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# Convenience yield and the theory of storage: applying an option-based approach

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We propose that an options-based approach is a superior alternative to the traditional cost-of-carry method to model both the behaviour of convenience yields and the commodity price responses to changes in inventory levels. This approach is shown to be more robust and avoids the simplifying assumptions embedded in cost-of-carry valuation which fully accounts for the non-negativity constraint on inventory. Unlike the cost-of-carry approach, the options-based approach does not treat the convenience yield as an exogenous factor. This offers a more natural measure of implied convenience yields in commodity trading strategies. We test the relationship between convenience yields and inventory levels for a number of liquidly traded base metals using both methods. Our results show that the relationship between convenience yields and inventory levels is strongly defined under the options-based approach in line with market beliefs. This result is consistent with other studies that have used the options-based approach in other nonmetals commodity markets.

**Key words:** base metals, convenience yields, cost of carry, futures markets, option pricing.

## 1. Introduction

Consumers of relatively durable but consumable commodities obtain a benefit from physically holding certain commodities for a given period. This benefit is not available to holders of forward or futures contracts held over the same commodities. Such benefits to the holder include the ability to profit from temporary shortages of the commodity as well as the ‘convenience’ benefits gained from ready access to supply for use in production processes or to meet certain other obligations. Scholars define these benefits collectively as a ‘convenience yield’, which aims to capture all the implicit benefits accrued to the owner due to having ready access to the commodity.

The notion of a convenience yield, first introduced by Kaldor (1939), is an instructive concept for understanding the theory of storage in the context of financial markets, because it serves to quantify the benefit accrued in the form of an incentive to buy and store certain commodities. In the Pindyck (1993) model of rational commodity pricing, convenience yields play the role of dividends in stock valuation for anticipating future changes in spot commodity prices. When

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the timing or the level of consumption is stochastic, holding commodity inventories acts as an insurance against unexpected price movements or demand shifts. Conceptually, the convenience yield (or the price of storage in the early literature) is a forward-looking variable that contains information not exclusive to future demands and has been shown to be inversely related to the inventory level of the commodity (Working 1948, 1949; Brennan 1958; Fama and French 1988). The intuition behind the inverse relationship between expected commodity price changes and convenience yields is that a permanent increase in the current and future commodity demand has differential impact on current and expected spot prices (Fama and French 1988). In particular, current spot prices should increase more than expected spot prices (futures prices), especially at low inventory levels. This is because the demand and supply responses of consumers and producers tend to partially offset the effect of the shock on expected prices. As a consequence, higher convenience yields and lower inventory levels are associated with lower expected spot prices (futures prices).

Convenience yields and their interaction through spot and futures price differentials have traditionally been modelled using variations of the cost-of-carry approach. Convenience yields are thus treated as an exogenous variable (Lin and Duan 2007; West 2012). The cost of carry is defined as the cost of storing the physical commodity for a period, while the 'carry' accounts for insurance, storage and interest on invested or forgone funds, as well as other incidental expenses. It also includes a convenience yield equivalent to the benefit enjoyed by holding or 'carrying' the commodity.

A commodity market in contango implies that there are surplus stocks, and so long as storage capacities are not saturated, the basis (spot – futures price differential) is stable and limited to the cost of storage. In a backwardated market, however, stocks are scarce, and the basis is almost solely determined by the spot price buyers are willing to pay to secure supply. There is no subjective limit to the basis, and moreover, because inventories are not sufficiently abundant to absorb demand fluctuations, the spot price becomes volatile. The elimination of arbitrage in commodity markets suggests that even if the convenience yield restores the no-arbitrage relationship between spot and futures prices, this relationship remains specific: the basis displays an asymmetric profile, due to the non-negativity constraint on inventory levels. While theoretically awkward, in practice, especially when the traditional cost-of-carry method is used, a negative convenience yield can exist if the spot price is heavily discounted relative to the futures price, after accounting for carrying amounts. Under the common assumption of independence between storage costs and inventory level, due to exploitation of arbitrage opportunities, negative outcomes should disappear. However, general inefficiencies and delays in the handling, transport and storage of commodities will naturally lead to a differential in the theoretical and practical basis. For example, this aberration occurred during the credit crisis in 2008/2009 in the trade of seaborne bulk commodities (iron ore, coal and

copper concentrate) where storage had been exhausted at all of the locations with appropriate plant and equipment to adequately accommodate them.

A progression from the cost-of-carry approach for measuring the value of convenience yields is a contingent claim or options-based valuation approach. This method treats convenience yields as a value contingent on an underlying asset, which mirrors the asymmetric valuation of option contract payoffs (Heinkel *et al.* 1990). This method eliminates the possibility of obtaining negative convenience yields which are theoretically possible but practically irrational. Consequently, we propose that an options-based approach improves both the accuracy and the precision of convenience yields and its relation with stored inventories because it uses the forward curve to incorporate *all* information about market expectations as well as catering for the implied volatility of commodity prices over the entire forward curve.

The purpose of this study is to determine whether the relation between convenience yield and inventory level for a portfolio of base metals can be better quantified using an options-based pricing approach relative to cost-of-carry approach. We derive a practical and empirical 'fair' value that is based on subtle changes in storage cost and capacity. We demonstrate that this method is more robust relative to the simpler, but more approximate, cost-of-carry approach without introducing an overly complex model for the purposes of estimation.

We examine the price behaviour and convenience yield of six major base metals: aluminium, copper, lead, nickel, tin and zinc. We apply actual storage cost data to the model to account for holding cost differentials through time. The principal finding of the analysis is that the relation between monthly changes on average convenience yields and inventories of base metals is better explained using the options approach relative to the cost-of-carry approach. An array of regression results show that the effect of inventory returns on convenience yield return is statistically significant across a model of base metals using the options-based method. This approach permits a greater understanding of the underlying dynamics of inventory relative to storage capacity in markets where the storage and shipment of commodities is cost-sensitive and a nontrivial activity.

## 2. Background

Commodities share similar characteristics with other financial assets in that they can be held for everyday use, they can be stored or they can be used in a variety of ways as an asset (e.g. collateral, securitisation, etc.). Deaton and Laroque (1992) classified demand for commodities as arising from one of three sources: (i) as an element of immediate consumption and production; (ii) as an element of anticipatory demand reflecting future needs; and (iii) as an element of holding-period risk compensated by returns in the form of capital gains (speculative demand). The market clearing price for

commodities is thus a function of the availability of new production plus inventories relative to expected total demand, based on the aggregate of current demand, anticipatory demand and asset demand (Deaton and Laroque 1992).

