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# Best subsets variable selection in nonnormal regression models

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**Abstract.** We present a new program, gvselect, that helps users perform variable selection in regression. Best subsets variable selection is performed and provides the user with the best combinations of predictors for each level of model complexity. The leaps-and-bounds (Furnival and Wilson, 1974, *Technometrics* 16: 499–511) algorithm is applied using the log likelihoods of candidate models. This allows the user to perform variable selection on a wide variety of normal and non-normal regression models. Our method is described in Lawless and Singhal (1978, *Biometrics* 34: 318–327).

Keywords: st0413, gvselect, regress, vselect, variable selection

# 1 Theory and motivation

Redundant predictors in a regression can yield an increase in the log likelihood and less biased predictions, but they may increase the variance of predictions. In this section, we will discuss the problem of variable selection and decide which predictors to use in a regression. Then, we will introduce a new command for performing variable selection, gvselect. We will follow this with examples of gvselect using real datasets.

In settings with few predictors, the likelihood-ratio (LR) test can be used to determine whether certain groups of predictors should be included in the model. We divide the predictors into two groups. One group, "the base group", will be included in our model. The other group, "the suspect group", may or may not be included in the model. We are not yet sure. We call the regression model containing all predictors in both the base and suspected groups "the full model". The regression model containing only the base predictors is called "the reduced model".

Let  $L_f$  and  $L_r$  be the log-likelihood values associated with the full and reduced models, respectively. Let  $n_f$  and  $n_r$  be the respective number of parameters in the full and reduced models. The LR test statistic is  $LR = -2(L_r - L_f)$ . Under the null hypothesis that the reduced model is true (all the predictor coefficients for the suspected group are zero), LR has an  $\chi^2$  distribution with  $n_f - n_r$  degrees of freedom. Accepting the null hypothesis leads us to use the reduced model as our regression model.

Rejecting the null hypothesis indicates that we should not ignore the predictors in the suspected group (at least one of the predictor coefficients is not zero). We can then reperform the test using subsets of the suspected group to determine which predictors to include in the model. The LR test may be easily performed in Stata via lrtest (see [R] lrtest).

This article is about settings with a large number of predictors. When the suspected predictor list grows large, it is not feasible to use the LR test method to determine the best regression model.

#### 1.1 Information criteria

The definition of the best, or the optimal, model will vary with the information criteria used for evaluating models. An information criterion is a function of a regression model's explanatory power and complexity. The model's explanatory power (goodness of fit) increases the criterion in the desirable direction, while the complexity of the model counterbalances the explanatory power and moves the criterion in the undesirable direction. Information criteria help us determine an optimal tradeoff between prediction accuracy and precision.

Akaike's (1974) information criterion (AIC) is a popular criterion for comparing different models. As AIC decreases, the model becomes more desirable. The explanatory power of the model is measured by the maximized log-likelihood of the predictor coefficients (assuming a normal model) and error variance. The complexity penalization comes from an addition of the number of parameters p.

$$AIC = 2(-L_r + p)$$

The Bayesian information criterion (BIC) was proposed by Schwarz (1978). Raftery (1995) provides another development and motivation for the criterion. BIC is similar to AIC but adjusts the penalty term for complexity based on the sample size.

$$BIC = -2L_r + p\log n$$

AIC and BIC can be estimated within Stata using estat ic. There is debate over whether AIC should be used in preference to BIC. A comparison of page 46 of Simonoff (2003) and page 235 of Hastie, Tibshirani, and Friedman (2009) demonstrates this. We find that selection based on BIC yields more parsimonious models here, but that does not mean these models are superior to the more complex models that are selected based on AIC.

The gvselect command calculates only AIC and BIC. Those interested in other information criteria and linear regression models should investigate the vselect command. This command was introduced in Lindsey and Sheather (2010b). It performs linear regression variable selection using the information criteria AIC, corrected AIC (Hurvich and Tsai 1989), BIC,  $R^2$  adjusted (Sheather 2009), and Mallows's C (Izenman 2008).

