties.

The rest of this paper is organized as follows. In section 2, I discuss the main attributes of groundwater briefly and then proceed to pose a groundwater management problem in a queuing context. I then explain the two kinds of queues that I propose to study. In section 3, I develop the two queuing models and then pose the management problem mentioned above as a simple optimization problem involving the choice of two control variables. Finally, in section 4, I present my salient findings, and I discuss some of the many directions in which my basic models may be extended.

To clarify any potential confusion, let me state at the outset that my problem is *not* the dam problem which has been analyzed by queuing theorists such as Moran (1959) and Prabhu (1965) at some length. First, I am not interested in determining how much water should be stored by a groundwater manager. The issue of storage is not germane to my problem. My problem is to determine how much water to supply and at what rate in some specified time period. Second, I am not interested in determining the optimal size of a dam; my secondary problem is to determine the optimal quota on water use within the aforementioned time period. Third, I am interested in the management problem, *inter alia*, due to the common property nature of groundwater use and the corresponding inefficiencies arising from unregulated use (see Dasgupta 1982, Chapter 6). As such, my principal motivation for studying this problem is not to make water supply more predictable in some statistically known manner.

2. Preliminaries

Water occurs as a stock and as a flow. Surface water, i.e., the flows in lakes, rivers, etc., is typically the result of runoffs from precipitation and/or snowpack melt; both of these physical processes are stochastic. On the other hand, groundwater exists in aquifers as a stock subject to stochastic recharge. This suggests why the supply of groundwater is uncertain. The demand uncertainty associated with groundwater is principally a function of intertemporal market and climatic conditions. Depending on these conditions, the demand for groundwater will exhibit temporal fluctuations. In a year with good rainfall and, hence, plentiful supplies of surface water, the demand for groundwater will typically be less than in time periods in which there is a drought. In these latter times, the demand for groundwater will increase. From a management perspective, what is important is that the demand for groundwater is stochastic.

An additional feature characterizing and providing a rationale for the regulation of groundwater is the fact that groundwater is a *res communes*, or a common property resource. That is, there are typically no well- developed property rights to the aquifers containing groundwater. Even in a developed country like the USA, different states have a loose patchwork of rules governing the use of groundwater. For instance, in Texas, the common law system regulates groundwater use; in New Mexico, groundwater rights are defined by the prior appropriation doctrine. In California, a mixed bag of riparian rights and rights according to the prior appropriation doctrine govern groundwater use. As a result, in the absence of regulation, each individual water user, in isolation, finds it profitable to exploit the aquifer until the price received from selling (using) the water obtained from the aquifer equals the average cost of obtaining the water.[1] At this point of resource use, all economic rents from the aquifer are dissipated and the total revenues from the use of groundwater equal the total cost of pumping the groundwater from the aquifer. The inefficiencies associated with this rent dissipation are well understood (see Dasgupta 1982, Chapter 6); hence, I shall not pursue this issue any further. A related problem concerns the intertemporal misallocation of pumping due to the *res communes* nature of aquifers (Brown 1974).

The dynamic and stochastic features of groundwater use make the management problem amenable to analysis via queuing theoretic methods. By management, I am referring to a situation in which a social planner (hereafter manager) who is assigned property rights to an aquifer solves an optimization problem. In solving this problem, the manager explicitly takes into account the social benefit and the social cost stemming from the provision and use of groundwater. In my models, the manager's optimization problem involves the maximization of the difference between monetary inflows (benefit) and monetary outflows (cost) from the provision and use of groundwater. In what follows, I shall refer to this difference as the residual benefit.

The uncertain demand for water over time is modeled by an independent and identically distributed (i.i.d.) stochastic arrival process of water users (hereafter users). The uncertain supply of water is modeled by an i.i.d. stochastic supply process of the water manager. The stochastic processes representing demand and supply are assumed to be independent of each other. More specifically, there is a single manager in charge of dispensing groundwater from an aquifer to the different users who pay a fee for the water that they receive. The users arrive at some central water-dispensing facility in accordance with some stochastic

[1]This is yet another manifestation of the so-called Isolation Paradox. See Sen (1967) for a discussion.

