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Chemical Control of Invasive Species**

**Alex L. Marten and Christopher C. Moore**

**Working Paper Series**

**Working Paper # 10-06  
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# An Options Based Bioeconomic Model for Biological and Chemical Control of Invasive Species

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May 25, 2010

## Abstract

The management of non-native invasive species is a complex but crucial task given the potential for economic and environmental damages. For many invasions the development of socially optimal control strategies requires more than is offered by the single-species, single-control models that have dominated this area of research. We develop a general stochastic optimal control framework that accommodates multiple interacting species while accounting for uncertainty in the temporal population dynamics. This extension to the current line of bioeconomic control models allows for the design of optimal management strategies that utilize both chemical and biological controls in an environment of uncertainty and irreversibility. We demonstrate the benefits of combining chemical and biological controls in long term management strategies through a case study of the hemlock woolly adelgid infestation in the eastern United States. In this application we find that the introduction of natural predators is usually sufficient to manage the infestation, though the availability of chemical controls is important when the detection of an invasion or the subsequent response is delayed.

*Keywords:* invasive species, real options, integrated pest management, biological control

*JEL Classification:* Q57, Q23, D81

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# 1 Introduction

The management of non-native invasive species (NIS) is a complex but crucial task given the potential for economic and environmental damages. Pimentel et al. [2000] finds that the 50,000 NIS in the United States cause \$115 billion in damages and losses annually, and that around \$21 billion is spent on NIS population control each year. It is therefore no surprise that considerable effort has been exerted in order to better understand optimal NIS management policies (see Lovell et al. [2006], Olson and Roy [2006], and Gren [2008] for a detailed review). While the literature on the optimal NIS management is substantial, economists have yet to rigorously consider optimal integrated pest management (IPM) strategies, despite the support this approach has received from practitioners, federal agencies, and academics. There exists some ambiguity in the term IPM, but in this paper we adopt the broad interpretation of ecologically conscious control of a target species through the coordinated use of biological and non-biological means.

In the case of some NIS infestations a policy of eradication and prevention may be adopted in order to eliminate the current population and deter reestablishment in the future. However, in many cases the ability to find and remove infestations will decrease with the size of the population often making eradication impossible or impractical [Kotani et al., 2009]. In these situations a control strategy that maintains a positive but economically efficient level of the NIS population is necessary. This need for long term management efforts creates an opening for substantial economic and ecological benefits through the use of IPM strategies and the accompanying reduction in pesticide use. Building on the foundation of traditional bioeconomic models that integrate NIS population dynamics with welfare economics, we develop a framework that is capable of considering optimal IPM strategies for NIS population management.

The initial work of Higgins et al. [1997], Eiswerth and Johnson [2002], Olson and Roy [2003], and Hastings et al. [2006] developed a set of dynamic models that consider the optimal control of a single NIS invasion using various non-biological controls. These studies have convincingly demonstrated that bioeconomic models have substantial value in informing the design of efficient NIS management strategies. While important for laying the foundation of bioeconomic models in NIS management, the absence of biological uncertainty in these deterministic models can lead to significant bias in their solutions. To this end Olson and Roy [2002] have shown how stochastic shocks to the population growth of a NIS can substantially affect the choice of eradication versus control as a management strategy. Similarly, Leung et al. [2002] demonstrate the importance of uncertainty in a study of prevention expenditures needed to obtain economically efficient levels of invasion risk in the context of aquatic invasive species.

Saphores and Shogren [2005] provide an important extension to this line of research by utilizing a real

options framework to incorporate both uncertainty and managerial flexibility into the traditional dynamic model of NIS management. They consider the case of a single species whose population dynamics are subject to uncertainty with an expectation of exponential growth. To control the NIS population the resource manager is considered to have the ability to eliminate the vast majority (90%) of the population at a fixed cost. The solution to the model is a target population level, that when reached will trigger the resource manager to exert their direct control on the population.

In this paper we develop a real options framework under similar theoretical constructs as Saphores and Shogren [2005] but we consider a more general form which is capable of incorporating more complex and realistic management strategies. While the increased complexity requires a more creative numerical solution, it allows us to analyze optimal IPM approaches that employ both chemical and biological controls. In addition to multiple, interacting controls, we further generalize the model to solve for the optimal timing and flow of the controls rather than restricting the solution to a fixed level of control.

We use this continuous time and continuous state stochastic dynamic programming model to analyze control strategies for managing the hemlock woolly adelgid (HWA) infestation in western North Carolina. In this case we find that introduced predators are sufficient to maintain acceptable NIS population levels as long as the infestation can be detected and controlled without substantial delay. When the NIS population is allowed to reach relatively high levels before controls are implemented a more costly strategy of simultaneous chemical and biological control is optimal.

This paper proceeds by first laying out a general stochastic dynamic programming model of NIS management with multiple controls and multiple populations of interest. We then impose some structure on the general model by providing specific population dynamics for the NIS and an introduced predator along with damage and cost functions. The structured model is then used to examine the HWA infestations that are affecting forests in the eastern United States, using empirical observations to calibrate the bioeconomic model. The paper concludes with a discussion of our method and results.

## 2 Optimal Control Framework

We consider the case of a resource manager charged with developing a management policy to control the NIS population,  $X_t$ . The objective is to find the optimal policy that will minimize the overall social cost defined as the discounted sum of damages from the NIS population and the costs of control. We assume the manager has a set of two controls at her disposal. The first has an immediate one-time effect on the NIS

population, such as a chemical pesticide or physical removal. If the control is exerted at time  $t$  it will be the case that

$$X_t = X_t^- - \omega,$$

where  $X_t^-$  is the level of the NIS population just prior to the application of the control and  $\omega$  is the level of the control exerted. We denote the fixed cost associated with the direct control as  $C_x$  and the marginal cost as  $\gamma(X)$ . Therefore the total cost associated with applying  $\omega$  units of the direct control is

$$C_x + \int_{X_t^- - \omega}^{X_t^-} \gamma(x) dx.$$

Also at the manager's disposal is a biological control which could be a predator, parasite, or pathogen that reduces the growth of the NIS population. Exerting this control has the effect of directly increasing the current population of the biocontrol agent,  $P_t$ , such that if the control is applied at time  $t$

$$P_t = P_t^- + v,$$

where  $P_t^-$  is the population of the biocontrol agent just prior the control and  $v$  is the amount introduced. The cost of exerting the biological control is assumed to have a form similar to that of the direct control, specifically the total cost will be

$$C_p + \int_{P_t^-}^{P_t^- + v} \pi(p) dp,$$

where  $C_p$  and  $\pi(P)$  are the fixed and marginal cost, respectively. It is important to note that the controls,  $\omega$  and  $v$ , are variable and *will be determined optimally*.

In the absence of controls the population dynamics of the two species are defined by a system of stochastic differential equations, such that

$$dX_t = a(X_t, P_t)dt + b(X_t)dW_t^P, \quad (1)$$

and

$$dP_t = c(X_t, P_t)dt + d(X_t)dW_t^X, \quad (2)$$

where  $W^P$  and  $W^X$  are standard Brownian motions and  $E[dW^X dW^P] = \sigma_{XP} dt$ . We do not impose a specific form for the drift and diffusion functions at this stage to emphasize the generality of our framework. However, the drift and diffusion function must still satisfy the standard conditions that rule out explosive growth (see Øksendal and Karsten [1998]).