#### 1.2 Variable selection algorithms

A variety of algorithms have been created to perform variable selection using information criteria like AIC and BIC. These variable selection algorithms take the specification of the full model and output a reduced model.

Stepwise selection algorithms use backward elimination or forward selection and add or remove predictors iteratively. They may yield a reasonable model but are not guaranteed to select an information criterion's optimal model.

The algorithm implemented in gvselect is guaranteed to select the best models for BIC and AIC. At each level of complexity (number of parameters p), the optimal model is the model with the largest log likelihood. gvselect applies the leaps-andbounds algorithm (Furnival and Wilson 1974) to the log likelihoods of the candidate models. The algorithm outputs the model with the largest log likelihood for each level of complexity. This approach is described in Lawless and Singhal (1978). The AIC and BIC of these final models can then be compared. Furnival and Wilson (1974) focus on doing variable selection for linear regression models, and residual sums of squares are used instead of log likelihoods. Lawless and Singhal (1978) generalized their method to nonnormal regression models, using log likelihood instead of residual sum of squares to choose models.

As Furnival and Wilson (1974) explain, the leaps-and-bounds algorithm organizes all the possible models in tree structures and scans through them, skipping (or "leaping") over those that are definitely not optimal. The original description of the algorithm is done with large amounts of Fortran code. Ni and Huo (2005) provide an easier description of the original algorithm. They use a pair tree where each node has two subsets of predictors.

When the algorithm examines a node, it compares the regressions of each pair of predictor lists with the optimal regressions of each predictor size that have already been conducted. Depending on the results, all or some of the descendants of that node can be skipped by the algorithm. The initial ordering of the predictors and their smart placement in sets within the nodes ensures that the algorithm completes after finding the optimal predictor lists and examining only a fraction of all possible regressions.

The predictor lists in the pair tree are created based on an automatic ordering of all the predictors by their  $\chi^2$  test statistic value in the original regression. The first node,  $(\phi, \{1, \ldots, k\})$ , contains the empty set and all k predictors. Let  $(\Omega_1, \Omega_2)$  be a node in the pair tree. If  $(\Omega_1, \Omega_2)$  is not the first child of its parent, then it has children. Denote the first child of  $(\Omega_1, \Omega_2)$  as  $(\Omega_{11}, \Omega_{21})$ . The subset  $\Omega_{11}$  is obtained by removing the last predictor from  $\Omega_2$ . The subset  $\Omega_{21}$  is obtained by removing the second-to-last predictor from  $\Omega_2$ . For  $i = 2, \ldots, k$ , denote child i of  $(\Omega_1, \Omega_2)$  as  $(\Omega_{1i}, \Omega_{2i})$ . The subset  $\Omega_{1i}$  is obtained by removing the *i*th from the last through the last predictor of  $\Omega_2$ . So two predictors are removed for a second child, three for a third, etc. The subset  $\Omega_{2i}$  is obtained by removing the i+1th from the last predictor of  $\Omega_2$ . Figure 1 shows the pair tree for a regression with five predictors.

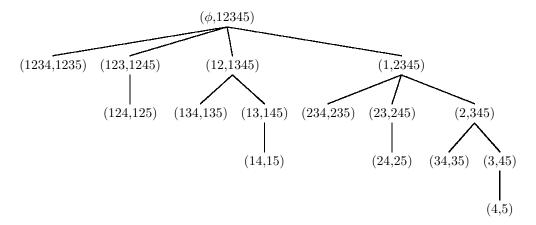


Figure 1. Pair tree for five predictors

We traverse the tree from the root node down and left to right. Let  $L_i$  be the maximum log likelihood for *i* predictors that has so far been calculated in the traversal. We denote the log likelihood under the predictors in  $\Omega_a$  by  $L_{\Omega_a}$ . If  $\Omega_b \subseteq \Omega_a$ , then  $L_{\Omega_b} \leq L_{\Omega_a}$ . Using this inequality and the pair tree definition, we obtain two rules of traversal. In the first rule, if  $L_{\Omega_2} < L_{|\Omega_1|}$ , then we can skip all the descendants of  $(\Omega_1, \Omega_2)$ . None of the descendant models have larger likelihoods than those that have already been found. In the second rule, we can skip the first *i* descendants of  $(\Omega_1, \Omega_2)$  if  $L_{|\Omega_2|-i-1} < L_{\Omega_2} \leq L_{|\Omega_2|-i}$ . Using these rules, we can find the models with the highest log likelihood for every complexity level and avoid fitting all possible models.

