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# Testing for spectral Granger causality

Hüseyin Tastan  
Department of Economics  
Yildiz Technical University  
Istanbul, Turkey  
tastan@yildiz.edu.tr

**Abstract.** In this article, I introduce a command (`bcgcausality`) to implement Breitung and Candelon’s (2006, *Journal of Econometrics*, 132: 363–378) Granger causality test in the frequency domain.

**Keywords:** `st0417`, `bcgcausality`, Granger causality, Geweke measure, VAR

## 1 Introduction

Granger (1969) developed a concept of causality between two variables (or two blocks of variables) that is based on two assumptions: 1) cause occurs before effect and 2) cause contains information about effect.<sup>1</sup> The concept of Granger causality implies that the causal variable can be used to forecast the future values of the effect variable. Denoting the cause and effect variables by  $y_t$  and  $x_t$ , respectively, we say that  $y_t$  Granger causes  $x_t$ , given all the information at time  $t$  on both series, and that the past of  $y_t$  helps predict the future value of  $x_t$ .

Geweke (1982) introduced the measure of linear feedback between  $y_t$  and  $x_t$ , which decomposed the information content into three components: feedback from  $y_t$  to  $x_t$ , feedback from  $x_t$  to  $y_t$ , and instantaneous feedback between  $x_t$  and  $y_t$ .<sup>2</sup> Similar measures of feedback in both time domain and frequency domain were also provided by Hosoya (1991). Geweke (1984) further extended and developed conditional measures of linear feedback allowing for the presence of a third series, and he showed that directional measures can be decomposed by frequency. Frequency domain or spectral analysis aims at decomposing variability in a time series into its periodic components, allowing us to determine relatively more important frequencies that contribute to fluctuations in the variable. Moreover, spectral causality or feedback measures may be very useful if causal links between variables change according to frequency (that is, short run or long run). Spectral measures of linear feedback (or causality) are defined using the spectral density of the “effect” variable, which is based on the moving-average representation of the vector autoregression (VAR). As a result, frequency-wise measures and the resulting Wald statistics are complicated nonlinear functions of the parameters of the VAR model. This, in turn, complicates the statistical inference for feedback measures in the frequency

---

1. In his Nobel lecture, Granger notes that the two components of the definition of causality (the precedence of cause before effect and the information content) were based on the definition by Norbert Wiener (Granger 2003).

2. A Stata implementation of Geweke’s measures in time domain is provided by Dicle and Levendis (2013).

domain because one must apply the bootstrap method or other numerical approximation methods, which can be computationally burdensome.

Building on Granger (1969), Geweke (1982), and Hosoya (1991), Breitung and Candelon (2006) developed a Granger causality test in frequency domain that is easier to implement. The test can be used to determine whether a particular component of the “cause” variable at frequency  $\omega$  is useful in predicting the component of the “effect” variable at the same frequency one period ahead.

In this article, I suggest a command (`bcbgcausality`) to implement the test, and I illustrate its usage with examples. Following the exposition and notation in Breitung and Candelon (2006), we let  $\mathbf{Y}_t = (x_t, y_t)'$  be a covariance-stationary vector time series that can be represented by a finite-order VAR( $p$ ) process,

$$\Theta(L)\mathbf{Y}_t = \varepsilon_t \quad (1)$$

where  $\Theta(L) = \mathbf{I}_2 - \Theta_1 L - \Theta_2 L^2 - \dots - \Theta_p L^p$  is a  $2 \times 2$  lag polynomial with the back-shift operator  $L^i \mathbf{Y}_t = \mathbf{Y}_{t-i}$ ;  $\mathbf{I}_2$  is a  $2 \times 2$  identity matrix;  $\Theta_i$   $i = 1, 2, \dots, p$ , is a  $2 \times 2$  coefficient matrix associated with lag  $i$ ; and  $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$  denotes a vector white-noise process, with  $E(\varepsilon_t) = \mathbf{0}$  and positive-definite covariance matrix  $\Sigma = E(\varepsilon_t \varepsilon_t')$ . Applying Cholesky factorization,  $\mathbf{G}'\mathbf{G} = \Sigma^{-1}$  (where  $\mathbf{G}$  is a lower-triangular matrix), we can write a moving-average representation of the system in (1) as

$$\begin{aligned} \begin{bmatrix} x_t \\ y_t \end{bmatrix} &= \Phi(L)\varepsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \\ &= \Psi(L)\eta_t = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} \end{aligned}$$

where  $\eta_t = \mathbf{G}\varepsilon_t$ ,  $E(\eta_t \eta_t') = \mathbf{I}$ ,  $\Phi(L) = \Theta(L)^{-1}$ , and  $\Psi(L) = \Phi(L)\mathbf{G}^{-1}$ .