To help understand the notion of how convenience yields relate to commodity prices, we first construct a simple model that largely mirrors existing terminology and notation. Let  $S_{(t)}$  and  $F_{(t,T)}$  denote the spot and futures price of a commodity for delivery at time  $T$  and let  $R_{(t,T)}$  be the nominal interest earned between period  $t$  and  $T$ . The basis is defined as the difference between the futures and the spot price  $F_{(t,T)} - S_{(t)}$ . Under the theory of storage, inventories are held only if the expected returns are positive; thus,  $S_{(t)}$  and  $F_{(t,T)}$  are both functions of the level of inventory. Kaldor (1939) showed that a negative basis  $F_{(t,T)} - S_{(t)}$  consists of two components. The first is the opportunity cost of forgone interest from having to borrow and buy the commodity. The second is the convenience yield  $CY_{(t,T)}$  (net of insurance and storage costs). Fama and French (1987) formalised the relation between convenience yields and the basis as

$$F_{(t,T)} - S_{(t)} = S_t R_{(t,T)} - CY_{(t,T)}. \quad (1)$$

Kaldor (1939) used the convenience yield to reflect the benefit from using a stored commodity whenever desired.

Although the convenience yield is unobserved, it can be computed from the observed spot price, futures price and interest rate using Equation (1). If the spot price is approximated by the price of the nearest futures contract, the basis has the form of a futures price spread. Convenience yields with negative sign are referred to as interest-adjusted basis (Fama and French 1988). Gorton *et al.* (2013) provide empirical evidence that convenience yields are a decreasing nonlinear function of inventories, since high inventory levels directly implies a low probability of a stock-out, and vice-versa. Forward (or futures) contracts, however, help to lower excess demand in the spot market, which reduces the financial benefit of holding the commodity (Brennan 1958; Fama and French 1988).

Fama and French (1987, 1988) tested whether convenience yields were inversely related to inventory using a cost-of-carry approach for several base metals. Their main finding is that the variability of interest-adjusted basis is higher when the values are negative (as a proxy for low inventory levels), which implies that the volatility of convenience yield increases as inventories diminish (Fama and French 1988). Their conclusions, however, are based on the relative variation in spot and forward prices but neglect to account for both the aggregate inventory data and the actual cost of storage. They assume marginal warehousing costs for metals are roughly constant over the relevant range of inventory. In practice, however, actual storage costs of commodities often alter depending on inventory level. Furthermore, their

approach to modelling convenience yield as an explicit variable does not adequately explain the dynamic relation between spot and forward prices implied by inventory levels. While the conclusions drawn from their empirical paper provide a powerful argument supporting the theory of storage, we believe the method needs to be refined to more accurately account for aggregate inventory, changes in storage costs and the asymmetric profile of convenience yields, especially when inventories are abundant. The need to account for asymmetric convenience yield profiles is discussed below.

A range of alternative literature has modelled convenience yields directly in a number of different ways. Its simple use as a tool to restore the no-arbitrage relationship gives rise to the suggestion that the convenience yield should be a negative function of the level of inventory. An intuitive way to represent convenience yields was to regard them as a positive and deterministic function of the spot price (Brennan and Schwartz 1985; Brennan 1991; Gibson and Schwartz 1991). For some commodities it was shown that convenience yields are not perfectly correlated to the spot price and could instead be regarded as a mean reverting stochastic variable (Gibson and Schwartz 1990; Schwartz 1997, 1998; Cortazar and Schwartz 2003). However, the need to account for the non-negativity constraint on inventory limits the universality of these methods.

Convenience yields have an asymmetrical profile; that is, convenience yields, net of storage costs, cannot be less than the inverse value of the cost of storage. Routledge *et al.* (2000) suggest that the correlation between spot prices and the convenience yields is higher in backwardation than in contango. While maintaining the stochastic model approach, they model the convenience yield as an endogenous variable, driven wholly by storage dynamics. While we believe this approach provides a rich explanation for convenience yield dynamics, it still fails to fully account for the non-negativity constraint on inventory. However, since an option comprises an asymmetrical payoff of an asset, then representing the convenience yield for a commodity as a call option on traded futures, partly implied by the level of inventory, would offer a more robust explanation for convenience yield behaviour, especially at the extremes of either abundant or scarce inventory levels. The option explicitly emerges because of the opportunity profit from an unexpected rise in the demand for inventory. In addition, storage costs are often assumed supposed to be constant for a large range of prices, as long as the storage capacity for a particular commodity is not saturated. We believe this assumption is too strict, and that actual storage costs have a direct effect on convenience yield values.

Other literature has explored convenience yields from a number of important perspectives. Ng and Pirrong (1994) used London Metals Exchange (LME) warehouse rental data as a proxy for storage costs and found that the volatility of spot and forward prices directly correlates with inventory levels. However, the study did not use inventory data, and hence, its conclusions are largely formulated on the same basis as of Fama and

French (1988). Geman and Smith (2013), building on the notion developed by Geman and Ohana (2009), used interest- and storage-adjusted spreads as a proxy for the convenience yield. They empirically tested the validity of the theory on metals and employed inventory inputs, but still failed to incorporate appropriate storage costs. The paper estimated the historical storage costs from 2011 LME warehouse data by adjusting them with the US CPI index. However, as Table 1 illustrates, the warehousing costs of base metals have increased much more than US inflation since 2003. This observation may introduce bias when using this approach.

Stepanek *et al.* (2013) used storage costs obtained from the LME to estimate convenience yields for base metals. Negative convenience yields, and therefore an indication of an arbitrage opportunity, were obtained in some instances. They added a constant term to rescale the smallest value to zero; however, to rationalise this rescaling, other incidental costs were assumed to be constant and equivalent to an absolute value of the largest negative convenience yield obtained. Their study revealed that the traditional approach, which attempts to estimate convenience yields from the difference between spot and futures prices (after adjusting interest forgone) and available storage costs, is limited by the output of negative convenience yields.