#### 1.3 Model validity and cross-validation

Note that the output of the leaps-and-bounds algorithm and the LR test are not very meaningful unless the full model is a valid regression model. A regression model is valid if the assumptions to perform its significance tests are met. The assumptions for linear regression models can be assessed using residual plots, scale-location plots, etc. Details can be found in Sheather (2009). Diagnostics for logistic regression models are discussed in Hosmer, Lemeshow, and Sturdivant (2013). Diagnostics for categorical models are discussed in Agresti (2013).

Marginal model plots were proposed by Cook and Weisberg (1997). They are a graphical diagnostic tool that can be applied to many regression models. They allow visual comparison of the parametric model fit of the conditional mean with a non-parametric estimate of the conditional mean. The proper specification of the model is corroborated when the two estimates correspond. When they differ, it suggests the model is misspecified. The mmp command, developed in Lindsey and Sheather (2010a), can be used to draw marginal model plots in Stata.

We must also note that inference on the models produced by the leaps-and-bounds algorithm is not equivalent to the inference on the same models that the users find independently without consulting the algorithm. Each step of a variable selection algorithm will fit one or more models and then make an inference on the next step using information from these models. So, in addition to inferences made using the final model, many preliminary inferences are made during variable selection.

This will affect the significance levels of the final model. The situation is similar to performing multiple comparisons on the factor means after an analysis of variance tells you there is a significant effect. Each of these comparisons should be evaluated at a different significance level than that of the original factor effect.

Cross-validation methods can be used to handle this multiple-inference difficulty. These methods generally perform variable selection on subsets of the data and then use an average measure of the results on these subsets to find the final model. They may also split the data into two parts, performing variable selection on one part (train) and using the other for evaluating the resulting model (test). Details of this method and a general discussion of the multiple-inference problem in variable selection are given in Sheather (2009). The variable selection method that we use here may be applied under certain cross-validation techniques.

In the next section, we will provide the full syntax of the gvselect command. Then, we will demonstrate how to use gvselect with two real-world examples. First, we determine the optimal linear regression model for measuring diabetes progression. Second, we find the best Poisson regression model for predicting the number of doctor visits an individual will have in a two-week period.

# 2 Use and examples

The gvselect command has the following syntax:

```
gvselect <term> varlist \ [, \underline{nm}odels(\#)]: est_cmd
```

*est\_cmd* is a call to an estimation command that returns a log-likelihood numeric result. *est\_cmd* contains instances of <term>. These are replaced by variables in *varlist* to determine the best subsets of *varlist* for fitting the model of interest. If the nmodels() option is specified with value #, the best # models are reported for each level of model complexity.

The gvselect command is straightforward in use. We will first demonstrate on a dataset highlighted in Ni and Huo (2005), the diabetes data, which were introduced in Efron et al. (2004). Then, we will use gvselect on data from Cameron and Trivedi (1986) to perform variable selection for a Poisson regression predicting the number of doctor visits made in a two-week period.

#### 2.1 Diabetes data

The diabetes study data (Efron et al. 2004) contain information on 442 diabetes patients. They are measured on 10 baseline predictor variables and a measure of disease progression. The predictors are age, sex, body mass index (bmi), blood pressure (bp), and six serum measurements (s1-s6). The progression variable, prog, is our models' response and was recorded a year after the 10 baseline predictors. We will estimate the parameters of the linear regression of prog on the 10 predictors.

