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An economic model of aboriginal fire-stick farming*

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Australian Aborigines faced a resource management problem, which they addressed though burning regimes, referred to as fire-stick farming. While dynamic economic analysis is clearly applicable, to date there have been no attempts to use it to model burning regimes. This paper develops a delayed-response optimal-control model to describe Aboriginal fire-stick farming. The model explains a collective welfare maximizing burning regime that successfully controlled wildfires, protected the resources essential to survival, and, incidentally, produced a biodiverse landscape and limited greenhouse gas emissions. When the parameters of the model are changed to reflect the current institutional realities of reduced access to the land, and less direct dependence on it, traditional Aboriginal burning is prevented or delayed, fuel loads build up and uncontrolled fires are more likely to occur, damaging previously protected species. If Aboriginal burning is to be used to control fires successfully in a modern resource management context, it is necessary to adjust for changes in the institutional incentive structure. Payments for carbon offsets are an example of replacing lost incentives with new ones.

Key words: Aborigine, catastrophe, fire-stick farming, optimal control, wildfire.

1. Introduction

This paper develops an economic model of the traditional institution of fire-stick farming among Australian Aborigines. Fire-stick farming has received considerable attention outside the economics literature (Jones 1969; Singh *et al*. 1981; Rose 1992; Kohn 1996). Aborigines burned vegetation at regular intervals to ensure their food supply. This meant opening up dense vegetation to facilitate new growth, which directly provided plant foods and indirectly provided animal foods. According to Vigilante *et al*. (2009), it also meant protecting fire sensitive rainforest patches, which contained other food sources, from uncontrolled fire damage. Over a long period of time, burning became part of Aboriginal religion and culture (Rose 1992). We use an optimal-control model with the burning interval as a control variable and a delayed response in the growth of the vegetative stock. Both food needs and cultural factors influence the choice of burning interval.

The success of traditional Aboriginal regimes in controlling wildfire and producing biodiversity (Williams *et al*. 2002; Bird *et al*. 2008), and the increase in uncontrolled fires and loss of biodiversity following their demise,

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have motivated consideration of their reintroduction (Whitehead *et al*. 2003). Interest has been further spurred by the recognition that large uncontrolled fires contribute a significant portion of Australia's greenhouse gas emissions (Russell-Smith *et al*. 2009; Heckbert *et al*. 2011). Reviewing work on the carbon storage potential of Northern Australia's fire-prone savannas, Murphy *et al*. (2009) conclude that frequent severe fires result in decreased woody biomass and increased carbon emissions, while frequent mild fires have little effect. It has also been shown that strategic burning to reduce fuel loads reduces other greenhouse gases such as nitrous oxide and methane.

However, reintroduction of traditional Aboriginal fire regimes will not necessarily successfully control wildfire. When the parameters of the model are changed to reflect the current institutional realities of reduced access to the land and less direct dependence on it, Aboriginal burning is prevented or delayed, fuel loads build up, uncontrolled fires occur, greenhouse emissions increase and fire-sensitive vegetation is hastened toward elimination. The lesson is that the cultural codes, which traditional societies used for resource allocation, were developed in the context of the existing constraints and parameters. Applying these codes in a different context can produce different, and sometimes undesirable, results. If Aboriginal burning is to be used to control fires successfully in a modern resource management context, where biodiversity preservation and reduced greenhouse gas emissions are broader societal goals, it is necessary to adjust for changes in the institutional incentive structure. This means active involvement by outside agents or the state to provide appropriate incentives (Altman 2004).

Section 2 provides background on traditional fire management by Aborigines. Section 3 discusses uncontrolled fire as a function of fuel load. Section 4 presents the optimal-control model of traditional Aboriginal fire management, Section 5 translates it to the modern context and Section 6 concludes.