The potential for negative convenience yields can be overcome by considering convenience yield as a value of contingent claim for a payoff that can be earned from storing an asset and is equivalent to the value of a call option (Heinkel *et al.* 1990; Milonas and Thomadakis 1997). This

**Table 1** Average metal price versus warehouse costs and inflation rate

	Copper	Zinc	Nickel	Lead	Aluminium	Tin
Spot price (USD/MT)						
Average 2013	7744	1984	16 880	2223	1965	23 425
January to April						
Annualised cost (2013)/price	2.0%	8.0%	1.1%	7.0%	8.9%	0.7%
Annualised warehouse cost (USD/MT)						
2013	156.4	159.4	186.3	156.4	174.5	174.1
2003	83.4	82.0	100.4	78.3	122.9	87.7
Rate of cost increase since 2003	87.6%	94.3%	85.5%	99.9%	41.9%	98.4%
US inflation index (CPI base)						
2003–2013						27.0%

The table compares the spot price against annualised warehouse cost and the rate of increase in warehouse costs and inflation rate in the US. MT, metric tonne. Source: Datastream, London Metal Exchange, United States Department of Labor.

approach does not consider convenience yield as an exogenous variable and also helps avoid the generation of negative convenience yields. The payoff of the option is computed as  $\max(S_T - K, 0)$ , where  $S_T$  is spot price at time  $T$  and  $K$ , the exercise price (Black and Scholes 1973), so theoretically the option payoff is bounded by zero when it is appropriately estimated. Milonas and Thomadakis (1997) applied this approach to explain the convenience yield of some commodities, but they did not consider actual storage costs. In another study, Lin and Duan (2007) used an options-based approach to show that the convenience yield of crude oil is weakly negatively correlated to inventory levels. In addition, Litzenberger and Rabinowitz (1995) used an options price for crude oil to explain the backwardation (positive interest-adjusted basis) of the commodity. They obtained some significant results. Hence, in this analysis, we partly apply the pioneering work of these studies to support our approach for viewing convenience yields as an option on inventory levels.

### 3. Research question

As convenience yields cannot be directly observed, their value must be estimated. Using the cost-of-carry approach, the convenience yield is the residual value for a spot contract after meeting necessary costs (Liu and Tang 2010). The theoretical convenience yield should therefore be directly affected by either changes in spot or forward (futures) prices. It is reasonable to assume that market participants pay close attention to the position of spot prices relative to forward prices while determining the implicit value (by implicit, we mean derived from spot and forward price dynamics) of possessing the physical asset. In this sense, the convenience yield is implicitly, as opposed to explicitly, related to price (that is, the convenience yield cannot be defined without consideration for relative spot and forward prices).

We pose two key research questions: (i) Are convenience yields inversely related to inventory level and if so under what conditions? (ii) Is an options-based approach more robust than the cost-of-carry approach to model convenience yield and spot inventory? The first question is designed to validate the use of an options-based approach to confirm the inverse relationship between convenience yields and inventory levels. We then compare this result with the more conventional cost-of-carry approach to determine which method yields more robust results across commodities. This forms the basis for answering the second research question. We apply this analysis to a portfolio of base metal spot and futures contract prices.

### 4. Methodology and data

We follow the method developed in West (2012) to model the convenience yield using a contingent-claim approach. The contingent-claim convenience yield value  $CY_{t,T}^{\text{Option}}$  can be estimated as

$$CY_{(t,T)}^{\text{Option}} = S_{(t)}^* N(d_1) - F_{(t,T)} N(d_2), \quad (2)$$

where,

$$S_{(t)}^* = S_{(t)} + W_{(t,T)}$$

$$d_1 = \frac{\ln(F_T) + \sigma_c^2 \tau / 2}{\sigma_c \sqrt{\tau}} \text{ and } d_2 = d_1 - \sigma_c \sqrt{\tau} = \frac{\ln(F_T) - \sigma_c^2 \tau / 2}{\sigma_c \sqrt{\tau}}$$

and

$$\sigma_c^2 = \sigma_{S^*(t)}^2 + 2\sigma_{S^*(t)}\sigma_{F(t,T)}\rho_{S^*(t)F(t,T)} + \sigma_{F(t,T)}^2$$

where  $CY_{t,T}^{\text{Option}}$  is the convenience yield from time  $t$  to  $T$  estimated by the options-based approach,  $W_{(t,T)}$  is the storage cost from time  $t$  to  $T$ ,  $F_T = S_{(t)}^*/F_{(t,T)}$ ,  $\sigma_c$  is the volatility of convenience yield,  $\sigma_S$  and  $\sigma_F$  are the volatility of spot and futures contracts,  $\rho_{S^*(t)F(t,T)}$  is correlation coefficient between spot and futures contracts and  $\tau$  is the duration of time from time  $t$  to  $T$ . The futures prices are adjusted for time value of money, following the notion of Milonas and Thomadakis (1997). We apply an exponentially weighted moving average (EWMA) scheme with decay parameter  $\lambda = 0.94$  to estimate volatilities of spot and futures contracts to allow for historical data to be retained in the estimation with higher weights assigned to the older data.<sup>1</sup> For the purpose of conducting regression analysis daily, convenience yields are estimated and then averaged to obtain monthly averages.

To estimate convenience yields using the cost-of-carry approach, we refine the formulation in Equation (1) as follows:

$$F_{(t,T)} - S_{(t)} = S_{(t)} R_{(t,T)} + W_{(t,T)} - CY_{t,T}^{\text{cost-of-carry}} \quad (3)$$

which reduces to

$$CY_{t,T}^{\text{cost-of-carry}} = S_{(t)} + S_{(t)} R_{(t,T)} + W_{(t,T)} - F_{(t,T)} \quad (4)$$

where,  $CY_{t,T}^{\text{cost-of-carry}}$  is the convenience yield from time  $t$  to  $T$ ,  $S_{(t)}$  is the spot price at time  $t$ ,  $F_{(t,T)}$  is the forward price at time  $t$  which matures at time  $T$ ,  $R_{(t,T)}$  is the risk-free interest rate from time  $t$  to  $T$  and  $W_{(t,T)}$  is the storage cost from  $t$  to  $T$ . The convenience yields based on the cost-of-carry approach

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<sup>1</sup> We considered the use of implied volatility estimates from observed option prices. However, since the turnover of most option contracts traded on the LME is less than 10 per cent of the turnover in associated futures contracts (for tin it is less than 1 per cent of total turnover (London Metal Exchange 2014)), the chances of obtaining unbiased estimates for implied volatility were considerably lower.

are also estimated at a daily basis and then averaged to obtain monthly estimates.