Using the Fourier transformations of the moving-average polynomial terms, we can write the spectral density of  $x_t$  as

$$f_x(\omega) = \frac{1}{2\pi} \{ |\Psi_{11}(e^{-i\omega})|^2 + |\Psi_{12}(e^{-i\omega})|^2 \}$$

Geweke's (1982) measure of linear feedback from  $y_t$  to  $x_t$  at frequency  $\omega$  is defined as

$$M_{y \rightarrow x}(\omega) = \log \left\{ \frac{2\pi f_x(\omega)}{|\Psi_{11}(e^{-i\omega})|^2} \right\} = \log \left\{ 1 + \frac{|\Psi_{12}(e^{-i\omega})|^2}{|\Psi_{11}(e^{-i\omega})|^2} \right\}$$

If  $|\Psi_{12}(e^{-i\omega})| = 0$ , then  $M_{y \rightarrow x}(\omega)$  will be 0. This means that  $y_t$  does not Granger cause  $x_t$  at frequency  $\omega$ . Because the asymptotic distribution of the Wald statistic is a complicated nonlinear function of the VAR parameters, testing the null hypothesis

$$H_0: M_{y \rightarrow x}(\omega) = 0$$

may be difficult in practice. Breitung and Candelon (2006) proposed a simple approach to test  $H_0$ . They show that when  $|\Psi_{12}(e^{-i\omega})| = 0$ , we also have  $M_{y \rightarrow x}(\omega) = 0$ , which

means we can say that  $y_t$  does not Granger cause  $x_t$  at frequency  $\omega$  if the following condition is satisfied:

$$|\Theta_{12}(e^{-i\omega})| = \left| \sum_{k=1}^p \theta_{12,k} \cos(k\omega) - \sum_{k=1}^p \theta_{12,k} \sin(k\omega)i \right| = 0$$

Here  $\theta_{12,k}$  is the  $(1, 2)$ -element of  $\Theta_k$ . In this case, necessary and sufficient conditions for  $|\Theta_{12}(e^{-i\omega})| = 0$  are

$$\begin{aligned} \sum_{k=1}^p \theta_{12,k} \cos(k\omega) &= 0 \\ \sum_{k=1}^p \theta_{12,k} \sin(k\omega) &= 0 \end{aligned}$$

Breitung and Candelon (2006) reformulated these restrictions by rewriting the equation for  $x_t$  in the VAR( $p$ ) system,

$$x_t = c_1 + \alpha_1 x_{t-1} + \cdots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p} + \varepsilon_{1t} \quad (2)$$

where  $\alpha_j = \theta_{11,j}$  and  $\beta_j = \theta_{12,j}$ . The null hypothesis of  $M_{y \rightarrow x}(\omega) = 0$  is equivalent to

$$H_0: \mathbf{R}(\omega)\boldsymbol{\beta} = 0 \quad (3)$$

where  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)'$  and  $\mathbf{R}(\omega)$  is a  $2 \times p$  restriction matrix.

$$\mathbf{R}(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \dots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \dots & \sin(p\omega) \end{bmatrix}$$

Because these are simple linear restrictions, the usual Wald statistic can be used. Let  $\boldsymbol{\gamma} = [c_1, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_p]'$  be  $q = (2p + 1) \times 1$  vector of parameters, and let  $\mathbf{V}$  be a  $q \times q$  covariance matrix from the unrestricted regression (2). Then the Wald statistic is

$$W = (\mathbf{Q}\boldsymbol{\gamma})'(\mathbf{Q}\mathbf{V}\mathbf{Q}')^{-1}(\mathbf{Q}\boldsymbol{\gamma}) \sim \chi_2^2 \quad (4)$$

where  $\mathbf{Q}$  is  $2 \times q$  restriction matrix such that

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0}_{2 \times (p+1)} & \vdots & \mathbf{R}(\omega) \end{bmatrix}$$

To prevent spurious or indirect causality, we can extend the framework to the case of additional variables. In this case, the frequency test is computed conditional on these variables. A natural way of conditioning is to include lagged values of additional variables in the test regression, as suggested by Geweke (1984). For simplicity, assume that there is only one additional variable,  $z_t$ . To test the hypothesis  $H_0: M_{y \rightarrow x|z}(\omega) = 0$ , we can run the following regression:

$$x_t = c_1 + \sum_{j=1}^p \alpha_j x_{t-j} + \sum_{j=1}^p \beta_j y_{t-j} + \sum_{j=1}^p \delta_j z_{t-j} + e_t$$