2. Aborigines and fire

Australia has long been a land of aridity and fire. Fire is dependent on the availability of fuel. In dry climates, with few biological agents to break down the litter, fire recycles it. If the fuel load is great enough, very intense fires are likely to develop. High intensity fires can spread unpredictably and uncontrollably, causing widespread damage.[1]

Controlling fire has been a dominant factor in Aboriginal resource management in Australia (Jones 1975; Pyne 1991; Flannery 1994; Gammage 2011). Traditional Aboriginal management changed the fire regime from one

[1] For example, in high intensity fires, firebrands are blown upward and may be transported considerable distances to spot new fires. The filigree bark strips from stringybark or candlebark varieties of Eucalyptus make excellent firebrands (Luke and McArthur 1978). In savanna regions, a broken canopy with a continuous dense vegetation mat can carry fires for considerable distances (Heckbert *et al*. 2011).

of less frequent high intensity fires to one of more frequent low intensity fires. Aborigines knew that without frequent fire, many landscapes would become dense thickets, impeding access by humans, and creating the fuel necessary for uncontrollable fire that would destroy the resources on which they relied. Early European accounts of the Australian landscapes created by Aborigines often described parklands though which movement was easily possible (O'Brian 1987). Jones (1969) listed a number of uses of fire by Aborigines. These included expanding their habitat by clearing rainforest, regeneration of plant food for themselves and kangaroos, facilitating travel and increasing awareness of poisonous snakes. Some areas, such as fire-sensitive rainforest pockets, were normally not burned (Haynes 1985; Lewis 1989; Head 1994; Russell-Smith *et al.* 1997). These areas were sources of food (such as fruit and yams) and had religious significance. Although a pocket of rainforest would be unlikely to be completely destroyed by fire, it could certainly be damaged by all but the most carefully controlled fire. Regular disturbance by fire would eventually eliminate it (Pyne 1991).

In Aboriginal culture, ownership provides rights to the resources of the land, as well as responsibilities for managing (burning) the land. The clan's lands and resources were owned in common, and there were numerous rules about the use of fire. These were designed to prevent the over-exploitation or destruction of resources by fire. Over time they became part of a fire-related cultural code. Burning to keep the country clean was one of the most important responsibilities that Aborigines had to the land (Jones 1969; Singh *et al.* 1981; Rose 1992; Kohn 1996). A number of authors (Haynes 1985; Lewis 1989; Head 1994; Russell-Smith *et al.* 1997) have provided evidence that Aborigines retain a part of their fire-related cultural code in the form of a cleaning-up ethic. Even in a situation of dispossession, the cultural motive for burning remains. The greatest concern to displaced Aborigines is not that they cannot burn for specific management reasons, but rather that the country needs cleaning-up. Previous burning patterns have been interrupted for a variety of reasons. Areas that were previously deliberately burned now are rarely burned. But, the cleaning-up ethic remains a motive to burn.

In areas where management by Aboriginal owners has been continuous, it has been shown that fuel suppression and coordinated fire use has reduced the incidence of uncontrolled fire and maintained diverse wildlife habitats (Yibarbuk *et al.* 2001). Although the cleaning-up ethic may remain sufficiently strong to motivate occasional burning, it has not been strong enough to prevent delays in burning and the associated fuel build-up. With greater fuel loads, uncontrolled fires are more likely, and fire-sensitive vegetation such as rainforest is likely to suffer (Head 1994). A cultural code, which could limit uncontrolled fires and sustain a resource base, now, cannot achieve these goals without assistance. A recent recommendation for northern Australia, where uncontrolled fire is a serious threat to biodiversity preservation, and remoteness is a great challenge for fire managers, is to actively support the skills and interests of the Aboriginal people to achieve more effective fire management and the associated goal of biodiversity preservation (Whitehead *et al.* 2003).