process with finite mean and form a queue. If the manager is idle, then the first user to arrive proceeds to obtain his/her water,[2] otherwise the user waits in queue. Users arrive one at a time and are served one at a time in order of arrival. In the two cases that I analyze, I shall assume that the arrival process of the users is Poisson and that this fact is known to the manager. As a result, the interarrival times are exponentially distributed and the distribution of interarrival times has the Markovian property of being memoryless. I will denote its distribution function by $M(\bullet)$, its mean by $1/\gamma$, and its rate by γ. I shall model the supply uncertainty in two ways. In the first case I shall assume that the time taken by the manager in supplying water can be represented by an exponential distribution. In the second case, I shall assume that the manager's time to supply water is represented by some arbitrary distribution whose cumulative distribution function is $G(\bullet)$. That is, the supply times, denoted by S, have a cumulative distribution function denoted by $G(\bullet)$. In both cases—Markovian in the first and general in the second—I assume that the mean of the cumulative distribution function is finite; I denote this mean by $1/\epsilon$.

The manager's task is to choose the queue capacity, i.e., the number of people he/she will supply water to in a specified time period and the rate at which he/she fills requests for water so as to maximize the residual benefit arising from the provision and use of groundwater. In economic parlance, the manager chooses a quota on water allocation in a certain time period and the rate at which he/she will distribute the available supply of water. In the language of queuing theory, I am studying, in turn, the M/M/1 and the M/G/1 queues, both with finite capacity. In this three-letter designation, the first letter refers to the fact that the interarrival times of the users has the Markovian property. The second letter, M and G, respectively, refers to the fact that the manager's supply times distribution has the Markovian property in the first case, whereas this distribution is general in the second case. Finally, the number 1 refers to the fact that there is a single groundwater manager.

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[2]This does not have to involve actual physical collection. Conceptually, obtaining could also mean arranging to have a certain quantity of water released by way of pipelines or canals.

I now proceed to a formal discussion of the queuing theoretic approach to the aforementioned water management problem.

3. The Analytical Framework

3a. The Water Management Regime as a M/M/1 Queue with Finite Capacity

Since my analysis is being conducted from a long-run perspective, I first have to determine the stationary probabilities for this type of queue. I now introduce three sets of probabilities that I shall work with. If $X(t)$ denotes the number of water users in the queuing system at an arbitrary time t, then let

$$P_k \equiv \lim_{t \to \infty} Pr\{X(t) = k\} \tag{1}$$

be the long-run probability that there are exactly k users in the system. Let $\{a_k: k \geq 0\}$ be the proportion of users who find k in the system when they *arrive*. Finally, let $\{d_k: k \geq 0\}$ be the proportion of users who leave behind k users in the system when they *depart*. It is important to note that P_k can be interpreted as the proportion of *time* that the system contains exactly k users. For my purposes, the relevant stationary probabilities are the $\{P_k\}$. However, since it will not be possible to obtain the $\{P_k\}$ directly in section 3b, I shall exploit some well-known relationships between the $\{a_k\}$, $\{d_k\}$, and $\{P_k\}$ to obtain the $\{P_k\}$. Since I will be working with finite capacity queues, let me denote the capacity of the queue by K. Thus, the state space of the queue can be indexed by k, where $k = 0, \ldots, K$. That is, when there are K users in the system, the manager will not provide water to any more users in a certain time period. K is, in fact, one of two choice variables for the manager. Later I will solve for the K which maximizes the residual benefit arising from the use of water.

To determine the $\{P_k\}$, I shall follow Ross (1985) and solve a set of balance equations. These equations make use of the basic principle that the rate at which the queue enters state k equals the rate at which it leaves state k. Using this rate equality principle, the rate at which the stochastic process (of arriving users) leaves state *0* must equal the rate at which the stochastic arrival process enters state *0*. So, for state *0*, I have

$$\gamma P_0 = \epsilon P_1 \ , \tag{2}$$

where γ and ϵ are the parameters of the two exponenti al distributions, as discussed in section 2. Similarly, for any state *k*, $k = 1, \ldots, K - 1$, I get

$$\{\gamma + \epsilon\} P_k = \gamma P_{k-1} + \epsilon P_{k+1} \ . \tag{3}$$

Finally, for state *K*, the rate equality principle gives me the following balance equation

$$\epsilon P_K = \gamma P_{K-1} \ . \tag{4}$$

The equation for state *K* requires some explanation. Note that state *K* can be left by means of a departure because once the system is in state *K*, no other users can enter the system. Analogously, state *K* can be entered only from state *K-1*, since there is no state *K+I*.