The current population of the invasive species,  $X_t$ , results in damages described by the convex function  $F(X_t)$ ,  $F'(X_t) \geq 0$ ,  $F''(X_t) \geq 0$ . The goal of the manager is to minimize the expected discounted flow of damages plus any costs associated with exerting the controls, such that

$$V(x, p) = \min_{\nu} E \left[ \begin{array}{l} \int_0^{\infty} e^{-\rho t} F(X_t) dt + \sum_{j=1}^J e^{-\rho \tau_j} \left[ C_x + \gamma(X_{\tau_j}^-) \omega_j \right] \\ + \sum_{i=1}^I e^{-\rho \xi_i} \left[ C_p + \pi(P_{\xi_i}^-) v_i \right] \end{array} \middle| X_0 = x, P_0 = p \right],$$

where  $V(x, p)$  is the value function,  $\rho$  is the discount rate, and  $\nu$  is the control policy. The control policy  $\nu$  will be a possibly finite sequence of control times and levels, such that

$$\nu = (\tau_1, \tau_2, \dots, \tau_j, \dots, \omega_1, \omega_2, \dots, \omega_j, \dots, \xi_1, \xi_2, \dots, \xi_i, \dots, \pi_1, \pi_2, \dots, \pi_j, \dots). \quad (3)$$

This sequence represents times,  $\tau_j$ , at which the manager exerts a control of size  $\omega_j$  on the invasive species population as well as times,  $\xi_i$ , at which point  $\pi_i$  biocontrol agents are introduced.

This model represents a two dimensional impulse control problem with two controls for which the solution is defined by a quasi-variational inequality (QVI), representing the Hamilton-Jacobi-Bellman equation and the necessary value matching conditions [Øksendal and Sulem, 2005]. In this case the conditions of the QVI are

$$\rho V(X_t, P_t) \geq -F(X_t) + a(X_t, P_t)V_X + c(X_t, P_t)V_P + \frac{1}{2}b^2(X_t)V_{XX} + \frac{1}{2}d^2(P_t)V_{PP} + \sigma_{XP}b(X_t)d(P_t)V_{XP}, \quad (4)$$

$$V(X_t, P_t) \geq \max_{\omega} V(X_t - \omega, P_t) - C_x - \gamma(X_t)\omega, \quad (5)$$

and

$$V(X_t, P_t) \geq \max_{\pi} V(X_t, P_t + v) - C_p - \pi(P_t)v, \quad (6)$$

where one of the conditions must hold with equality. Which condition holds with equality will determine the optimal control policy. If (4) holds with equality then it is optimal for the policy maker to wait before applying any of the controls. To see this we consider the conditions in (5)-(6). If both are assumed to hold with strict inequality it is implied that the value at the current state of nature,  $V(X_t, P_t)$ , is greater than any other which could be obtained when taking into account the cost of exerting the controls. On the other hand if (5) or (6) is to hold with equality then it is optimal to exert the control on the invasive species or

the predator population respectively since a higher value may be obtained if at least one control is used. In some cases there will be a region of the state space in which it is optimal for the resource manager to pursue a strategy that utilizes both the chemical and biological controls simultaneously. For points at which a dual control strategy is optimal both (5) and (6) will hold with equality.

We have introduced the theoretical framework in a fairly general fashion in order to emphasize the fact that the numerical algorithm developed to solve the model is not dependent upon any specific functional forms. Therefore this methodology may easily be applied to the management of any invasive species where both chemical and biological agents are viable control options. In the subsequent section we focus the model on a particular invasive species infestation and present specific functional forms for the biological and economic expressions.

### 3 Hemlock Woolly Adelgid Case Study

Native to Asia, where it is a mostly benign insect-parasite of hemlock trees, the hemlock woolly adelgid (*Adelges Tsugae*; henceforth HWA) was first documented in western North America in the 1920s and was discovered in the eastern United States near Richmond, Virginia in 1951. Similar to those found in Asia, the type of hemlock that grows in western North America exhibits a natural resistance to HWA infestations. However, hemlock species found in the eastern part of the continent are highly susceptible and rarely survive even moderate infestations. The eastern hemlock (*Tsuga Canadensis*) is a slow-growing evergreen that can live for 800 years or more and reach heights of more than 50 meters. The Carolina hemlock (*Tsuga Caroliniana*) is a closely related but less common species only found on the slopes of the southern Appalachian mountains.

Hemlock ecosystems provide critical habitat for several species of birds [Howe and Mossman, 1995] and in the southern parts of their range their dense foliage shades streams keeping waters cool enough to support native brook trout populations [Ross et al., 2003]. In addition, groves of old growth hemlocks provide a unique setting for outdoor recreation in local, state, and national parks and forests. While hemlocks tend not to be a commercially important species they are widely used for horticultural purposes. For these reasons federal, state, and local governments have been working to control the spread of HWA and mitigate their impacts on the landscape.

There are a number of control methods available to resource managers. The first line of defense is quarantine. Several states are threatened by HWA but no established infestations have been discovered. For these

states regulations that prevent the importation of nursery stock and timber from states with documented infestations provides an initial line of defense. In instances where small infestations are discovered the NIS is eradicated by removing and burning the host trees.

For at least 14 states with large established infestations eradication and protection is not a viable mitigation strategy. Instead they rely largely on chemical control of the pest. Spraying the foliage of an infested tree with horticultural soaps and oils kills virtually all of the HWA on a tree. This approach is relatively inexpensive but limited to trees that are less than 30 meters tall and accessible to the vehicles carrying the spraying apparatus. Other trees can be treated with more expensive systemic pesticides. These pesticides are typically injected into the soil at the base of a tree where the root system draws the chemical into the foliage killing any pest feeding on the tree.

Resource managers are increasingly relying on biological forms of control to overcome the limitations of chemical insecticides [Venner, 2006]. Several beetle species that prey on HWA in their native range have fared well in laboratory breeding programs, prey on HWA during each of their life cycles, and tend to survive winters of the northeastern United States making them excellent candidates for biocontrol agents. It is still too soon to tell if they can be effective enough in the field to reverse the damages to hemlock forests from HWA. However, resource managers agree that predator introduction is the most promising option for long term control of HWA.

### 3.1 Model Specification and Parameterization

In order to apply the real options framework presented in Section 2 we specify functional forms for the evolution equations and cost functions for control. To represent the dynamic interaction between the introduced predator and the NIS we adopt a model similar to Harper [1991] with a modification such that the carrying capacity of the predator is a function of the prey population density<sup>1</sup>. Adding a diffusion term to the growth equation for each species generates

$$dX_t = \left[ \alpha \left( 1 - \frac{X_t}{K} \right) X_t - \beta P_t X_t \right] dt + \sigma_X X_t dW_t^X, \quad (7)$$

and

$$dP_t = \eta \left( 1 - \frac{\epsilon P_t}{X_t} \right) P_t dt + \sigma_P P_t dW_t^P, \quad (8)$$

---

<sup>1</sup>It is worth noting that this structure assumes the predator is unable to sustain itself on other native species, implying that there is no risk of the biocontrol agent eventually becoming a pest itself. However, the expansion of this framework to incorporate the probability that the biological agent adapts into pest population is a non-trivial problem which may warrant further research.

where  $E[dW^X dW^P] = \sigma_{XP} dt$ . This predator-prey system has several desirable properties. In the absence of a predator the prey population will exhibit logistic growth bounded by the constant  $K$ . The population of the predator also exhibits logistic growth but is bounded by  $X/\epsilon$ , which is an increasing function of the prey population. In the absence of stochastic shocks a stable non-trivial equilibrium exists [Berryman, 1992].

While this model of predator-prey population dynamics fits a variety of situations, it should be noted that it does not reflect the dynamics of all interspecies competition. This relationship will have an important impact on the optimal control policy and care should be taken to ensure the accuracy of the biological model. Fortunately the approach we develop to solve the optimal control problem does not depend on specific forms for the drift or diffusion functions and therefore this framework remains general enough to handle alternative cases. These might include the case of Malthusian style growth as suggested in the one-species model of Saphores and Shogren [2005] where geometric Brownian motion describes the dynamics.