3. Uncontrolled fire and catastrophe

The modern literature on controlling wildfires describes wildfires as having two states of intensity: a controlled state and an uncontrolled state. In the first state, fire burns under environmental control at relatively low intensity level (<500 kilowatts per meter) (Chapman 1999). If the intensity level reaches a higher threshold level, there can be a sudden transition to an uncontrolled state in which the fire's spread becomes unpredictable and uncontrollable. The intensity of a fire, and hence its controllability, is influenced by the available fuel, as well as other factors, including the heat content of the fuel, wind velocity, the slope of the terrain and the moisture content of the fuel (Andrews 1988; Trevitt 1991; Chapman 1999). Hesseln *et al.* (1998) have argued that the controllability of wildfires can be analyzed as a cusp catastrophe. Three of the characteristics of a cusp catastrophe are bimodality, sudden jumps (catastrophes) and inaccessibility. Biomodality means that there are two distinct stable equilibrium states. Sudden jumps from one to the other of these equilibrium states can occur. Other equilibria exist, but are unstable and hence inaccessible. Building on the basic insight of Hesseln *et al.* (1998), wildfires can be modelled using a mathematical catastrophe model based on the spruce budworm model (Ludwig *et al.* 1978; Murray 1989).

Let X represent fire intensity. Its dynamics are represented by equation (1).

$$\dot{X} = RX\left(1 - \frac{X}{M}\right) - \kappa(X). \tag{1}$$

Here, R is the available fuel, and M is the maximum attainable intensity, given the fuel's heat and moisture content. The term $\kappa(X)$ represents the fact that at low intensity levels, the fire is controllable by environmental factors such as wind velocity and the slope of the terrain. Let $\kappa(X) = BX^2/(A^2 + X^2)$. At low intensity levels, $\kappa(X)/X$ is increasing, and at high intensity levels, it is decreasing. At high intensity levels, its influence is minimal, and the fire is uncontrollable. Then (1) becomes:

$$\dot{X} = RX\left(1 - \frac{X}{M}\right) - \frac{BX^2}{(A^2 + X^2)}. \tag{2}$$

The steady state solutions are obtained by setting the right hand side (RHS) of (2) equal to zero. This gives $X = 0$ (unstable) as one of the steady states, with the rest determined by $R\left(1 - \frac{X}{M}\right) = \frac{BX}{(A^2+X^2)}$. Holding M, A and B constant, there can be one, three or again one, steady state equilibrium, depending on the level of the fuel load. As Figure 1 shows, with a low fuel load, R_a, there will be one

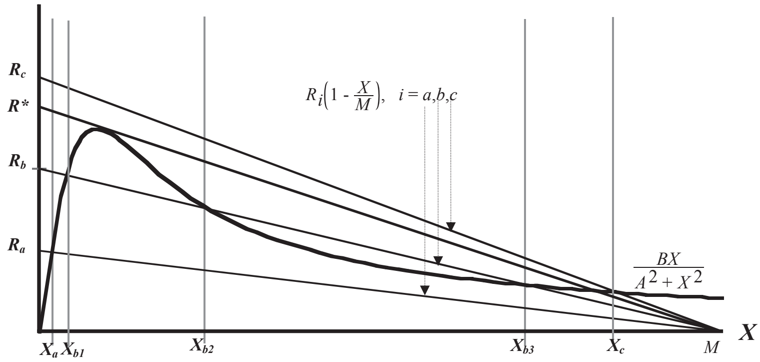

Figure 1 Equilibrium states for fire intensity model.

stable equilibrium at a low fire intensity of X_a. With a fuel load R_b, there are stable equilibria at X_{b1} and X_{b3}, and an unstable one at X_{b2}. If we start with a fuel load R_a and gradually increase it, the equilibrium fire intensity will monotonically increase from X_a. As the fuel load increases to R_b, the low intensity equilibrium will move to X_{b1}. However, at fuel loads larger than R^*, there is a jump to a high intensity equilibrium. With fuel load R_c, there will be a high intensity equilibrium at X_c.[2] Whereas X_a and X_{b1} are controlled fire equilibria, X_c is an uncontrolled fire equilibrium.