I now solve (2)-(4) in terms of the stationary probability that the manager is idle, i.e., in terms of P_0. I get

$$\begin{aligned} P_1 &= (\gamma/\epsilon) P_0 \ , \\ P_2 &= (\gamma/\epsilon)^2 P_0 , \ldots , \\ P_K &= (\gamma/\epsilon)^K P_0 \ . \end{aligned} \tag{5}$$

I can now use the fact that $\sum_{k=0}^{k=K} P_k = 1$ to solve for P_0 explicitly. I get

$$P_0 = \left\{ \frac{1 - (\gamma/\epsilon)}{1 - (\gamma/\epsilon)^{K+1}} \right\} . \tag{6}$$

Using (5) and (6) I get

$$P_k = \left[\frac{(\gamma/\epsilon)^k \{1 - (\gamma/\epsilon)\}}{1 - (\gamma/\epsilon)^{K+1}} \right], \quad k = 0, \ldots, K \ . \tag{7}$$

This accomplishes my first task. The stationary probabilities for the water management regime which I have modeled as a *K* state *M/M/1* queue are given by (7).

I can now pose the manager's optimization problem. In my model, the manager's task is (a) to choose the *number* of users who will be supplied water during some time period and (b) to choose the *rate* at which he/she will supply water to the different users. The rate at which users arrive to demand water in a certain time period—which I take to be a month—is given by $Int[y(1 - P_K)]$ where $Int[\bullet]$ denotes the integer part of the term inside the square brackets. Further, if the i th user demands x_i units of water for which he/she pays $a_i = a_i(x_i)$ dollars, then the manager's monetary inflows per month equal $\gamma\left(1 - P_K\right) \sum_{i=1}^{i = Int\left[\gamma\left(1 - P_K\right)\right]} a_i$. Two additional interpretations are possible for the a_i. First, they can be viewed as each individual groundwater user's tax arising from the use of groundwater. Second, the a_i can also be viewed as a net payment, i.e., a payment which includes a return for groundwater use. I assume that the manager incurs fixed costs of $F to supply water and that he/she incurs variable costs which are a function of the rate at which he/she chooses to supply water and the number of users to whom he/she supplies water. I denote this variable cost function by $C = C(\epsilon, K)$. The manager's objective is to maximize the total residual benefit per month arising from the provision of water, which I assume to be the difference between monetary inflows (benefit) and outflows (cost) as described above. Thus, the manager solves

$$max_{\epsilon,K}\ \gamma\left(1 - P_K\right) \sum_{i=1}^{i = Int\left[\gamma\left(1 - P_K\right)\right]} a_i(x_i) - C(\epsilon,K) - F\ . \tag{8}$$

The first-order necessary conditions for an interior maximum are [3]

$$\left(\sum_{i=1}^{= Int\left[\gamma\left(1 - P_K\right)\right]} a_i\right) \gamma^K\left\{\gamma^{K+2} + \gamma^K\epsilon^{K+1}(K+1) - \epsilon^{K+1}\gamma - \gamma^2\epsilon^K(K+1)\right\} = \left\{\partial C(\bullet)/\partial\epsilon\right\}\left\{\epsilon^{K+1} - \gamma^{K+}\right. \tag{9}$$

and

$$\left(\sum_{i=1}^{i = Int\left[\gamma\left(1 - P_K\right)\right]} a_i\right) \gamma^{K+1}\left\{\gamma \ln\gamma + \epsilon \ln\epsilon - \epsilon \ln\gamma - \gamma \ln\epsilon\right\} = \left\{\partial C(\bullet)/\partial K\right\}\left\{\epsilon^{K+1} - \gamma^{K+1}\right\}^2 . \tag{10}$$

[3]I assume that the second-order conditions are satisfied.

Equation (9) says that the weighted marginal cost of providing water per month (the RHS of the equation) must equal the weighted total sum of monetary payments for water use. Similarly, (10) says that the weighted marginal cost—with respect to K—of providing water (the RHS of the equation) must be set equal to a different weighted sum of the total monetary payments. The weights themselves are functions of the parameters of the two different exponential distributions. The solutions to (9) and (10) give us the optimal values of the choice variables ϵ and K. Since equations like (9) and (10) cannot, in general, be solved analytically, one will have to resort to numerical methods in order to obtain the optimal ϵ and K.

Two quantities which have a bearing on the efficiency of the management function and, hence, are of considerable interest to the manager are the average number of groundwater users in the system, i.e., the expected system size (which I shall denote by L) and the average number of groundwater users waiting in queue or the expected queue size (which I shall denote by L_Q). The quantities L and L_Q are useful for planning purposes. They provide the manager with summary statistics about the water users. Since the manager is a social planner vested with property rights to the aquifer, he/she will want to alter his/her choice variables over time if, for instance, the average number of users being supplied water is deemed to be too few.