In addition to the drift and diffusion functions it is necessary to specify the functional form of the damage function,  $F(X_t)$ . To this end we follow Saphores and Shogren [2005] and Kotani et al. [2009] in utilizing a quadratic form

$$F(X_t) = F X_t^2,$$

where  $F > 0$ .

The costs of applying the chemical control include the cost of the insecticide itself and the labor required to find infested trees and inject the insecticide into the soil. As the population density of the invasive increases less labor would be required to find infested trees [Olson and Roy, 2008]. As such, we assume that the marginal cost of applying chemical control is decreasing in the population density of the invasive,

$$\gamma(X_t) = \frac{\gamma}{X_t},$$

where  $\gamma > 0$ .

The advantage of the biological control, and the reason it is considered by many to be the only long-term solution to the HWA infestation, is because once the beetles are released there is little for the managers to do. If successful, the beetles will establish a reproducing population that seeks out HWA and keeps their population at an acceptable level. To apply biological control, the manager has to find only one infested tree on which to release up to 2,000 beetles. Thus, the inverse relationship between population density of the invasive and marginal cost of chemical control does not arise with biological control and the marginal cost

function for biological control is simply

$$\pi(P_t) = \pi,$$

where  $\pi > 0$ . While it is certainly conceivable that low levels of population density would make it difficult to find one infested tree on which to release biological control agents, in the interest of clarity and tractability we assume a constant marginal cost over all levels of  $X$ .

In 2006 the US Forest Service conducted a contingent valuation (CV) survey to estimate willingness to pay (WTP) for a mitigation program on public lands in the southern Appalachian Mountains. Analysis of the survey data shows a positive WTP for hemlock protection that is sensitive to scope [Moore, 2008]. Further, the results show that the costs of current mitigation program are justified and an expansion of the program may be warranted due to the large benefit-cost ratio. We use these data to calibrate the damage function of our optimization routine.

In addition to the damage function, the model requires parameterization of the cost functions and the SDEs defining the temporal dynamics of the NIS and predator populations. The data available to assign values to the economic and biological parameters of the model are sparse. Thus we draw as much as we can from empirical observations but make some assumptions to complete the calibration of our model. We test these assumptions with a sensitivity analysis over these parameters. Table 1 lists the parameters of the biological model, the data used to calibrate each parameter, and any key assumptions required. Table 2 presents the same information for the economic portion of the model.

### 3.2 Numerical Results

Given the complex nature of this continuous time, two state, two impulse control stochastic dynamic programming problem closed form solutions for the value function and the related control policy do not exist. Instead we utilize a numerical solution that approximates the value function with the tensor product of two piecewise linear functions with 300 breakpoints. Using upwinding finite difference methods to approximate the derivatives of the value function we solve for the approximating coefficients by ensuring that the approximation satisfies the QVI in (4)-(6) on a discrete version of the state space with 300 nodes in each direction. For this example we terminate the state space at 2 for both species, which represents a level of twice the carrying capacity for the non-native invasive species. Based on our testing extending the boundary beyond this point does not alter the results within the range of interest. The specifics of the numerical method are presented in Appendix A.

Parameter	Symbol	Value	Comments on Calibration
HWA carrying capacity	K	1.0	Normalized to one to set scale of biological model.
Intrinsic growth rate of HWA	$\alpha$	0.3	Observations of HWA spread and time from introduction to heavy infestation; Identified through simulation of the biological model.
Intrinsic growth rate of predator	$\eta$	0.3	It generally takes six to ten generations of the prey species for a predator population to become established [DeBach et al., 1974]. Identified through simulation of the biological model.
Predation parameter	$\beta$	2.0	Experiments conducted at six separate sites revealed that releasing relatively few adult beetles (2,400-3,600) into an infested hemlock forest reduced adelgid densities by 47-88% on release trees in five months with a median reduction of 52.9% [McClurel et al.]. Identified through simulation of the biological model.
Sustenance parameter		1.0	Set to 1.0 by assumption. The implication being that a beetle population at carrying capacity requires a prey population also at carrying capacity to sustain that population level.
Volatility parameters	$\sigma_i$	0.25	Assumption tested in sensitivity analysis.
Brownian Motion Correlation	$\sigma_{xp}$	0.5	Assumption tested in sensitivity analysis.

Table 1: Parameterization of the Biological Model

Parameter	Symbol	Value	Comments on Calibration
Discount rate	$\rho$	0.05	Midpoint of the 0.03 and 0.07 rates recommended for discounting federal projects in OMB Circular A4.
Damage function scaling parameter	$F$	10 <sup>a</sup>	Estimated using CV data (Moore [2008]). We assume that forest services (the good valued by the CV survey) are inversely related to HWA density (the state variable in our damage function).
Fixed cost of chemical control	$C_X$	0.15 <sup>a</sup>	From Appendix F of Jacobs [2005].
Fixed cost of biological control	$C_P$	0.01 <sup>a</sup>	From Appendix F of Jacobs [2005].
Marginal cost parameter for chemical control	$\gamma$	3.0 <sup>b</sup>	From Appendix F of Jacobs [2005] which reports average variable costs. Identified assuming the prevailing HWA density is 0.5 and multiplying by 200 to scale from per site cost to entire study area.
Marginal cost parameter for biological control	$\pi$	4.0 <sup>b</sup>	From Appendix F of Jacobs [2005]. Identified by multiplying per site variable costs by 200 to scale to entire study area.

<sup>a</sup> Reported in millions of 2006\$

<sup>b</sup> Reported in millions of 2006\$ per unit of control

Table 2: Parameterization of the Economic Model

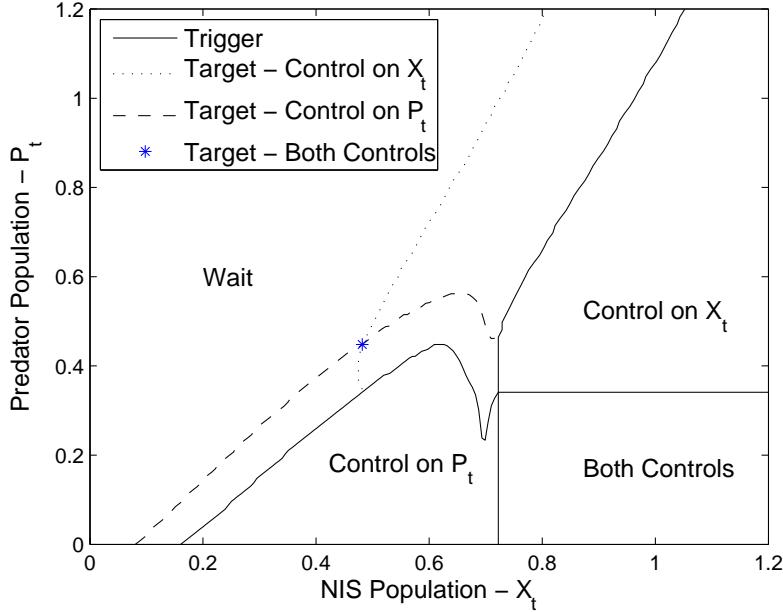


Figure 1: HWA Optimal Management Policy Under the Baseline

Since the problem involves two state variables, the NIS and predator populations, any value of the state vector can be represented by a single point in two dimensional space. Figure 1 presents the optimal management policy based on the default biological and economic parameter values listed in Tables 1-2. Depicted in the plot are regions indicating the optimal control response to a given value of the state vector. At any point in an action region the optimal level of the control to be exerted is described by the distance to the nearest point on the target curve in the appropriate dimension for the control. When the value of the state vector places it in the “Wait” region it is optimal to apply neither control and allow the state variables to change according to the evolution equations 7 and 8. If as the populations evolve over time they hit one of the triggers that bound the control regions it will be optimal for the resource manager to exert the corresponding control. If the manager is able to observe the states and respond without delay the state vector will never find the interior of a control region but “bounce” immediately to the corresponding target. If, however, the invasive population is introduced and grows undetected for some time, or if the manager is not able to respond to the infestation with controls immediately, the state vector may be allowed to reach the interior of a control region. In this case the manager will respond by applying the necessary controls to reach the appropriate target. These cases are explored further in the next section.