We can then apply the testing procedure on the parameters of lagged  $y_t$ , as described above. Hosoya (2001), on the other hand, argued that Geweke's approach would not eliminate a third-series effect and suggested an alternative conditional measure of causality. Let  $w_t$  be the projection residuals from the regression of  $z_t$  on  $x_t, x_{t-1}, \dots, x_{t-p}, y_t, y_{t-1}, \dots, y_{t-p}$ , and  $z_{t-1}, \dots, z_{t-p}$ . According to Hosoya (2001), conditional Granger causality can be tested in the following model:

$$x_t = c_1 + \sum_{j=1}^p \alpha_j x_{t-j} + \sum_{j=1}^p \beta_j y_{t-j} + \sum_{j=0}^p \delta_j w_{t-j} + e_t$$

As mentioned by Breitung and Candelon (2006), in Hosoya's (2001) approach, the variable  $w_t$  carries the contemporaneous information in  $z_t$ , which may not fit well with Granger causality. Furthermore, ignoring the contemporaneous information in  $z_t$ , as in Geweke's approach, may also potentially lead to spurious causality.

So far, the variables in the system were assumed to be  $I(0)$  and can be represented by a stationary VAR. If the variables in the system appear to be nonstationary, then one must establish integration and cointegration properties of the data. If each variable is  $I(1)$ , then the system must be tested for cointegration, for example, by using the Johansen test. If the variables are cointegrated, then at least one-directional Granger causality exists between variables. If the system appears not to be cointegrated, then we can estimate a VAR in first-differences and implement Granger causality tests.

Toda and Yamamoto (1995) and Dolado and Lütkepohl (1996) proposed a modified Wald test for Granger causality that does not rely on pretesting for cointegration. They suggested that the usual Wald test statistic will be valid for Granger causality on  $p$ -lags of a variable in an overfitted  $\text{VAR}(p + d_{\max})$  model where  $d_{\max}$  is the highest order of integration in the system. Breitung and Candelon (2006) suggested that this approach can also be used for the frequency domain test. Assuming that  $d_{\max} > 0$ , we can write the test regression as

$$x_t = c_1 + \sum_{j=1}^p \alpha_j x_{t-j} + \sum_{j=1}^p \beta_j y_{t-j} + \sum_{k=p+1}^{p+d_{\max}} \alpha_k x_{t-k} + \sum_{k=p+1}^{p+d_{\max}} \beta_k y_{t-k} + e_t$$

The null hypothesis of  $M_{y \rightarrow x}(\omega) = 0$  [see (3)] involving only  $\beta_j$ ,  $j = 1, \dots, p$  can be tested using the Wald statistic (4). Note that the coefficients on the additional lagged variables are not included in the computation of the Wald statistic. For example, for a bivariate system with  $I(1)$  variables, suppose that the optimal lag order is three (chosen by a data-dependent information criterion, such as the Akaike information criterion). Then, one fits a  $\text{VAR}(4)$  model but conducts the Granger causality tests using only three lags.

## 2 The `bcgcausality` command

### 2.1 Syntax

The calling syntax to test the null hypothesis that there is no Granger causality from *causevar* to *effectvar* conditional on `condvars` at a particular frequency is

```
bcgcausality effectvar causevar [condvars] [if] [in] [, varlag(integer)
    frequency(#) condtype(string) exog(varlist) nograph]
```

### 2.2 Options

`varlag(integer)` specifies the number of lags. The default is `varlag(3)`. The minimum number of lags is 2.

`frequency(#)` specifies one frequency at which the Granger causality test statistic is to be computed (between 0.01 and 3.14). The default is to compute all frequencies.

`condtype(string)` specifies the type of conditioning, either `geweke` or `hosoya`. The default is `condtype(geweke)`.

`exog(varlist)` lists exogenous variables.

`nograph` requests no graphical output. The default is to graph test statistics over all frequencies.

## 3 Usage

### ► Example

In this example, we illustrate the method with the quarterly U.S. macroeconomic data covering the period 1959q1–1998q4 used by Breitung and Candelon (2006) in their empirical application. The dataset contains two variables: `growth` (logarithmic first-difference of real gross domestic product) and `spread` (difference between long-run and short-run interest rates).

```
. use bcdta.dta
. bcgcausality growth spread, varlag(6)
```

---

```
Granger-causality from spread to growth
```

This command computes test statistics for all frequencies in the interval  $(0, \pi)$  and produces the graphical output in figure 1. The test statistics are significant at the 5% level for frequencies less than 0.68 and at the 10% level for frequencies less than 0.79. These frequencies correspond to a wavelength of more than  $2\pi/\omega \approx 2$  years. The test statistics are also significant in the range  $\omega \in [1.77, 2.33]$  corresponding to a wavelength between 2.7 and 3.5 quarters.