Evaluation of the residual plots and other diagnostics do show the full model is valid. But, as we see in the variance inflation factors, there are serious multicollinearity problems. In particular, s1-s5 all exhibit variance inflation factors that exceed 8.

. use diabetes	3						
. regress prog	g age-s6						
Source	SS	df	MS	Numb	er of obs	=	442
				- F(10	, 431)	=	46.27
Model	1357023.32	10	135702.332	2 Prob	> F	=	0.0000
Residual	1263985.8	431	2932.68168	8 R-sq	uared	=	0.5177
				- Adj	R-squared	=	0.5066
Total	2621009.12	441	5943.33135	5 Root	MSE	=	54.154
prog	Coef.	Std. Err.	t	P> t	[95% Cont	f.	Interval]
age	0363613	.2170414	-0.17	0.867	4629526		.3902301
sex	-22.85965	5.835821	-3.92	0.000	-34.32986		-11.38944
bmi	5.602962	.7171055	7.81	0.000	4.193503		7.012421
bp	1.116808	.2252382	4.96	0.000	.6741061		1.55951
s1	-1.089996	.5733318	-1.90	0.058	-2.21687		.0368782
s2	.7464501	.5308344	1.41	0.160	296896		1.789796
s3	.3720042	.7824638	0.48	0.635	-1.165915		1.909924
s4	6.533831	5.958638	1.10	0.273	-5.177772		18.24543
s5	68.48312	15.66972	4.37	0.000	37.68454		99.28169
s6	.2801171	.273314	1.02	0.306	257077		.8173111
_cons	-334.5671	67.45462	-4.96	0.000	-467.148		-201.9862

. estat vif

Variable	VIF	1/VIF
s1 s2	59.20 39.19	0.016891
s2 s3	15.40	0.064926
s5	10.08	0.099246
s4	8.89	0.112473
bmi	1.51	0.662499
s6	1.48	0.673572
bp	1.46	0.685200
sex	1.28	0.782429
age	1.22	0.821486
Mean VIF	13.97	

Now, we invoke gvselect on the data. Our model choices match those of Ni and Huo (2005). The choices of best model predictor sizes were 5 for BIC and 6 for AIC. The 6-predictor model seems like a prudent choice, given the closeness of the optimal BIC value to the BIC value under 6 predictors.

```
. gvselect <term> age-s6: regress prog <term>
Optimal models:
   # Preds
                          AIC
                                    BIC
                 LL
        1 -2454.019 4912.038 4920.221
        2 -2411.199 4828.398 4840.672
        3 -2402.613 4813.226
                               4829.591
        4 -2397.481 4804.963
                               4825.419
        5 -2390.132 4792.264
                               4816.811
        6 -2387.302 4788.603 4817.243
        7 -2386.66
                     4789.32 4822.051
        8 -2386.12 4790.241 4827.062
        9 -2386.007 4792.015 4832.928
        10 -2385.993 4793.986
                                4838.99
predictors for each model:
1 : bmi
2 : bmi s5
3 : bmi bp s5
4 : bmi bp s5 s1
5 : bmi bp s5 sex s3
6 : bmi bp s5 sex s1 s2
7 : bmi bp s5 sex s1 s2 s4
8 : bmi bp s5 sex s1 s2 s4 s6
9 : bmi bp s5 sex s1 s2 s4 s6 s3
10 : bmi bp s5 sex s1 s2 s4 s6 s3 age
```

Using the 6-predictor model on the 442-patient dataset, we still find some high variance-inflation factors between the first and second serum variables. Note that they are far lower in magnitude than under the full model.

. estat vif		
Variable	VIF	1/VIF
s1	8.81	0.113561
s2	7.37	0.135750
s5	2.20	0.454745
bmi	1.47	0.678813
bp	1.34	0.743677
sex	1.23	0.815832
Mean VIF	3.74	

If we are concerned about this multicollinearity, we can try the 5-predictor model, which BIC chose.