Each control has a direct effect on just one of the state variables and therefore the application of a single

control results in either a horizontal or vertical movement from the current point, based on whether it is the chemical or biological control respectively. For example, if the state vector hits the trigger associated with the chemical control the optimal application of the pesticide would reduce the NIS population moving it horizontally until the target for chemical control is reached. Likewise, reaching the biological control region would trigger a response in which the predator population is increased to the optimal level, represented by a vertical movement to the target for biological control. In some cases it is optimal to apply both chemical and biological controls simultaneously, this represents a unique case with special considerations which we will discuss shortly.

The distance between the trigger and target curves is due to the presence of a fixed cost associated with exerting either control. Due to the stochastic nature of the two populations the manager will want to avoid a situation in which the point reached after exerting the control has a high probability of hitting the trigger again in the very near future. Such an event would require the resource manager to incur the fixed cost of the control again. Therefore, the distance between the two curves will depend upon the size of the fixed cost. As that cost approaches zero so will the distance between the trigger and target curves.<sup>2</sup>

The shape and position of the control regions are determined by the biological and economic characteristics of the system. Fixed and marginal costs of the controls and the impact that controls have on inter- and intra-species dynamics interact to form the optimal policy response. When the NIS population is low the marginal cost of chemical control is high and the logistic growth function means they could recover very quickly from the one-time effect of the chemical insecticide. Biological control on the other hand has a persistent effect on the NIS population and exhibits constant marginal cost. Further, when the predator population is low introducing more of them increases their chance of success because the population is still climbing the logistic growth curve. Together these characteristics make the biological control the optimal management solution when both populations are relatively low.

When the NIS population reaches higher levels chemical control becomes a cost effective control. For one, the marginal cost of chemical control is low because the NIS are easy to find. And because damages are also high the immediate impact of the insecticide is preferred to the sustained effect of predator introduction. If the predator population is also high then introducing more of them will suppress their natural reproduction because the population is now beyond maximum sustainable yield on the logistic growth curve. All of these conditions conspire to produce the ‘lower triangular’ appearance of the optimal solution in Figure 1 in which the control regions tend to be below the 45 degree line.

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<sup>2</sup>The absence of fixed costs would move the model into a singular control framework, we refer the interested reader to Miranda and Fackler [2002] for more information on the distinction between impulse and singular control models.

In certain parts of the state space it is optimal for the manager to employ a mixture of the two controls simultaneously. In Figure 1 this region of the state space is labeled as “both controls”. To apply both controls simultaneously is to increase the predator population and at the same time reduce the prey available to them. This seemingly counterproductive strategy is only optimal in the extreme case of a high NIS population and a low predator population. Under these conditions the chemical control can be applied at relatively low marginal cost and have an immediate impact on the high damages being caused by the large NIS population. At the same time the predator population is small enough that even after some of the NIS have been removed conditions remain favorable for introducing more predators.

Perhaps more interesting than the conditions that make it optimal to apply both controls simultaneously is the singular point in the state space representing the target for a combined strategy. Because the controls are being applied simultaneously and because they have an immediate impact on their respective state variables, the optimal target does not depend on the value of the state vector before the controls are applied  $(X^-, P^-)$ , but rather the target of the other control. That is the optimal target for predator population depends on  $(X, P^-)$  and the optimal NIS population target depends on  $(X^-, P)$  leading to the one point in the state space where the targets intersect. That is also why the “Both Controls” region of Figure 1 is bounded by one horizontal and one vertical line indicative of the fact that the optimal does not depend on the current levels of the state variables, only the targets. The optimal level of the control will be determined by the distance from the current point in the state vector to the target.

Notice that the biological control trigger makes an abrupt downward turn before it meets the chemical control trigger, carving a ‘wedge’ out of the biological control region. This wedge is also the product of fixed costs and uncertainty in future NIS population. As the NIS population approaches levels at which chemical control becomes a cost effective solution the resource manager will want to release fewer biological control agents because it is becoming less likely that they will be sufficient to avoid the fixed cost of chemical control. In other words, a positive shock to the NIS population immediately after releasing biological control could cause the resource manager to incur the costs of biological control plus costs of chemical control all in a short amount of time. If the resource manager observes the state vector inside this wedge she will wait to see how the NIS population evolves before deciding how many biological control agents to release. If the NIS population falls she will release more, confident that the predator population will sufficiently suppress the NIS population to prevent hitting the chemical control. If however, she observes an increase in the NIS population she will employ a smaller release of predators and expect to augment that release with chemical control in the near future.

### 3.3 Monte Carlo Simulation

To demonstrate how the optimal management strategy would be implemented over time we conduct a Monte Carlo simulation of the bioeconomic model under four different assumptions. First we prevent any action by the forest manager in order to establish a baseline path of NIS population growth and the resulting damages. In the second case we assume that the forest manager is aware of the introduction shortly after it occurs and is able to respond immediately. We also consider the case where any action is delayed until the NIS population reaches a critical level, representing delayed discovery or a lag between discovery and initial response. Finally, we consider a case in which the biological control is not available, restricting the strategy to the use of chemical insecticides only. In each case we utilize 10,000 simulated paths with a 50 year horizon for each of the populations, using a monthly time step for the discretization of the continuous time SDEs.

To demonstrate the baseline case we introduce the NIS in the first period of the planning horizon by setting the population equal to 5% of carrying capacity. Figure 2 shows two paths of the NIS population. One is the mean of the simulated paths representing the expected NIS population over time. The other path represents one of the 10,000 paths chosen at random, demonstrating the effect that stochastic shocks can have on a single path. The expected path shows NIS population growing gradually in the years immediately following introduction, reaching a period of rapid growth and eventually settling down and asymptotically approaching carrying capacity. This is precisely what we would expect from a logistic growth function with carrying capacity. Since no controls are exerted the predator is never introduced and its population will therefore be zero over the entire time horizon.

Now we consider the case in which the manager is aware of the introduction and is able to respond immediately thereafter. At the time of introduction/discovery the state vector is in the 'Wait' region of Figure 1 and so no control action is taken. Figure 3(a) presents the expected path of NIS and predator populations over the 50 year time horizon when the resource manager adheres to the previously defined optimal control policy. In relatively few cases the NIS population grows quickly enough that the manager is introducing predators in the second year of the simulation, and therefore the expectation is that the predator population will remain close to zero for the first couple of years. The stochastic nature of the NIS infestation means that the control triggers will be reached at different times leading to an expectation that suggests a gradual introduction of biological control. However, when we examine a single set of simulated paths in Figure 3(b) the discrete jumps in the predator population resulting from the application of the biological control become apparent. During years 3, 4, and 28 the state vector hits the 'Control on  $P_t$ ' trigger and predator population is increased as a result. It is interesting to note that none of the simulated paths results