We conduct two separate regression analyses to evaluate the two models. The respective regressions are represented as

$$CY_{t,T}^{\text{Option}}/CY_{t-1,T-1}^{\text{Option}} - 1 = \beta_0 + \beta_1[I_{t-1}/I_{t-2} - 1] + \varepsilon_t \quad (5)$$

$$CY_{t,T}^{\text{cost-of-carry}}/CY_{t-1,T-1}^{\text{cost-of-carry}} - 1 = \beta_0 + \beta_1[I_{t-1}/I_{t-2} - 1] + \varepsilon_t \quad (6)$$

where  $\beta_0$  and  $\beta_1$  are the regression parameters,  $[I_{t-1}/I_{t-2} - 1]$  represents the monthly change in inventory levels and  $\varepsilon_t$  is the error term. The dependent variable in each regression is the monthly convenience yield return, while the independent variable is the monthly return on inventory levels. Negative convenience yields estimated by the cost-of-carry approach are eliminated from the data set. Both equations are designed to measure how month-on-month percentage changes in monthly convenience yield average are related to 1-month lagged relevant return of inventory. The purpose of applying return figures is to overcome unit root issues inherent in most estimated convenience yields and inventory estimates.

We obtained cash prices as a proxy for spot prices and 3-month LME futures prices for six base metals: aluminium, copper, lead, nickel, tin and zinc, from Thomson Reuters Datastream (Datastream) for the period January 2000–May 2013. The rolling 3-month futures contract is chosen because it represents the most liquid futures rate for each of the six base metals (Fama and French 1988; Geman and Smith 2013). The rationale for the selection of the LME data is as follows: (i) the LME is the largest nonferrous exchange in the world where over 80 per cent of on-exchange business is handled (London Metal Exchange 2013), (ii) the LME imposes no limit on spot and futures price changes, which more or less enables full price discovery, and (iii) the LME data provide simultaneous spot and futures prices, for fixed forward maturities, for each business day in the period of interest.

Inventory volume data are obtained from the LME and combined with volume data from the Shanghai Futures Exchange (SHFE) and the Commodity Exchange (COMEX) for each of the base metals. This is aligned with the approach used by Geman and Smith (2013) and provides a global measure of inventory stock data instead of relying solely on the narrower inventory stock data of LME-compliant warehouse facilities. We obtained inventory data from Datastream. For storage cost data, daily LME warehouse rental prices provided by the LME are used to proxy the storage cost of each of the base metals used in the analysis. Anecdotal evidence from other commodity exchanges suggests that base metals storage cost data are relatively well correlated across warehouses due to the high level of

geographic spread of such facilities globally. We obtained storage cost per day data for each metal. We then converted this into arithmetic average rents of various warehouse locations. Previous research only estimates historical storage costs by adjusting the LME warehouse rents for a certain year with the US inflation rate. However, Table 1 shows warehouse rents have increased more rapidly than the inflation rate over last decade. Therefore, we account for actual storage costs rather than approximate ones. We used 3-month US Treasury bills as a proxy for near-term interest rates from Datastream. Spot and futures metals price data are thus synchronised with inventory, storage cost and interest rate data on daily basis.

## 5. Results

First, we apply the Augmented Dickey–Fuller (ADF) test (Dickey and Fuller 1979) to conduct a unit root test. The test statistics are estimated based on Akaike information criterion (AIC). The results in Table 2 show that number of the estimated monthly average convenience yields and inventory series are nonstationary (based on nonzero constant with trend assumption). We therefore convert each series into month-on-month percentage change figures. The same ADF test (zero constant with no trend assumption is selected by observing the data set) was conducted on new data set which eliminated the effect of non-stationarity and the presence of a unit root at a 5 per cent significance level.

Table 3 presents the results of the options-based approach regression analyses for month-on-month percentage changes in convenience yields against the corresponding return figures for the inventory levels of each of the six base metals. White's robust standard error was used to estimate *t*-values for the metals that are affected by heteroscedasticity. The residuals were also tested for the presence of autocorrelation, but no evidence of this was detected. The results show that we obtained low  $R^2$  values for the single factor regression (inventory lagged by one period) to explain the variation in convenience yield. Nevertheless, negative coefficients are statistically significant for aluminium, nickel, tin and zinc (for models with LME and LME plus SHFE). The model for copper is insignificant when regressed against LME and LME plus SHFE inventory figures. The inclusion of COMEX inventory onto LME and SHFE achieves significance at the 10 per cent level. Lead is the only metal for which we fail to reject the null hypothesis of there being no relation between convenience yields and inventory returns. A number of base metals used in the analysis are highly substitutable by other metals (e.g. aluminium for copper in the cabling industry and chrome for nickel in the stainless industry) and because of this the value of holding physical metal may be altered by the consumption and production patterns of other metals. Along with other industry-specific factors this may also affect the behaviour of convenience yields. In addition, the cost of aluminium production is highly dependent on electricity prices, but for this analysis, we

**Table 2** Unit root tests on the convenience yields and inventories

Augmented Dickey–Fuller tests: based on AIC max lag (12)				
	Monthly average		Monthly return of monthly average	
	Test stat.	P-value	Test stat.	P-value
Convenience yield (COC)				
Al	-4.214***	0.005	-9.663***	0.000
Cu	-1.588	0.793	-10.398***	0.000
Ni	-3.361*	0.061	-8.876***	0.000
Pb	-3.660**	0.028	-12.579***	0.000
Tn	-3.664**	0.028	-18.515***	0.000
Zn	-2.694	0.241	-3.630***	0.000
Convenience yield (Op)				
Al	-3.484**	0.045	-10.464***	0.000
Cu	-0.889	0.954	-9.371***	0.000
Ni	-2.687**	0.024	-1.476***	0.000
Pb	-2.552	0.303	-9.936***	0.000
Tn	-4.203***	0.006	-12.338***	0.000
Zn	-1.953	0.622	-11.811***	0.000
Inventory				
Al				
LME	-2.574	0.293	-3.557***	0.000
LME and SHFE	-2.497	0.329	-3.827***	0.000
Cu				
LME	-1.850	0.676	-5.123***	0.000
LME and SHFE	-2.055	0.566	-4.617***	0.000
LME and SHFE and COMEX	-1.608	0.786	-4.647***	0.000
Ni				
LME	-3.038	0.125	-5.488***	0.000
Pb				
LME	-1.789	0.706	-7.878***	0.000
LME and SHFE	-1.639	0.773	-7.805***	0.000
Tn				
LME	-2.631	0.267	-8.366***	0.000
Zn				
LME	-3.386*	0.057	-1.937*	0.051
LME and SHFE	-3.170*	0.095	-2.552**	0.011