Now $L = \sum_{k=0}^{k=K} kP_k$. For the K state *M/M/1* queue, this simplifies to

$$L = \frac{\gamma\left[1 + K(\gamma/\epsilon)^{K+1} - (K+1)(\gamma/\epsilon)^{K}\right]}{(\epsilon-\gamma)\left[1-(\gamma/\epsilon)^{K+1}\right]} . \tag{11}$$

The expected queue size, $L_Q = \gamma_a W_Q$, where γ_a = expected arrival rate of users = $\gamma(1-P_K)$ and W_Q = expected time a user spends waiting in queue = $\left[L/\{\gamma(1-P_K)\} - \{1/\epsilon\}\right]$. Thus, I get

$$L_Q = \gamma(1-P_K)\left[\frac{L}{\gamma(1-P_K)} - \frac{1}{\epsilon}\right] \tag{12}$$

where P_K is given by (7) and L is given by (11).

Thus far, I have characterized a groundwater management regime as a *M/M/1* queue with finite capacity. In democratic polities, society can choose how to use the maximized residual benefit. It can be used to maintain water supply facilities so as to make the management regime self-financing. It can be used to pay for water imports in times of emergency such as a drought. If one were to view the a_i as net payments by users, then some of this benefit could be retumed to the individual groundwater users. Finally, if society desires that the manager break even financially, then the manager's problem becomes one of choosing ϵ and K so as to equate monetary inflows (benefits) and outflows (costs).

I now discuss an alternate queuing theoretic characterization of a groundwater management regime.

3b. The Water Management Regime as a M/G/1 Queue with Finite Capacity

I now model the water management regime as a M/G/1 queue with finite capacity. Before obtaining the stationary probabilities for the K state *M/G/1* queue, I first have to obtain the stationary probabilities for the *M/G/1* queue with a countable state space. Because the manager's supply time distribution is arbitrary for this queue, the methods cannot be applied here that were employed in section 3a to obtain the stationary probabilities for $\{X(t): t \geq 0\}$ where $X(t)$ denotes the number of users in the system at time t. In fact, $\{X(t): t \geq 0\}$ is not a discrete time Markov chain. As is usually done in this case, I proceed by analyzing the embedded Markov chain $\{X(n): n \sim l, 2, . . .\}$ where $X(n)$ refers to the number of users left behind by the nth departure from the system (see Ross 1983, pp. 100-112). Let $P = [p_{ij}]$ denote the chain's transition probability matrix and let the one-step transition probabilities be [4]

$$P_{ij} = \int_0^\infty \frac{e^{-\gamma t}(\gamma t)^{j-i+1}}{(j-i+1)!} dG(t), \; j \geq i-1, \; i \geq 1 \tag{13}$$

where G(•) is the distribution function of the manager's supply times. If I let π_n = *Pr{n arrivals during a supply time S = t}*, then I get

$$\pi_n = \int_0^\infty \frac{e^{-\gamma t}(\gamma t)^n}{n!} \, dG(t) \; . \tag{14}$$

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[4]I use the Stieltjes integral to avoid problems arising from the potential nonexistence of the density function.

In order to obtain the stationary probabilities for the chain {X(n): n = l, 2, . . .}, I shall assume that $\delta = \sum_{n=1}^{\infty} n\pi_n < 1$. That is, the mean number of arrivals in a supply time period is assumed to be less than

unity. This assumption ensures the ergodicity of the chain and hence the existence of the stationar y probabilities. To determine the stationary probabilities $\{d_k\}$, I need to solve the stationary equations . These are

$$d_t = d_0\pi_i \sum_{j=1}^{j=i+1} d_j\pi_{i-j+1}, \quad i = 0, 1, 2, \ldots \tag{15}$$