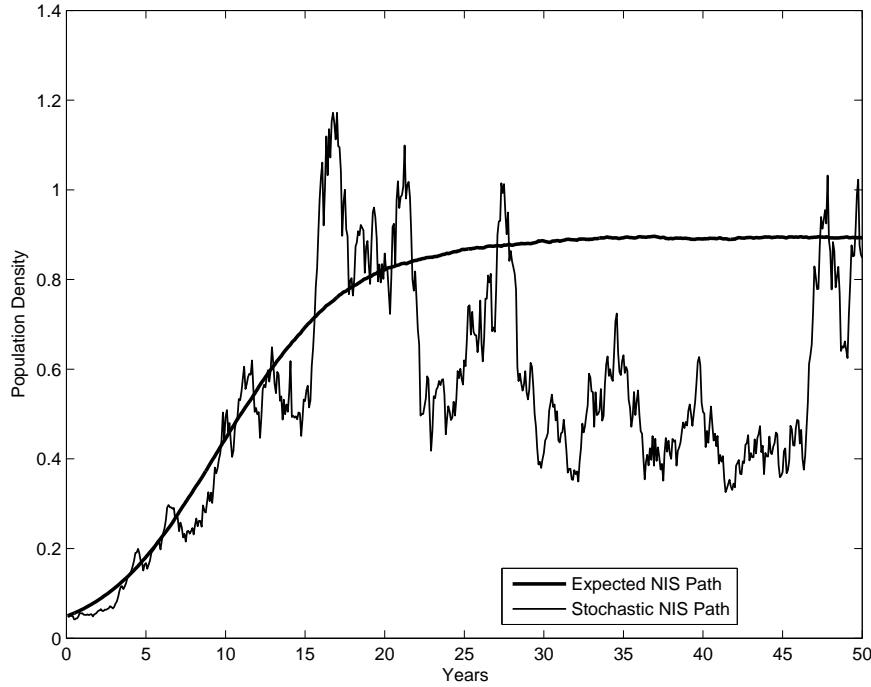


Figure 2: Expected and Stochastic NIS Population Paths with no Control

in a single application of chemical control. It appears that the biological and economic parameters specified for this model are such that the biological control is a cost effective response to the evolution of the NIS population. In the following section we test the sensitivity of our results to this specification and show that under different conditions the chemical control will play a more substantial role in controlling the NIS infestation.

Next we consider the case in which control of the infestation does not occur until the NIS population reaches a critical level, at which point the manager then proceeds with an optimal control program. This could happen because the infestation is difficult to detect until damages become obvious or because circumstances exist that prevent the immediate implementation of a control strategy, due for example to funding or planning lags. All of these issues appear to be present in our reference case of hemlock woolly adelgid infestations in the southern Appalachian Mountains. To conduct this simulation we restrict action by the manager until after the NIS population reaches 50% of carrying capacity. Figure 4(a) shows the expected paths of NIS and predator populations. It is worth noting that the expected NIS population will not reach 50% of its carrying capacity since the individual paths will reach this point in different time periods at which the population will be subsequently controlled, leaving the average below 0.5. Figure 4(b) presents a single set of stochastic paths, taken from the 10,000 generated. In this case the NIS population reaches 0.5 in year 11

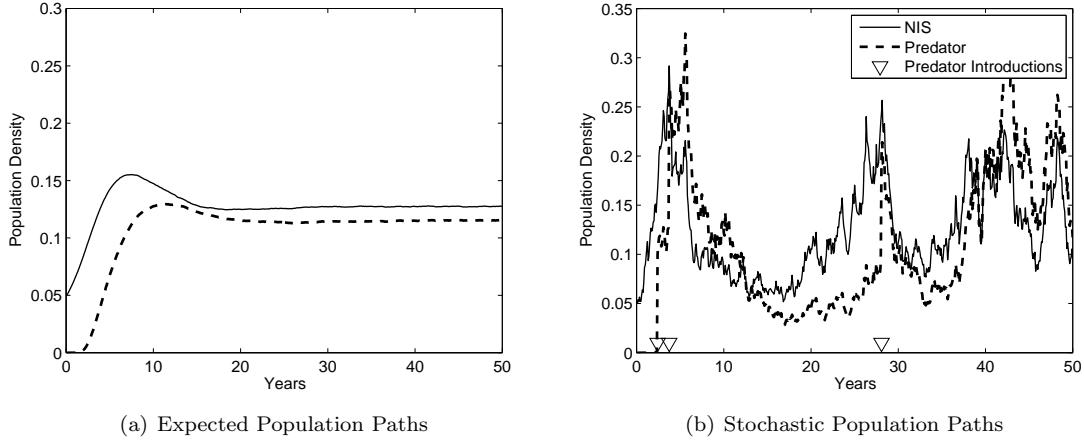


Figure 3: Expected and Stochastic NIS Population Paths with Control

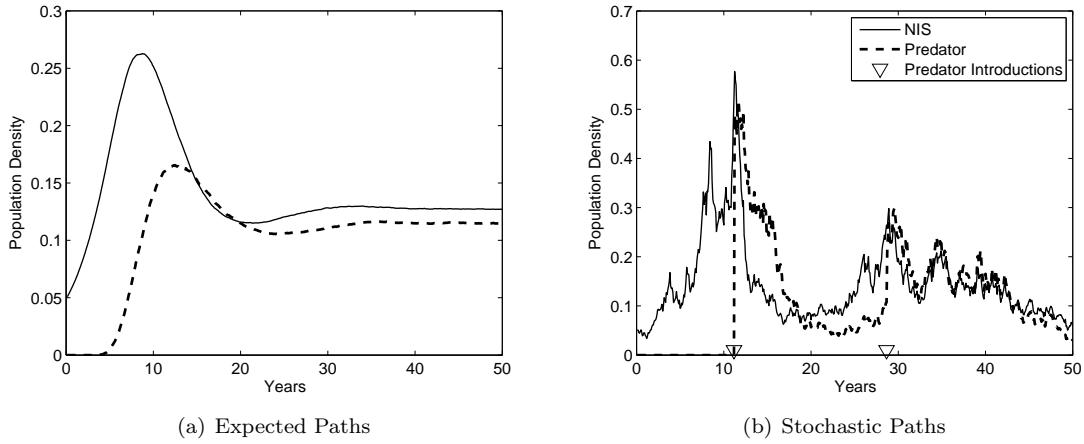


Figure 4: Expected and Stochastic NIS Population Paths with Delayed Control

of the infestation, at which point it is optimal to introduce a large number of predators to control the pest. Notice that despite using the same stream of stochastic shocks as the previous case, the delayed response leads to just two predator introductions during the time horizon. The initial introduction is large enough that the state vector does not enter the “Control on  $P_t$ ” region again until year 28 of the simulation.

Finally, we consider the case in which biological control is not available. To conduct this simulation we compute the optimal control policy using a simplified version of the model in Section 2 which has only the chemical control and assumes that the predator population is zero throughout the time horizon. Figure 5(a) shows the expected paths of NIS and predator populations. Due to the fact that the control does not place persistent downward pressure on the NIS, the control will need to be exerted more frequently in order to maintain an acceptable population level. This increase in application and the increasing marginal cost result

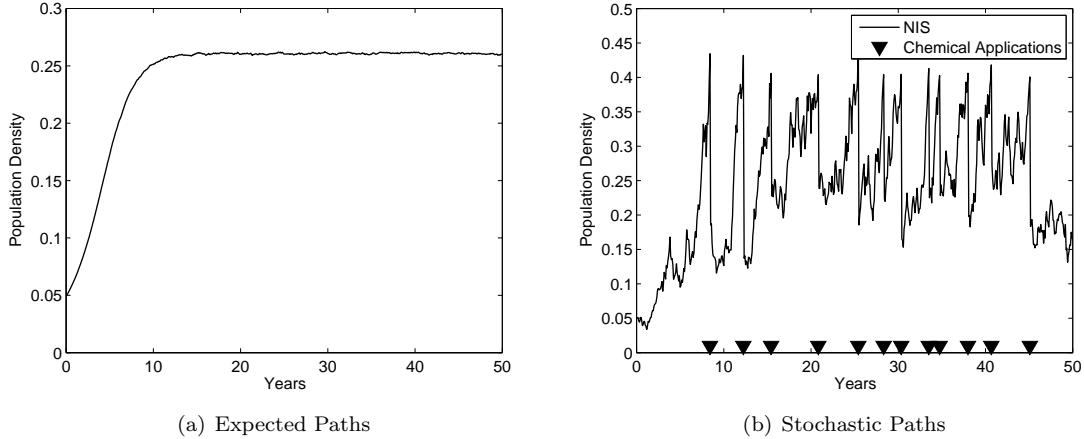


Figure 5: Expected and Stochastic NIS Population Paths with a Chemical Control Only

in an expected population that levels off well above the cases where the biological control is included in the management strategy. Figure 5(b) presents a single set of stochastic paths, taken from the 10,000 generated. The lack of a predator population to control the NIS over the long term results in more frequent chemical control. For this specific path the chemical control is applied 12 times over the 50 year horizon or at an average of every 4 years.