Note: The table presents results of unit root test (ADF test) on the convenience yield and inventory of six base metals. Number of lag is automatically selected by EViews with maximum lag of 12. AIC is used to select the lag number in the test. Rejection of the null hypothesis ( $H_0$ : the variable contains a unit root) with each significance level is displayed by \*\*\*1%, \*\*5% and \*10%. For monthly average figures assumption of nonzero constant with trend is used and for the series of month-on-month percentage change in monthly average zero constant with no trend is used. LME, London Metals Exchange; SHFE, Shanghai Futures Exchange. Source: Datastream, London Metal Exchange.

have assumed that power prices are likely to remain constant over each 3-month period in the sample data due to producers preferring fixed long-term power offtake contracts. Some metals are highly recyclable by remelting the discarded products such as batteries (for lead and nickel alloys). Other metals are amenable to recycling which means they are remelted into a form of alloy, instead of a pure metal, for use directly in a different type of finished

**Table 3** Regression analysis results:  $CY_{t,T}^{\text{Option}}/CY_{t-1,T-1}^{\text{Option}} - 1 = \beta_0 + \beta_1[I_{t-1}/I_{t-2} - 1] + \varepsilon_t$ Convenience yields (options approach) versus inventory ( $t - 1$ )

	Aluminium		Copper		
	LME <sup>rob.</sup>	LME + SHFE <sup>rob.</sup>	LME	LME + SHFE	LME + SHFE + COMEX <sup>rob.</sup>
$R^2$	0.079	0.067	0.008	0.010	0.018
$\beta_0$	0.05**	0.05**	0.03*	0.03*	0.03*
( <i>p</i> -value)	(0.039)	(0.044)	(0.055)	(0.054)	(0.052)
$\beta_1$	-1.11***	-1.02***	-0.12	-0.17	-0.23*
( <i>p</i> -value)	(0.000)	(0.001)	(0.254)	(0.208)	(0.082)
( <i>t</i> -value)	(-3.74)	(-3.25)	(-1.14)	(-1.26)	(-1.75)
SE	0.30	0.31	0.10	0.14	0.13

Convenience yields (options approach) versus inventory ( $t - 1$ )

	Lead		Nickel	Tin	Zinc	
	LME	LME + SHFE	LME	LME <sup>rob.</sup>	LME <sup>rob.</sup>	LME + SHFE <sup>rob.</sup>
$R^2$	0.010	0.011	0.033	0.019	0.042	0.042
$\beta_0$	0.04**	0.04**	0.03*	0.05*	0.03*	0.03*
( <i>p</i> -value)	(0.037)	(0.034)	(0.093)	(0.051)	(0.090)	(0.082)
$\beta_1$	-0.16	-0.17	-0.25**	-0.28*	-0.50***	-0.53***
( <i>p</i> -value)	(0.202)	(0.181)	(0.022)	(0.070)	(0.005)	(0.006)
( <i>t</i> -value)	(-1.28)	(-1.35)	(-2.31)	(-1.82)	(-2.86)	(-2.77)
SE	0.13	0.13	0.11	0.15	0.17	0.19

Note: The table presents the regression analysis results of convenience yields estimated by the options approach versus 1-month lagged inventory. Convenience yields are first estimated on daily basis and then converted into monthly average. All the data are converted into month-on-month percentage change in monthly average figures. Data pertain to the period between January 2000 and May 2013 (monthly). The results are tested for heteroscedasticity by Breusch–Pagan test, and White's robust standard error is used for those models failed to accept homoscedasticity hypothesis (indicated as 'rob.' in the table). Significance level displayed by \*\*\*1%, \*\*5% and \*10%. LME, London Metals Exchange; SHFE, Shanghai Futures Exchange. Source: Datastream, London Metal Exchange.

product (Eiji Hosoda Inter Seminar 2007). These alloys are not traded on-exchange since they no longer comply with future contract specifications. Both aluminium and lead involve high recycling rates; however, whereas primary metal still makes up almost three quarter of the world annual aluminium consumption, recycled metal represents more than half of the lead consumed globally (Melik and Kouzmenkov 2010; International Lead Association 2014). The differential in on- and off-exchange traded physical supply may therefore have a direct effect on our results. However, in summary, we offer the cautious conclusion that these results broadly support the hypothesis that convenience yields are inversely related to inventory levels for the majority of base metals over the observation period.

We next examine the validity of the cost-of-carry approach for explaining the relation between convenience yields and inventory levels. The results of regression analysis are presented in Table 4. Again, we apply White's

standard error to account for the presence of heteroscedasticity. For the cost-of-carry approach, we obtained even lower  $R^2$  values relative to the options-based approach results in Table 3. While previous studies seem to support the cost-of-carry approach, our results suggest that the effect of inventory returns on convenience yield returns is not statistically significant at the 10 per cent significance level, except for copper and tin (using LME inventories). In summary, almost all of the regression parameters are not statistically significant using the cost-of-carry approach, which is in stark contrast to the results for the options-based approach. We therefore fail to reject the null hypothesis of zero relationship between convenience yields and inventory levels in most of the metals studied. The inconsistency between previous results and ours is likely to be due to the use of monthly changes in order to

**Table 4** Regression analysis results:  $CY_{t,T}^{\text{cost-of-carry}} / CY_{t-1,T-1}^{\text{cost-of-carry}} - 1 = \beta_0 + \beta_1[I_{t-1}/I_{t-2} - 1] + \varepsilon_t$

Convenience yields (cost-of-carry) versus inventory ( $t - 1$ )