To solve the above set of equations, I shall use probability-generating functions as in Ross (1983 , pp. 111-112). To this end, let

$$D(z) = \sum_{i=0}^{\infty} d_i z^i, \; |z| \leq 1 \tag{16}$$

and let

$$\Pi(z) = \sum_{i=0}^{\infty} \pi_i z^I . \tag{17}$$

Multiplying the LHS and the RHS of (15) by z^i, summing over i, and then solving for $D(z)$ yields

$$D(z) = \frac{d_0(1-z)\Pi(z)}{\Pi(z) - z} . \tag{18}$$

Equation (18) can be further simplified by using L'Hopital's rule, noting that $\Pi(1) = 1$ and that $\Pi'(1) = \delta = \gamma E[supply\ time] = \gamma(1/\epsilon)$, where $E[\bullet]$ is the expectation operator. Performing thes e simplifications, I get $d_0 = 1 - \delta$ and

$$D(z) = \frac{(1-\delta)(1-z)\Pi(z)}{\Pi(z) - z} . \tag{19}$$

Equation (19) is as far as I can go in obtaining the stationary probabilities of the embedded Markov chain {*X(n): n = l, 2, . . .*}. So far, I have obtained the $\{d_k\}$ for the countable state Markov chain. However, I am actually interested in obtaining the $\{d_k\}$ for the K state embedded chain. I now proceed to obtain these probabilities.

I first have to truncate the state space to $k = 0 \ldots, K - 1$ states. This truncation necessitates a modification of the relevant stationary equations. These equations can now be written as

$$d_i = d_0 \pi_i + \sum_{j=1}^{j=i+1} d_j \pi_{i-j+1}, \; i = 0, 1, \ldots, K-2$$
$$d_i = 1 - d_0 \sum_{n=0}^{n=K-2} \pi_n - \sum_{j=1}^{j=K} d_j \sum_{n=0}^{n=K-j-1} \pi_n, \; i = K - 1 \; . \tag{20}$$

The solution to these K equations will give us the $\{d_k\}$ for the K state chain. To obtain the solution, I shall proceed heuristically.[5] Inspecting (15) and (20), I conjecture that the stationary probabilities for the K state embedded Markov chain must be proportional to the stationary probabilities for the countable state Markov chain which I now denote by $\{d_n^\infty\}$. That is,

$$d_k = L d_k^\infty \tag{21}$$

where L is the constant of proportionality. To verify that my conjecture is true, I first need to determine L. To obtain L I shall use the fact that $1 = L \sum_{k=0}^{k=K=1} d_k^\infty = \sum_{k=0}^{k=K=1} d_k$. This tells me that $L = 1/\sum_{k=0}^{k=K-1} d_k^\infty$. From this I conclude that the stationary probabilities for the K state Markov chain are given by

$$d_k = \frac{d_k^\infty}{\sum_{k=0}^{k=K-1} d_k^\infty}, \; k = 0, 1, \ldots, K - 1 \; . \tag{22}$$

I have now solved for the $\{d_k\}$, i.e., the proportion of users leaving behind k in the system when they depart the system. However, we are really interested in the $\{P_k\}$. To obtain the $\{P_k\}$, I shall first solve for the $\{a_k\}$. Now, in any queuing system in which users arrive one at a time and are supplied water one at a time, $a_k = d_k, \; k \geq 0$ (Ross 1985, p. 308). This property holds in the case that I have been studying. Further, by the PASTA property (see Wolff 1989, pp. 293-297), $a_k = P_k$. Hence, to obtain the $\{P_k\}$, it suffices to find the $\{a_k\}$. In my case, $a_k = d_k$, except that to obtain the $\{a_k\}$, I have to expand the state

[5] See Cohen (1982, pp. 570-572) for an alternate and considerably more rigorous approach to the solution.

space to include the *K* state. This can be done easily. First, note that d_t = *Pr{an arrival finds k users occupying the system/arrival does join the queue}* and, hence, $d_k = a_k/(1 - a_K)$. Thus,

$$a_k = P_k = (1 - a_K)\, d_k,\ k = 0, \ldots, K - 1 . \tag{23}$$

Finally, to find $a_K = P_K$, I use the rate equality principle alluded to in the derivation of (2)–(4). I have $\epsilon(1 - d_0) = \gamma(1 - a_K)$ and, hence,

$$a_K = P_K = \frac{(\gamma/\epsilon) - 1 + d_0}{\gamma/\epsilon} . \tag{24}$$

Also, using (24), then (23) can be simplified to

$$a_k = P_k = \frac{(1 - d_0)\, d_k}{\gamma/\epsilon} . \tag{25}$$

Thus, the required stationary probabilities for the *K* state *M/G/1* queue are given by (24) and (25).