Comparing the total losses under each of these cases can provide some insight to the net benefit from controlling hemlock woolly adelgid in the study area and the value of being able to respond quickly to the initial introduction of the pest. Figure 6(a) plots annual damages under all four cases. The baseline case in which no control action is taken and the NIS is allowed to approach carrying capacity results in large annual damages. To better illustrate the differences between the three control cases Figure 6(b) excludes the example where the NIS remains uncontrolled. Expected annual damages for the delayed case remain greater for many years after the manager is able to apply controls showing the value of being able to respond quickly to an NIS introduction. While the higher expected annual damages from the case where only the chemical control is considered highlights the importance of biological controls when developing management strategies for NIS infestations.

### 3.4 Sensitivity Analysis

Since a closed form solution for the optimal control policy is not available, we turn to sensitivity analysis in order to better understand the role of key parameters. While all of the parameters will have some effect on the optimal control policy and the value function, we focus on those that have the greatest impact, and

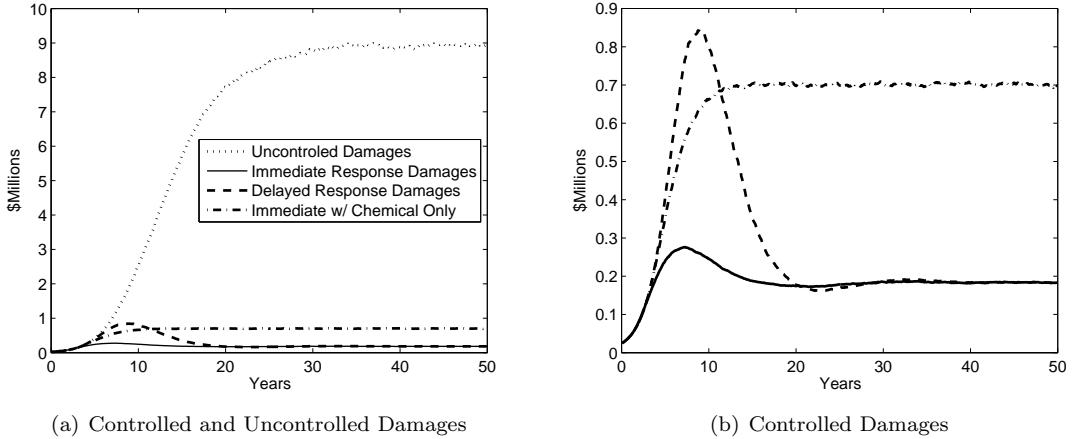


Figure 6: Expected Annual Damages

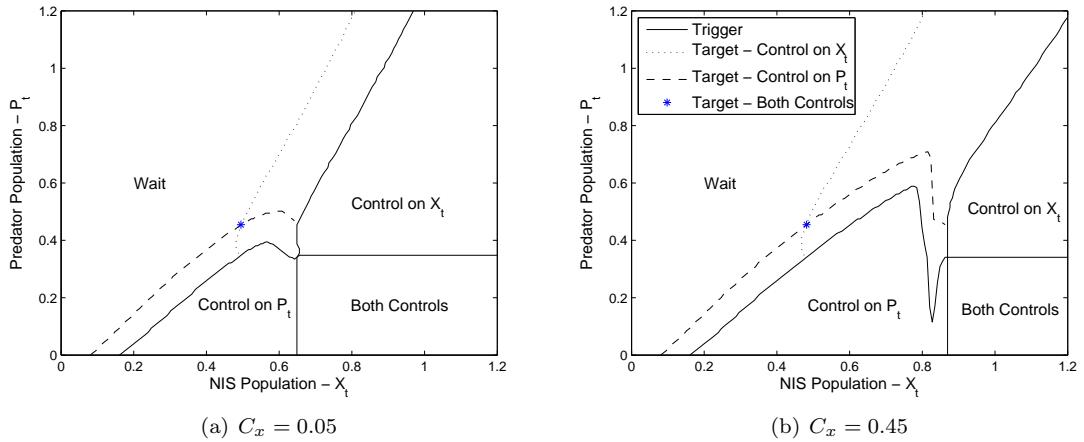


Figure 7: Sensitivity of Optimal Control Policy to the Relative Fixed Cost ( $C_x/C_p$ )

particularly those that are characteristic of bioeconomic analysis and have not thus far been explored in a real options setting.

We begin with the sensitivity of the optimal control policy to the parameters of the cost functions. Figure 7 presents the impact of the fixed cost associated with exerting the chemical control. An increase in the fixed cost of the control reduces the relative attractiveness of the control and therefore the probability that the control is utilized. Furthermore, raising the fixed cost will increase the distance between the trigger and target curves in order to avoid a situation in which the large fixed costs must be incurred more than once within a short period of time. The impact of changes to the fixed cost of the biological control are qualitatively the same.

Also of interest is the sensitivity of the solution to changes in marginal cost, particularly in the non-

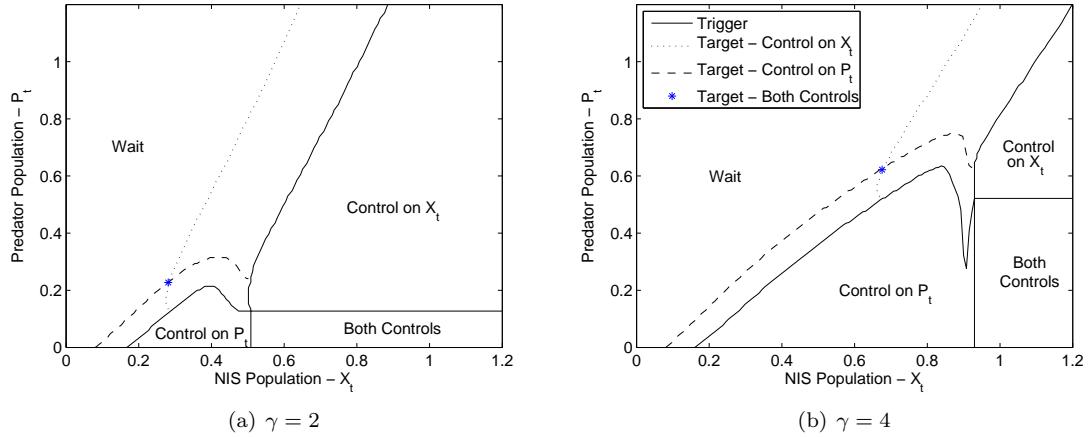


Figure 8: Sensitivity of Optimal Control Policy to the Chemical Control’s Marginal Cost ( $\gamma$ )

constant case of the chemical control. Figure 8 presents the effect of the scaling parameter,  $\gamma$ , in the marginal cost function for the chemical control. As expected changes in the marginal cost do not affect the distance between the triggers and their targets but they do affect the relative likelihood of applying one control rather than the other. This is strikingly apparent in Figure 8 in which the scaling parameter in the chemical control’s marginal cost function is set above and below the baseline. As the cost of applying the chemical control becomes relatively more expensive the probability of it being optimal to utilize the control decreases. Again qualitatively similar results may be obtained with respect to the marginal cost parameter for the biological control, however these figures are omitted in consideration of space.