	Aluminium		Copper		
	LME <sup>rob.</sup>	LME + SHFE	LME	LME + SHFE	LME + SHFE + COMEX
$R^2$	0.012	0.012	0.019	0.013	0.015
$\beta_0$	0.26***	0.26***	0.14***	0.14***	0.14***
( <i>p</i> -value)	(0.001)	(0.001)	(0.003)	(0.004)	(0.004)
$\beta_1$	-1.65	-1.57	-0.54*	-0.59	-0.63
( <i>p</i> -value)	(0.122)	(0.196)	(0.080)	(0.146)	(0.127)
( <i>t</i> -value)	(-1.56)	(-1.30)	(-1.76)	(-1.46)	(-1.53)
SE	1.06	1.21	0.31	0.41	0.41

Convenience yields (cost-of-carry) versus inventory ( $t - 1$ )

	Lead		Nickel	Tin	Zinc	
	LME <sup>rob.</sup>	LME + SHFE <sup>rob.</sup>	LME	LME <sup>rob.</sup>	LME <sup>rob.</sup>	LME + SHFE <sup>rob.</sup>
$R^2$	0.001	0.001	0.014	0.014	0.000	0.000
$\beta_0$	1.17	1.17	0.40***	0.29**	0.19**	0.19**
( <i>p</i> -value)	(0.269)	(0.268)	(0.010)	(0.048)	(0.014)	(0.013)
$\beta_1$	3.17	3.00	-0.95	-1.32***	-0.24	-0.18
( <i>p</i> -value)	(0.387)	(0.391)	(0.203)	(0.006)	(0.753)	(0.815)
( <i>t</i> -value)	(0.87)	(0.86)	(-1.28)	(-2.81)	(-0.32)	(-0.23)
SE	3.66	3.48	0.74	0.47	0.77	0.79

Note: The table presents regression analysis results of convenience yields estimated by the cost-of-carry approach versus 1-month lagged inventory. Convenience yields are first estimated on daily basis and then converted into monthly average. All the data are converted into month-on-month percentage change in monthly average figure. Data pertain to the period between January 2000 and May 2013. The results are tested for heteroscedasticity by Breusch–Pagan test, and White's robust standard error is used for those models failed to accept homoscedasticity hypothesis (indicated as 'rob.' in the table). LME, London Metals Exchange; SHFE, Shanghai Futures Exchange. Significance level displayed by \*\*\*1%, \*\*5% and \*10%. Source: Datastream, London Metal Exchange.

overcome the unit root issue observed in both convenience yields and inventories.

When the inventory level of commodities is high, it is possible, in practice, for the convenience yield to become temporarily negative because it is costly to own and carry forward stocks surplus to requirements. For instance, if a warehouse is operating at close to its capacity, it would require more time (perhaps days) to store and retrieve a commodity from the storage and hence costs to carry that asset increases. Holders that do so must be rewarded by a significantly upward-sloping futures curve (high contango). The convenience yield thus helps define the shape of the futures curve and implicitly reward or penalise holders of spare inventory accordingly. Asserting the existence of a negative convenience yield is akin to arguing that the forward price exceeds the spot price by more than the cost of storing a commodity (storage costs and transaction fees, etc.) on a risk-free interest-adjusted basis. Due to arbitrage, this will not occur unless storage is at capacity. The spread would then include the shadow price of storage capacity, and large spreads would increase the profitability of adding more capacity. Adding capacity, however, does not occur instantaneously and therefore the notion of ‘negative’ convenience yields while possible over brief time horizons are illusory in practice.

Descriptive results on estimated convenience yields presented in Tables 5 and 6 show that the options-based approach provides a higher convenience yield than the cost-of-carry approach across each of the six base metals. This outcome is also seen in the study of West (2012) who applied a similar analysis to seaborne thermal coal and Lin and Duan (2007) who examined the crude oil market. It is interesting to note that the gap between two different methods has become wider in the years leading into the global financial crisis. This may be explained as an increase in the price volatility amplified by uncertainty in reaction to a growth in inventory stockpiles in China and Korea, which resulted in the market severely discounting futures price against expected spot prices. This change in the market environment has been captured by the use of the options-based approach to model convenience yields but is not directly observed using the cost-of-carry approach.

The cost-of-carry approach generated a great number of negative convenience yield estimates. Most of the negative values were less than 1 per cent of spot price with some exceptions and mostly distributed over 2008–2009. This is consistent with the highly volatile commodity price fluctuations relative to inventory saturation levels observed during this period (Campbell *et al.* 2011). For some metals, even arithmetic annual average values dropped below zero (aluminium in 2008 and nickel in 2008–2010 and 2012). These results are presented in Table 6. Annual average LME nickel inventory in 2012 was below 2010 and similar to 2011 (Table 7). It is impractical to assume storage was at a capacity in 2012 based on the historical evidence. This suggests that storage capacity was not the primary cause of negative