. estat vif		
Variable	VIF	1/VIF
s5	1.46	0.684663
s3	1.46	0.685455
bmi	1.44	0.692867
bp	1.35	0.742260
sex	1.24	0.807833
Mean VIF	1.39	

### 2.2 Poisson model for doctor visits

The doctor visits dataset was examined in Cameron and Trivedi (1986). It contains data on the number of doctor visits in the past 2 weeks for a sample of 5,190 adults in the Australian Health Survey 1977–1978 (Australian Burea of Statistics 1978). We will model the number of doctors visits using a Poisson regression on 12 predictors, which include gender, mean-adjusted age, age squared, income, and insurance status. Private insurance is indicated by the binary variable **private**. Insurance provided by the government because of low income is indicated by the binary variable **inslow**. Insurance provided by the government for retirees, the disabled, and veterans is indicated by the binary variable **insrdv**. The variable **hscore** is a health questionnaire score. The number of illnesses reported in the last week (**illness**) and whether the individual has had days of reduced activity because of illness or injury (**daysred**) are also included as predictors. Finally, indicators of whether an individual has a chronic condition that either limits his or her activity (**chcondlim**) or does not (**chcondnlim**) are included as predictors.

We estimate the parameters of the full model with poisson and then use the mmp command to draw marginal model plots to check the model's validity. The mean prediction for poisson, n is specified in predict(). We use a lowess estimate of the conditional mean by specifying lowess in the smoother() option. We draw a marginal model plot for the linear form and all continuous predictors by specifying linear and predictors, respectively.

```
. use docvisits
(Doctor visits)
. poisson docvis sex age agesq income private inslow insrdv illness hscore
> chcondlim chcondnlim daysred
Iteration 0: log likelihood = -3358.2094
Iteration 1: log likelihood = -3310.9148
Iteration 2: log likelihood = -3310.8564
Iteration 3: log likelihood = -3310.8564
```

Poisson regres Log likelihood		1		Number LR chi2 Prob > Pseudo	2(12) = chi2 =	5,190 1344.68 0.0000 0.1688
docvis	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
sex	.115277	.0562067	2.05	0.040	.0051138	.2254401
age	.8193558	.4051036	2.02	0.043	.0253674	1.613344
agesq	.0164819	.7585332	0.02	0.983	-1.470216	1.50318
income	1828918	.0873301	-2.09	0.036	3540557	011728
private	.1019069	.0714919	1.43	0.154	0382146	.2420285
inslow	4774047	.1802192	-2.65	0.008	8306278	1241816
insrdv	.134944	.0922363	1.46	0.143	0458357	.3157238
illness	.1338196	.0188996	7.08	0.000	.096777	.1708623
hscore	.0527531	.009596	5.50	0.000	.0339453	.0715609
chcondlim	.1804489	.0812719	2.22	0.026	.021159	.3397389
chcondnlim	.0842826	.0661212	1.27	0.202	0453124	.2138777
daysred	1.442045	.0557248	25.88	0.000	1.332826	1.551264
_cons	-2.07373	.1174598	-17.65	0.000	-2.303947	-1.843513

. mmp, mean(n) smoother(lowess) linear predictors

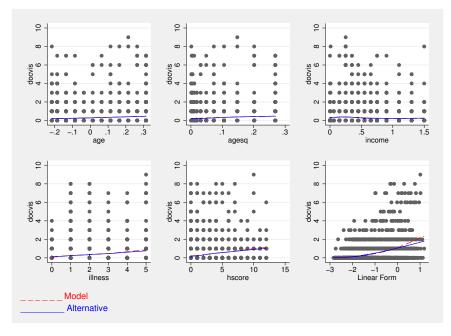


Figure 2. Marginal model plots for full model

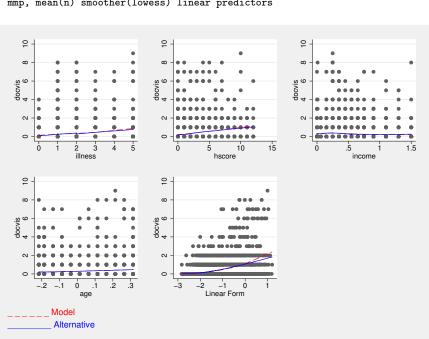
Figure 2 contains six marginal model plots. The model fit and the alternative fit are generally close for each continuous regressor and the linear form estimate. The model fit line is a lowess estimate of the conditional mean estimate from the model on the horizontal axis variable. The alternative fit line is a lowess estimate of the response (observed number of doctor visits) on the vertical axis variable. The closeness of the model and alternative fit lines corroborates the validity of the full model.