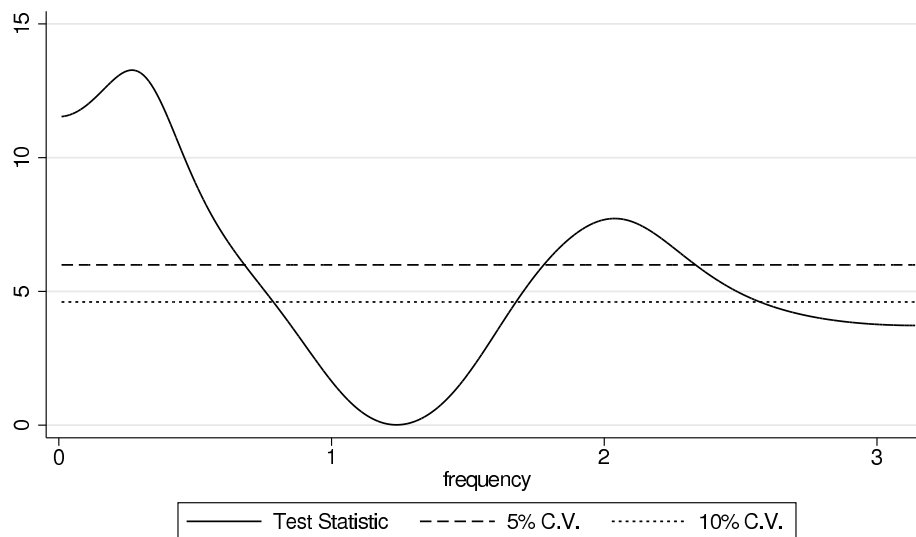


Figure 1. Breitung–Candelon (2006) frequency domain Granger causality test from spread to growth

```
. return list
macros:
      r(varlag) : "6"
      r(cmd)    : "bcgcausality"

matrices:
      r(W)      : 314 x 3

. matrix list r(W)
r(W) [314,3]
      frequency  teststat  pvalue
r1          .01  11.536505  .00312521
r2          .02  11.549343  .00310522
r3          .03  11.570684  .00307226
r4          .04  11.600444  .00302688
r5          .05  11.638498  .00296983
(output omitted)
r312        3.12   3.7278866  .15505997
r313        3.13   3.727229  .15511097
r314        3.14   3.7269678  .15513122
```

The `bcgcausality` command returns test results in the  $314 \times 3$  matrix `r(W)`. Frequency, Wald statistics, and  $p$ -values are organized in the columns as shown above. To perform the test for one frequency (for example,  $\omega = 0.78$ ), we type

```
. bcgcausality growth spread, varlag(6) freq(0.78)
```

---

Granger-causality from spread to growth

---

H0: No Granger-causality from spread to growth at frequency w = 0.78  
Wald test statistic = 4.6933  
pvalue = 0.0957

---

No graphical output

---

```
. return list
macros:
      r(varlag) : "6"
      r(cmd)    : "bcgcausality"
matrices:
      r(W)      : 1 x 3
. matrix list r(W)
r(W)[1,3]
      frequency  teststat    pvalue
r1             .78  4.6933035  .09568902
```

◀

### ► Example

Here we use the quarterly German macrodataset to compute the Breitung–Candelon test using a modified Wald test suggested by Toda and Yamamoto (1995). We first implement Granger causality tests in time domain.

```
. webuse lutkepohl2
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)
. tsset
      time variable: qtr, 1960q1 to 1982q4
      delta: 1 quarter
. generate trend = _n
```

Augmented Dickey–Fuller and Dickey–Fuller generalized least-squares tests (not reported to save space) suggest that each variable in the trivariate system (income, consumption, and investment in natural logs) is integrated of order 1. Having determined that  $d_{\max} = 1$ , we proceed to VAR order ( $p$ ) selection, which can easily be done using Stata’s `varsoc` command.

```
. varsoc ln_consump ln_inc ln_inv
Selection-order criteria
Sample: 1961q1 - 1982q4                Number of obs   =      88
```

lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	278.793				3.8e-07	-6.26801	-6.23399	-6.18356
1	724.27	890.95	9	0.000	1.9e-11	-16.1879	-16.0519*	-15.8501*
2	736.624	24.708	9	0.003	1.7e-11	-16.2642	-16.026	-15.673
3	748.837	24.426*	9	0.004	1.6e-11*	-16.3372*	-15.997	-15.4927
4	754.18	10.687	9	0.298	1.8e-11	-16.2541	-15.8118	-15.1562

```

Endogenous:  ln_consump ln_inc ln_inv
Exogenous:   _cons