One of the most important parameters in defining the optimal control policy is the predation parameter  $\beta$  which determines the effectiveness of the biocontrol agent. Figure 9 presents the optimal control policy for the cases of  $\beta \in (0, 0.5, 1, 3)$  as opposed to the baseline of  $\beta = 2$ . A value of  $\beta = 0$  implies that the second population,  $P$ , is not a predator of  $X$ . As such the level of the second population does not influence the point at which it becomes optimal to exert the chemical control, and for obvious reasons it is never optimal for the manager to increase the level of the second population. At a level of  $\beta = 0.5$  the biological control is not effective enough to be selected by the resource manager, but because the predator will have some effect on the NIS, predator population does impact the length of time to wait before utilizing the chemical control. Only in cases where the predation parameter is close to or greater than one is biocontrol part of the optimal strategy. However, at low values of  $\beta$  the chemical control plays a larger role in the optimal strategy than was the case in our default model.

An important but largely unexplored parameter is the correlation between the stochastic shocks of the

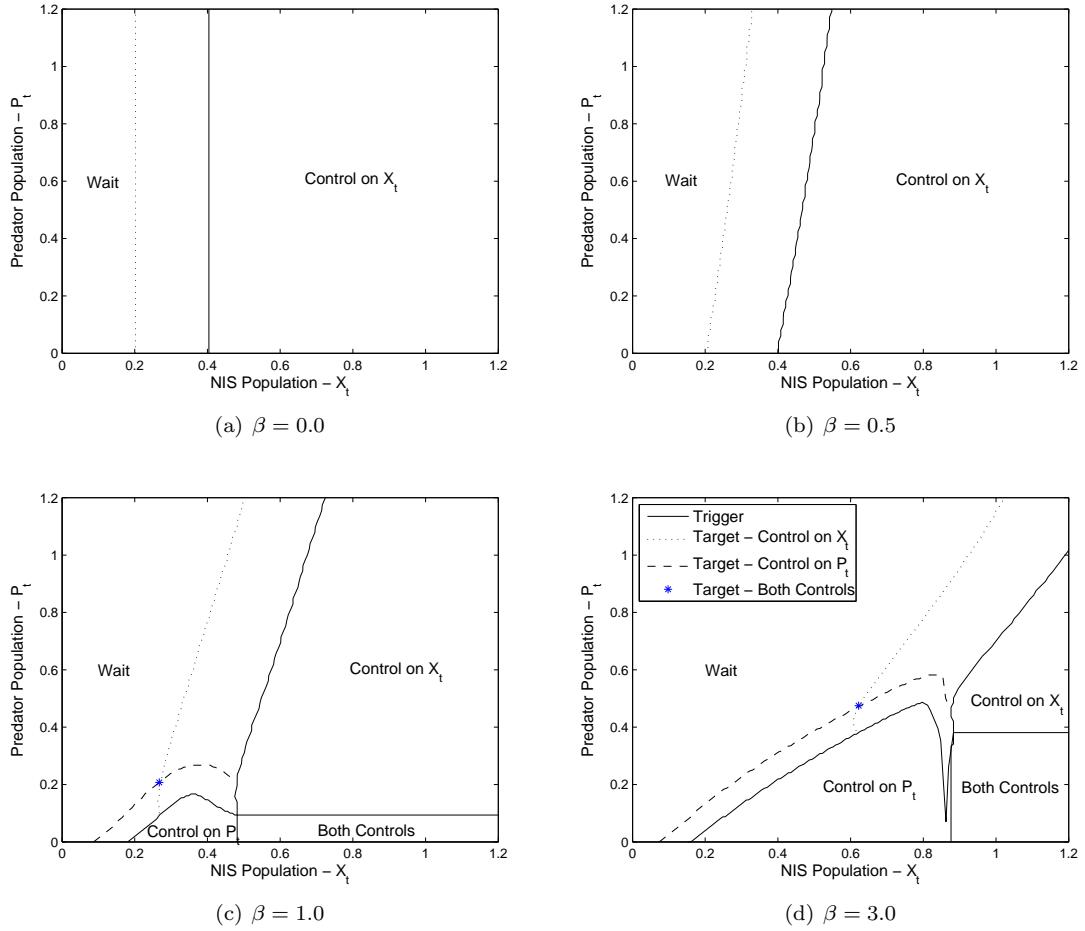


Figure 9: Sensitivity of Optimal Control Policy to the Predation Parameter ( $\beta$ )

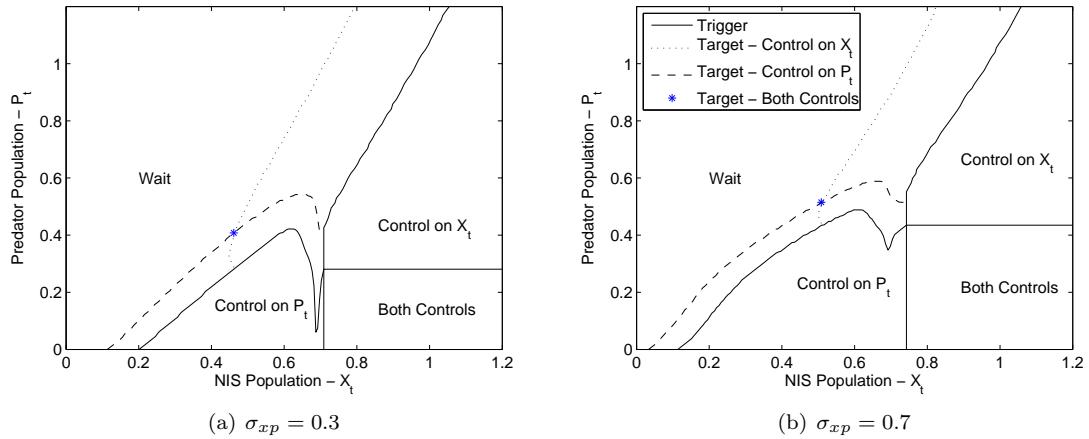


Figure 10: Sensitivity of Optimal Control Policy to the Brownian Motion Correlation Coefficient ( $\sigma_{xp}$ )

two populations. Figures 10(a) and 10(b) present case in which the  $\sigma_{xp}$  is equal to 0.3 and 0.5 respectively. A higher correlation between the exogenous shocks increases the use of the biological control because the predator population in part acts as a hedge against unexpected increases in the NIS population. In other words given a positive correlation a positive shock to the NIS population is likely to be accompanied by a positive shock in the predator population reducing the probability that a future response by the manager will be warranted in order to react to the shock. Therefore the larger this correlation the greater this “hedging” effect will be and the more attractive the biological control will appear relative to the chemical control.

## 4 Concluding Remarks

Scientists, resource managers, and federal agencies have been advocating integrated pest management (IPM) over the past half-century as a way to reduce the use of potentially harmful chemical insecticides. Some of the guiding principles of IPM are to use multiple forms of control in a complementary way in order to maintain acceptable pest levels rather eradicate the pest. Previous stochastic dynamic optimization efforts have been unable to accommodate IPM as a strategy. We have filled that void with a framework that allows multiple stochastic interacting populations in addition to a direct form of NIS control. Our solution is consistent with the aforementioned principles of IPM in that a positive but acceptable level of NIS population is maintained throughout the time horizon and the availability biological control reduces, and in many cases eliminates, the need for chemical insecticides. Based on our specification of the model, chemical control is only required when there is a substantial lag between the initial invasion and the control response. Even

when the negative externalities of chemical pesticides are ignored, we have shown that having a biological control as an additional option reduces may significantly reduce the total cost of pest management. In our case study of the HWA the total costs were reduced by about half when considering the use of natural predators in addition to a chemical control.