**Table 5** Descriptive statistics summary of monthly average convenience yield (options approach)

		2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
	<i>n</i>	12	12	12	12	12	12	12	12	12	12	12	12	12	5
Aluminium	Mean	39.46	38.31	26.34	38.52	46.94	58.86	93.55	73.09	68.32	52.65	62.14	59.11	45.78	39.60
	SD	7.09	21.12	6.00	12.71	18.60	13.18	21.21	41.21	14.25	11.05	9.69	9.12	10.90	7.26
	Min	29.20	18.87	19.59	22.32	22.92	39.91	55.40	32.90	44.56	37.13	52.39	47.10	23.93	31.95
	Max	51.90	87.64	36.80	61.05	90.44	75.43	132.42	153.77	90.82	77.18	81.45	73.13	60.16	48.84
Copper	Mean	37.94	32.70	28.13	39.27	144.31	236.15	344.94	291.51	305.39	204.21	215.71	247.87	190.29	140.74
	SD	6.05	7.91	2.40	11.99	40.39	35.90	133.28	59.12	45.81	18.30	35.98	55.79	44.27	49.26
	Min	26.02	21.20	24.35	26.74	66.31	186.95	200.18	203.06	197.38	164.13	164.31	189.98	121.98	106.61
	Max	45.30	51.66	32.10	66.41	202.54	289.42	562.26	353.24	352.52	241.98	271.61	365.57	261.25	226.27
Lead	Mean	13.67	18.45	14.34	23.78	67.18	66.43	68.69	146.52	119.77	92.98	86.34	103.64	65.73	55.04
	SD	5.08	4.73	3.75	9.20	16.87	8.00	22.01	50.97	20.99	20.23	9.94	15.94	7.91	8.52
	Min	7.75	13.18	10.70	11.88	38.71	46.81	37.94	83.62	81.18	71.10	70.20	75.81	48.79	46.07
	Max	22.50	26.30	25.05	39.50	96.44	76.71	103.05	221.85	152.50	136.86	103.98	127.78	78.02	63.56
Nickel	Mean	463.63	287.31	242.02	311.88	749.72	619.86	1740.76	2482.85	962.87	695.73	776.98	784.30	459.93	341.43
	SD	119.00	104.20	61.52	75.03	136.34	178.09	1142.26	1300.91	201.32	163.28	160.32	110.83	76.69	24.49
	Min	179.06	162.29	171.33	222.42	556.75	422.96	435.57	957.03	684.68	457.13	602.85	587.49	358.53	299.75
	Max	636.89	508.63	413.65	482.97	1014.97	1074.45	3860.40	4234.37	1401.94	991.78	1161.68	991.99	628.87	363.30
Tin	Mean	90.23	74.63	74.97	91.59	340.37	250.61	333.02	507.59	800.24	652.29	568.27	793.69	624.31	462.64
	SD	22.18	29.11	13.72	31.46	158.12	50.24	104.97	90.75	269.71	178.98	156.61	134.96	108.24	81.33
	Min	65.28	39.86	55.99	64.85	176.94	159.84	238.97	387.57	384.24	315.01	339.30	577.34	434.71	379.18
	Max	133.00	112.13	107.46	167.65	645.92	344.52	523.91	653.79	1232.87	992.82	763.40	1069.46	778.48	551.63
Zinc	Mean	33.15	16.85	15.55	18.84	30.72	43.38	176.52	159.28	96.11	72.96	85.63	74.66	53.99	41.71
	SD	15.97	4.72	1.54	4.54	6.84	7.63	61.77	28.31	18.53	12.38	10.81	8.23	9.62	5.59
	Min	21.11	12.02	13.31	13.83	22.01	34.47	57.74	115.47	64.67	55.10	70.82	59.73	40.83	34.76
	Max	80.83	27.48	18.33	28.73	42.28	54.81	246.76	210.82	126.11	90.60	104.05	89.64	66.98	48.17

Note: Unit = USD per metric tonne. Source: Datastream, London Metal Exchange.

Table 6 Descriptive statistics summary of monthly average convenience yield (cost-of-carry)

	<i>n</i>	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
		12	12	12	12	12	12	12	12	12	12	12	12	12	5
Aluminium	Mean	25.43	24.50	12.30	30.90	24.73	39.63	33.16	34.20	-6.08	0.56	9.70	14.25	9.74	5.54
	SD	9.49	24.75	5.69	15.37	18.31	14.08	19.47	51.37	7.31	3.50	8.67	9.13	11.20	5.31
	Min	11.04	6.10	6.38	12.43	7.26	22.00	12.15	-5.62	-18.88	-3.63	1.33	2.95	-3.52	-1.23
	Max	43.35	82.62	23.20	57.28	73.30	58.87	78.01	140.47	4.65	8.28	29.05	32.93	28.05	11.23
Copper	Mean	17.66	13.47	6.39	16.59	106.96	226.89	166.69	138.67	127.35	11.25	19.74	22.07	43.27	8.30
	SD	8.09	9.50	3.08	10.58	40.37	38.79	52.05	69.00	85.11	13.05	24.55	13.28	28.25	2.79
	Min	3.72	1.85	0.93	5.70	35.99	178.70	80.02	30.35	-2.58	-6.04	-0.17	12.65	14.53	4.87
	Max	29.86	33.79	12.16	38.67	165.94	286.04	254.90	235.30	265.68	29.68	81.98	53.82	103.29	12.30
Lead	Mean	9.63	14.51	10.78	18.00	58.20	62.16	42.75	78.00	27.67	12.87	7.60	40.32	24.18	21.00
	SD	4.82	4.95	3.68	6.27	18.21	8.32	25.35	28.28	10.77	9.70	6.08	22.52	11.73	4.59
	Min	4.13	9.74	7.64	8.66	23.65	39.18	7.40	40.14	16.48	-2.81	2.18	10.36	2.24	15.98
	Max	18.62	23.71	21.18	29.18	86.64	67.99	80.65	125.42	50.18	28.37	24.95	79.21	45.94	25.28
Nickel	Mean	343.66	157.20	76.03	76.63	174.71	347.41	1237.25	1476.83	-42.11	-26.59	-10.92	21.86	-13.65	-18.94
	SD	116.33	120.67	64.53	31.75	97.21	245.94	1142.87	1549.89	58.09	16.46	19.73	11.77	15.76	3.90
	Min	56.27	20.78	27.70	27.32	54.42	59.73	50.07	-79.28	-150.10	-41.83	-32.87	2.00	-43.60	-22.92
	Max	474.41	426.77	254.08	136.22	372.74	957.17	3558.14	3610.83	53.59	18.45	41.99	40.26	8.32	-13.50
Tin	Mean	66.96	21.72	3.68	21.45	195.54	126.19	121.76	173.65	150.36	236.08	10.59	5.81	31.75	10.58
	SD	30.29	21.34	2.56	26.54	149.86	42.34	71.83	117.89	82.52	153.89	41.34	32.56	38.88	15.67
	Min	32.42	0.89	0.15	3.41	42.25	66.82	48.65	-13.67	22.39	-36.96	-34.99	-38.13	-22.45	-6.15
	Max	130.25	81.55	9.19	87.79	535.92	214.88	260.47	316.23	286.10	527.36	79.96	83.57	83.62	34.55
Zinc	Mean	24.45	8.59	4.33	7.28	8.76	21.34	80.45	69.95	14.42	4.39	6.89	15.10	20.53	10.35
	SD	19.34	6.98	2.66	2.83	2.78	7.87	39.80	24.53	7.96	2.88	6.97	7.00	11.65	4.08
	Min	7.49	2.04	0.79	2.09	5.71	11.44	25.16	31.06	-0.25	-3.49	0.12	7.27	0.25	5.17
	Max	81.06	23.29	8.72	12.17	15.39	34.16	143.98	125.47	27.28	7.28	22.80	28.43	37.32	15.32

Note: Unit = USD per metric tonne. Source: Datastream, London Metal Exchange.