I am now in a position to discuss the manager's optimization problem. Reasoning analogous to that employed in section 3a reveals that the manager's optimization problem is given by (8). The relevan t first-order necessary conditions for an interior maximum are [6]

$$\left. {}_{-P_K)]} \; a_i \right) \left[\left\{ \sum_{k=0}^{k=K-1} d_k^{\infty} \right\}^2 - \sum_{k=0}^{k=K-1} d_k^{\infty} + \epsilon\partial \left\{ \sum_{k=0}^{k=K-1} d_k^{\infty} \right\} / \partial\epsilon - \gamma\partial \left\{ \sum_{k=0}^{k=K=1} d_k^{\infty} \right\} / \partial\epsilon \right] = \{ \partial C(\cdot)/\partial\epsilon \} \left\{ {}^{k=} \right. \tag{26}$$

and

$$\left[\left\{ \sum_{i=1}^{i=Int[\gamma(1-P_K)]} a_i \right\} (\gamma - \epsilon) \right] \left[\partial \left\{ \sum_{k=0}^{k=K-1} d_k^{\infty} \right\} / \partial K \right] = \{ \partial C(\cdot)/\partial K \} . \tag{27}$$

Equation (26) tells us that optimality requires the manager to set the weighted marginal cost of supplying water (the RHS of the equation) equal to the weighted sum of total monetary inflows. Further, (27) tells us that the marginal cost of supplying water (the RHS of the equation) should be set equal to a different weighted sum of monetary inflows. Alternately, the weighted sum of total monetary inflows can also be

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[6]I assume that the second-order conditions are satisfied.

interpreted as the marginal revenue from water use. The optimal ϵ and K are defined by (26) and (27) implicitly. As in section 3a, (26) and 27) will typically have to be solved numerically to obtain the desired ϵ and K.

To obtain the expected system size, L, and the expected queue size, L_Q,[7] for the K state M/G/1 queue, I will follow a slightly different route. As is well known, $L = \gamma_a W$, where W is the expected time that a water user spends in the system. $\gamma_a = \gamma(1 - P_K)$, as in section 3a. W can be computed from the relation $W = W_Q + (1/\epsilon)$. Some algebra reveals that $W_Q = \left\{\gamma\epsilon(1 - P_K)E\left[S^2\right]\right\} / \left\{2\left[\epsilon - \gamma\left(1 - P_K\right)\right]\right\}$ (see Ross 1985, p. 337). Finally, $L_Q = \gamma_a W_Q$. Using these relationships, I get

$$L = \frac{\left\{\gamma\left(1 - P_K\right)\right\}^2 \left\{\epsilon^2 E\left[S^2\right] - 2\right\} + 2\epsilon}{2\epsilon\left\{\epsilon - \gamma\left(1 - P_K\right)\right\}} \tag{28}$$

and

$$L_Q = \frac{\epsilon\left\{\gamma\left(1 - P_K\right)\right\}^2 E\left[S^2\right]}{2\left\{\epsilon - \gamma\left(1 - P_K\right)\right\}} \tag{29}$$

where S denotes a supply time and P_K is given by (24).

In addition to the ways suggested in section 3a of apportioning the residual benefit arising from the provision of groundwater, this benefit can be used for other purposes as well. This could include activities such as the financing of a monetary scheme which would reward water conservation during times of emergency.

4. Conclusions and Potential Extensions

In this paper I have proposed and analyzed a new framework for modeling a groundwater management problem. This framework applies the techniques of queuing theory. Specifically, I

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[7]These quantities were defined in section 3a.

characterized two simple water management regimes as two different kinds of queues. I then went on to show how the manager could choose a quota on water allocation and the rate at which he/she would supply water in a certain time period so as to maximize the residual benefit arising from the provision and use of groundwater.

The basic models discussed in this paper may be extended in many directions. In what follows, I suggest three possible extensions. One can make the models richer by considering bulk arrivals and/or bulk supply. While this would invalidate the $a_k = d_k$ result, the explicit incorporation of bulk arrivals and/or bulk supply will permit one to analyze management regimes more general than the ones that I have analyzed. A second line of extension would be to consider cases where the arrival process is arbitrary and the supply times process is either deterministic or some known stochastic process. This kind of setting would be in the spirit of the principal/agent paradigm of information economics. [8] In this paradigm, the principal (my manager) is generally assumed to know the characteristics affecting the discharge of his/her functions with certainty. However, the principal is assumed to know the characteristics of the agents (my users) only imperfectly. Finally, the use of queue networks would permit the analysis of interactions such as those between agricultural and nonagricultural uses of groundwater.

[8] See Kreps (1989, Chapters 16 and 17) for an informative discussion of the principal/agent paradigm.

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