```

The likelihood-ratio test, the Akaike information criterion, and the final prediction-error criteria suggest that  $p = 3$ , whereas the Hannan–Quinn information criterion and the Schwarz–Bayesian information criterion suggest that  $p = 1$ . Thus we fit a VAR( $p + d_{\max} = 4$ ) model but implement the test using  $p = 3$  lags. To accomplish this in Stata, we use the `var` command and add the fourth lags of variables as if they were exogenous.

```
. var ln_consump ln_inc ln_inv, lags(1/3) exog(L4.(ln_consump ln_inc ln_inv) trend)
(output omitted)
```

We also include a time trend in the VAR system.

```
. vargranger
Granger causality Wald tests
```

Equation	Excluded	chi2	df	Prob > chi2
ln_consump	ln_inc	24.143	3	0.000
ln_consump	ln_inv	5.8171	3	0.121
ln_consump	ALL	30.077	6	0.000
ln_inc	ln_consump	3.2211	3	0.359
ln_inc	ln_inv	5.9239	3	0.115
ln_inc	ALL	9.0822	6	0.169
ln_inv	ln_consump	5.5939	3	0.133
ln_inv	ln_inc	.78396	3	0.853
ln_inv	ALL	15.657	6	0.016

Results indicate that there is unidirectional Granger causality from income to consumption. To run the frequency domain test, we type

```
. bcgcausality ln_consump ln_inc ln_inv, varlag(3)
> exog(L4.(ln_consump ln_inc ln_inv) trend)
```

---

```

Conditioning type is = geweke
Granger-causality from ln_inc to ln_consump conditional on ln_inv

```

Similarly, to test Granger noncausality from consumption to income, we type

```
. bcgcausality ln_inc ln_consump ln_inv, varlag(3)
> exog(L4.(ln_consump ln_inc ln_inv) trend)
```

---

Conditioning type is = geweke  
Granger-causality from ln\_consump to ln\_inc conditional on ln\_inv

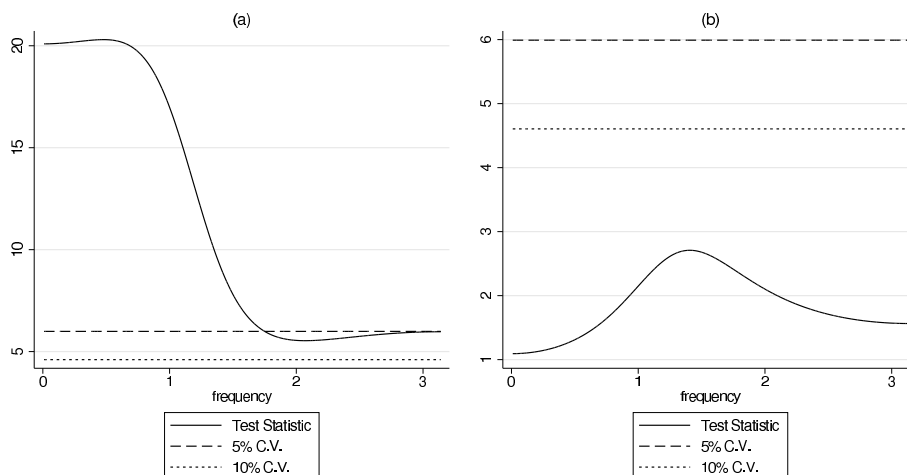


Figure 2. Frequency domain Granger causality test results conditional on investment: (a) from income to consumption, (b) from consumption to income

Figure 2 displays Wald statistics over all frequencies  $\omega \in (0, \pi)$ . Not surprisingly, test statistics for the Granger noncausality from income to consumption [figure 2(a)] are significant at the 10% level for all frequencies. The null hypothesis of no Granger causality is rejected at the 5% significance level for low-frequency components in the range  $\omega \in (0, 1.75)$  corresponding to wavelengths of more than 3.6 quarters or about a year. Consumption does not Granger cause income, conditional on investment, at all frequencies [figure 2(b)]. The test results according to Hosoya-type conditioning are not reported because they are qualitatively similar.

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**About the author**

Hüseyin Tastan is a professor of economics and econometrics at the Department of Economics, Yildiz Technical University, Istanbul, Turkey.