**Table 7** Descriptive statistics summary of monthly average London Metals Exchange (LME) inventory

	<i>n</i>	2000 12	2001 12	2002 12	2003 12	2004 12	2005 12	2006 12	2007 12	2008 12	2009 12	2010 12	2011 12	2012 12	2013 5
Aluminium	Mean	548.99	590.47	1148.32	1274.97	996.77	565.59	723.11	842.92	1229.06	4029.78	4454.08	4582.38	5010.40	5184.54
	SD	198.02	130.75	166.30	92.50	284.77	57.12	38.89	67.63	337.14	716.08	120.08	98.22	113.75	18.22
	Min	308.51	359.57	836.26	1129.64	700.76	500.76	674.21	712.33	943.71	2552.00	4280.64	4423.59	4826.39	5155.23
	Max	841.24	784.43	1295.63	1410.02	1436.23	687.89	784.99	930.84	2061.57	4614.67	4614.86	4810.43	5206.68	5204.12
Copper	Mean	570.40	548.45	893.02	676.27	156.13	55.27	119.59	159.26	175.97	389.02	447.10	436.75	260.79	500.61
	SD	178.78	169.22	42.88	139.17	107.26	15.36	22.57	36.75	65.43	88.46	70.36	35.17	39.49	120.53
	Min	344.62	339.36	819.24	449.38	53.69	28.11	94.90	101.89	113.71	268.52	361.25	378.70	217.42	341.29
	Max	804.69	786.28	965.30	850.23	399.04	78.48	173.01	212.45	317.89	524.52	547.56	472.68	351.98	616.54
Lead	Mean	165.07	118.52	164.88	164.15	58.61	41.53	75.97	37.15	62.12	94.12	184.64	320.45	340.82	273.59
	SD	32.71	17.58	29.22	18.05	19.54	9.07	25.80	7.30	18.93	33.87	16.65	40.99	28.32	21.93
	Min	111.93	96.65	119.04	122.03	38.30	32.52	40.95	22.85	43.32	48.05	151.80	241.66	288.95	245.16
	Max	205.82	141.60	194.86	183.91	98.75	59.39	113.84	45.98	95.08	142.53	206.19	385.90	375.82	299.93
Nickel	Mean	22.46	14.02	22.05	24.77	14.14	13.04	17.40	18.12	51.80	111.63	137.71	109.32	111.79	162.50
	SD	11.66	3.83	2.86	5.77	3.47	7.12	13.00	16.11	7.69	17.89	17.17	16.77	15.17	12.74
	Min	10.98	9.04	18.14	17.72	9.27	6.41	5.33	3.63	45.20	80.11	117.45	85.92	92.43	146.73
	Max	44.92	18.53	28.34	34.25	19.76	29.90	36.45	46.44	71.69	148.30	164.43	136.03	138.44	178.61
Tin	Mean	10.98	17.95	33.69	19.56	7.40	6.62	13.20	11.88	7.30	17.37	18.98	18.88	11.97	13.72
	SD	1.11	5.12	4.30	4.43	3.85	3.38	1.75	2.02	2.46	6.99	5.25	3.33	1.15	0.55
	Min	9.42	12.85	26.06	14.05	3.85	3.48	11.08	8.47	3.70	8.40	12.59	12.03	9.70	12.94
	Max	12.78	27.77	37.95	25.11	15.14	14.40	16.65	14.97	11.87	26.77	27.22	22.61	14.13	14.36
Zinc	Mean	237.06	305.89	583.86	691.41	721.84	549.40	218.69	81.53	150.66	379.37	589.69	796.56	981.39	1159.81
	SD	31.47	75.83	68.76	21.06	32.30	63.18	96.67	14.77	34.95	58.19	54.30	63.22	123.19	59.08
	Min	196.69	193.33	463.41	663.56	645.13	413.94	87.28	61.39	101.14	284.40	491.10	708.45	833.07	1077.09
	Max	287.40	420.27	651.52	740.72	769.46	382.61	629.02	102.98	231.08	466.01	677.29	883.94	1227.46	1216.03

Note: Unit = USD per metric tonne. Source: Datastream, London Metal Exchange.

convenience yields, but rather it is the failure to account for storage costs and inventory level dynamics across all warehouse facilities. There is also a possibility that price volatility in the estimation period partly contributes to the observation of negative convenience yields using the cost-of-carry approach. In contrast, the use of the options-based approach bounds convenience yields at zero by construction and thus avoids this fundamental error.

The empirical results in Tables 3 and 4 strongly support the theoretical view that convenience yields should more appropriately be viewed as options, rather than as an exogenous factor to be modelled (that is, as a factor used for setting an arbitrary external condition rather than seeking to achieve realistic model behaviour). Where the supply of certain commodities is confined within a rigid production cycle, realignment costs prevent holders of commodities from releasing inventories to maintain stable spot and futures price dynamics. The differences that emerge naturally correspond to a payoff attached to a claim with short maturity. We argue that this fundamental behaviour has the characteristics of a call option with stochastic exercise price (Milonas and Thomadakis 1997) and have shown that modelling convenience yields in this way provides a sensible alternative to the traditional approach. We also show that negative relationship between convenience yields and inventory holds when using the options-based approach, but is not maintained under the cost-of-carry method. These results demonstrate that the option-based model offers a more robust and consistent model for convenience yields relative to inventory levels than the cost-of-carry approach used by most commodity analysts.

## 6. Concluding remarks

The motivation for this paper was to model the dynamics of convenience yields using an options-based approach and to compare its performance with the more traditional cost-of-carry approach. It was also to determine which approach is more suitable to help understand the implied relation between inventories and convenience yields. For the options-based approach to modelling convenience yield, the value of the option is derived from a practical inability to rapidly adjust the level of inventory to meet demand. The nature and behaviour of the production processes of base metals largely prevents this degree of flexibility assumed in the cost-of-carry approach to commodity price modelling. But while the reduction of freight duration differentials between buyers and sellers across geographical regions as well as the reduction in other barriers (tariffs, quotas) is expected to result in lower option values over time, the inability to instantaneously replenish inventories at low marginal costs will result in at least some convenience yield.

We have estimated convenience yields of six major base metals using the cost-of-carry approach and the options approach and compare the strength

of the relationship with inventories through regression analysis. Our results support the hypothesis that the relationship between convenience yields and inventory levels of commodities is more appropriately explained using an options-based approach, given the asymmetric behaviour of inventory levels relative to market prices.

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