Through sensitivity analysis we have identified the biological and economic parameters that have the greatest impact on the solution. More precise identification of the predation parameter and better understanding of how the two populations respond to exogenous shocks would drastically improve the efficiency of the solution. While the trigger regions and target levels expand or shrink over the range of feasible values for a number of key parameters, the relative position of those regions is constant. If, however, the marginal cost functions, the damage function, or the predator-prey model took different forms we could see more fundamental changes in the solution. While such analysis is beyond the scope of this paper it may be a fertile area for future research.

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## A Appendix - Model Solution

The two maximization problems in the QVI that defines the solution to the impulse control problem present a significant computational problem, especially given the multi-dimensional nature of the state space. Therefore we take the approach of [Balikcioglu and Fackler, 2007] and transform the impulse control problem into the form of an optimal switching model. This transformation will replace the QVI in (4)-(6) with a set of

three variational inequalities (VIs) which may be more easily solved. The problem will now contain three separate value functions  $V^1$ ,  $V^2$ , and  $V^3$  which represent the “regimes” associated with waiting, applying the control on the invasive species, and applying the control on the predator population, respectively. The VI associated with being in regime 1 (the waiting regime) will be

$$\rho V^1(X_t, P_t) \geq F(X_t) + a(X_t, P_t)V_X^1 + c(X_t, P_t)V_P^1 + \frac{1}{2}b^2(X_t)V_{XX}^1 + \frac{1}{2}d^2(P_t)V_{PP}^1 + \sigma_{XP}b(X_t)d(P_t)V_{XP}^1, \quad (9)$$

$$V^1(X_t, P_t) \geq V^2(X_t, P_t) - C_x, \quad (10)$$

and

$$V^1(X_t, P_t) \geq V^3(X_t, P_t + \pi) - C_p, \quad (11)$$

where again one of the conditions must hold with equality. The interpretation of these conditions is quite intuitive, as (10)-(11) simply state that it is only optimal for the manager to exert a control if the value after the action has been taken net of the fixed cost is at least as large as the current value.

The VI associated with being in regime 2, that is the regime entered once the control on the invasive species is exerted, will be

$$V^2(X_t, P_t) \geq V^1(X_t, P_t), \quad (12)$$

$$\gamma(X_t) \geq -V_X^2(X_t, P_t), \quad (13)$$

and

$$V^2(X_t, P_t) \geq V^3(X_t, P_t) - C_p, \quad (14)$$

where again one of the condition must hold with equality. If it is optimal to continue to exert the control then it will be the case the (13) holds with equality. This has the intuitively appealing meaning that if it is optimal to exert the control then the marginal value must be equal to the marginal cost.

Similarly the VI for the third regime in which the manager is exerting the control on the predator population is defined as

$$V^3(X_t, P_t) \geq V^1(X_t, P_t), \quad (15)$$

$$V_X^3(X_t, P_t) \geq V^2(X_t, P_t) - C_x, \quad (16)$$

and

$$\pi(P_t) \geq V_P^3(X_t, P_t), \quad (17)$$

where again one of the condition must hold with equality. These conditions have a similar interpretation as the case of regime 2. Again the conditions state that the condition for stopping use of the control is that the marginal cost begins to exceed the marginal value.

In order to better facilitate the discussion of a numerical solution for the value functions and optimal control policies we note that the above set of VIs in (9)-(17) may be restated as a set of complementarity problems, such that

$$0 = \min [\rho V^1(X_t, P_t) - F(X_t) - \mathcal{L}V^1(X_t, P_t), V^1(X_t, P_t) - V^2(X_t, P_t) + C_x, V^1(X_t, P_t) - V^3(X_t, P_t) + C_p], \quad (18)$$

$$0 = \min [V^2(X_t, P_t) - V^1(X_t, P_t), \gamma(X_t) + V_X^2(X_t, P_t), V^2(X_t, P_t) - V^3(X_t, P_t) + C_p], \quad (19)$$

$$0 = \min [V^3(X_t, P_t) - V^1(X_t, P_t), V^3(X_t, P_t) - V^2(X_t, P_t) + C_x, \pi(P_t) - V_X^3(X_t, P_t)], \quad (20)$$

where

$$\mathcal{L}V(X_t, P_t) = a(X_t, P_t)V_X + c(X_t, P_t)V_P + \frac{1}{2}b^2(X_t)V_{XX} + \frac{1}{2}d^2(P_t)V_{PP} + \sigma_{XP}b(X_t)d(P_t)V_{XP}. \quad (21)$$

Since a closed form solution for the value functions in (18)-(20) does not exist we implement a collocation and projection method approach. We start by utilizing an approximation to the value functions  $V^i(X_t, P_t) \approx \phi(X_t, P_t)\theta^i$ , where  $\phi(X_t, P_t)$  is a family of  $n$  basis functions and  $\theta^i$  is a vector of  $n$  approximating coefficients for the value function in regime  $i$ . We solve for the  $3n$  unknown coefficients that define the value function approximations by insuring that the conditions in (18)-(20) hold for  $n$  sets of points in the state space. We denote the set basis function evaluated at the  $n$  points as the  $n \times n$  matrix  $\Phi$ . Then the complementarity problems in (18)-(20) evaluated at the  $n$  nodal points in the state space,  $(x, p)$ , may be rewritten as an extended vertical linear complementarity problem (EVLCP) of the form

$$0 = \min (M_1\Theta + q_1, M_2\Theta + q_2, M_3\Theta + q_3), \quad (22)$$

where  $\Theta = (\theta_1, \theta_2, \theta_3)^T$ ,

$$M_1 = \begin{pmatrix} \rho\Phi - L & 0 & 0 \\ -\Phi & \Phi & 0 \\ -\Phi & 0 & \Phi \end{pmatrix},$$

$$M_2 = \begin{pmatrix} \Phi & -\Phi & 0 \\ 0 & \Phi_X & 0 \\ 0 & -\Phi & \Phi \end{pmatrix},$$

$$M_3 = \begin{pmatrix} \Phi & 0 & -\Phi \\ 0 & \Phi & -\Phi \\ 0 & 0 & \Phi_P \end{pmatrix},$$

$$q_1 = \begin{pmatrix} F(x) \\ 0 \\ 0 \end{pmatrix},$$

$$q_2 = \begin{pmatrix} C_x i \\ \gamma(x) \\ C_x i \end{pmatrix},$$

$$q_3 = \begin{pmatrix} C_p i \\ C_p i \\ \pi(p) \end{pmatrix},$$

$$\begin{aligned} L_{jk} &= a(x_{jk}, p_{jk})V_X(x_{jk}, p_{jk}) + c(x_{jk}, p_{jk})V_P(x_{jk}, p_{jk}) + \frac{1}{2}b^2(x_{jk})V_{XX}(x_{jk}, p_{jk}) \\ &\quad + \frac{1}{2}d^2(p_{jk})V_{PP}(x_{jk}, p_{jk}) + \sigma_{XP}b(x_{jk})d(p_{jk})V_{XP}, \end{aligned}$$

and  $i$  is an  $n \times 1$  vector of ones. Applying the Fischer-Burmeister function to the EVLCP in (22) produces a semi-smooth function with the same roots as the complementarity problem therefore allowing for the use of Newton-type algorithms to obtain the approximating coefficients.