Now, we will use gvselect to fit the optimal model.

```
. gvselect <term> sex age agesq income private inslow insrdv illness hscore
 chcondlim chcondnlim daysred: poisson docvis <term>
Optimal models:
   # Preds
                 LL
                          AIC
                                    BIC
        1 -3482.672 6969.344 6982.453
        2 -3394.773 6795.546
                               6815.21
        3 -3343.585 6695.169 6721.387
        4 -3326.782 6663.564 6696.336
        5 -3322.142 6656.285 6695.611
        6 -3317.884 6649.768 6695.649
        7 -3315.092 6646.185 6698.621
        8 -3313.097 6644.194 6703.185
        9 -3312.132 6644.263 6709.808
        10 -3311.671 6645.343 6717.442
        11 -3310.857 6645.713 6724.367
        12 -3310.856 6647.713 6732.921
predictors for each model:
1 : daysred
2 : daysred illness
3 : daysred illness age
4 : davsred illness hscore age
5 : daysred illness hscore sex age
6 : daysred illness hscore inslow sex age
7 : daysred illness hscore inslow income sex age
8 : daysred illness hscore inslow chcondlim income sex age
9 : daysred illness hscore inslow chcondlim income sex age chcondnlim
10 : daysred illness hscore inslow chcondlim income sex age insrdv private
11 : daysred illness hscore inslow chcondlim income sex age insrdv private
    chcondnlim
12 : daysred illness hscore inslow chcondlim income sex age insrdv private
    chcondnlim agesq
```

The model with 8 predictors is favored by AIC, while the model with 5 predictors is favored by BIC. The AIC model includes the variables of the BIC model, income, the indicator for chronic conditions with activity limitation, chcondlim, and the indicator for government health insurance for those with low income, inslow.

We will examine the effect of the 8-predictor model chosen by AIC on the dataset with 5,190 adults. Recalling our discussion of cross-validation, we do not show the results of the model fit in **poisson**, because the significance levels would be misleading.



. quietly poisson docvis daysred illness hscore inslow chcondlim income sex age

. mmp, mean(n) smoother(lowess) linear predictors

Figure 3. Marginal model plots for 8-predictor model

The marginal model plots in figure 3 support the validity of the 8-predictor model.

Now, we examine the 5-predictor model chosen by BIC.

```
. mmp, mean(n) smoother(lowess) linear predictors
      10
                                                                 10
       ω
                                                                  ω
     docvis
4 6
                                                               docvis
4 6
       N
                                                                  2
       c
                                                                  c
                             2
                                                                                                                  15
                                      ż
                                                                                                   10
                                                                                     5
                               illness
                                                                                          hscore
                                                                 10
       10
       α
                                                                  ω
                                                               docvis
4 6
     docvis
                                                                  2
                                                                  C
                              Ó
                                              .2
                                                       .ż
                                                                           2
                                                                                                    Ó
                                                                                       Linear Form
                                 age
              Model
               Alternative
```

```
. quietly poisson docvis daysred illness hscore sex age
```

Figure 4. Marginal model plots for 5-predictor model

The marginal model plots in figure 4 support the validity of the model. If one does not favor BIC over AIC, or vice versa, you could choose this model for its parsimony.

# 3 Conclusion

We explored both the theory and practice of best subsets variable selection in regression. Using real datasets, we have demonstrated the use of the leaps-and-bounds algorithm in selecting regressors in normal and nonnormal models.

We fully defined the gvselect command as a method for performing regression variable selection in Stata and demonstrated its use with two different datasets.

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