While Aboriginal fire management took account of variables influencing M, A and B, fuel load was the variable on which their burning efforts were focused. Fires were largely controlled by adjusting the frequency of burning to keep the fuel load from becoming too large (Lewis 1989). In the delayed-response optimal-control model that follows, M, A and B are held constant. Burning regimes only influence the probability of a jump, from a controlled fire equilibrium to an uncontrolled fire equilibrium, through their influence on fuel load, R. We may not know the precise fuel load level at which a fire will become uncontrolled, but we do know that the higher the fuel load, the greater the probability that $R > R^*$, and that an ignited fire will become uncontrolled, easily spreading and causing damage to separate rainforest patches that would not be intentionally burned.

4. Optimal-control model

For a traditional Aboriginal society, which depended on the resources from its land, the problem was one of infinite horizon dynamic optimization in the

[2] This model exhibits hysteresis, which implies that the path from equilibrium X_{b1} to equilibrium X_c is not the same as that from X_c to X_{b1}. However, from a fire control perspective, our only concern is with preventing movements from X_{b1} to equilibrium X_c.

allocation of resources. The flow of net benefits from controlled burning must be maximized into the infinite future subject to constraints associated with the biological nature of vegetation/fuel load growth and the characteristics of fire. Though vegetation dynamics is often modelled as a continuous time problem, for analytical convenience, discrete time is used.

The model is an optimal-control model involving delayed response. Examples include the delayed-recruitment models in fisheries economics (Clark 1976) and the lagged entry models of limit pricing (De Bondt 1976; Kamien and Schwartz 1981). There is one stock variable in the model, the fuel load (R). It is related to the vegetation on a block of land of fixed size, L. The fuel load on L can be accidentally or deliberately ignited, with accidental fires having the potential to become uncontrolled.

There is also assumed to be a rainforest biomass containing important food items, which exists separately from L, is not part of the fuel load and is burned only if attacked by an uncontrolled wildfire originating on L. The time unit is 1 year. Traditional burning regimes varied according to climate and purposes, with burning frequencies anywhere from 1 to 5 years or more (Gammage 2011). Our model is a stylized one, mimicking conditions in parts of northern Australian's tropical savanna, where the fuel load on unburned land can reach its maximum level within 4 years (Gill *et al.* 1990; Williams *et al.* 1999; Russell-Smith *et al.* 2003). With this fuel load profile, burning every third year or never burning dominates any intermediate burning regime, giving four potential regimes: burning every year, every second year, every third year or never burning.[3]

The fuel load is characterized as the amount of vegetation on L. Initially assume no deliberate burning; only accidental burning in t. Between year t and year $t+1$, the fuel load will decrease on land that is accidentally burned and increase on land that is not. The proportion of L that will be accidentally burned in any year is ϕ, and the proportion remaining unburned is θ.[4] Accidental burning removes the entire existing fuel load on the land burned. Between t and $t+1$, the fuel load decrease on L due to accidental burning in t is ϕR_t tonnes. On land that was not burned in t, there will be growth in the fuel load. If a hectare of land was burned in $t-1$, and is not reburned in t, it will accumulate s_1 tonnes of fuel between t and $t+1$. If it was burned in $t-2$, and is not reburned in $t-1$ or t, it will accumulate s_2 tonnes between t and $t+1$. If it was burned in $t-3$, and is not reburned in $t-2$, $t-1$ or t, it will accumulate s_3 tonnes between t and $t+1$. Since we are applying the model in the context of northern Australia, we use a fuel load curve similar to Russell-Smith *et al.* (2003), and assume,

[3] Other fuel load profiles will be observed in other areas, or with changing climate and weather conditions. The model can be adapted to other areas or conditions with longer or more time periods and different accidental burning probabilities.

[4] ϕ could also be treated as the probability that L will be accidentally burned at any time, t.

following Gill *et al*. (1990) and Williams *et al*. (1999), that the fuel load on unburned land increases to a maximum within 4 years. Specifically, we assume that $s_1 > s_2 > s_3 > 0$, and that, after 3 years, there is no further fuel load accumulation.

Assume the initial condition that all of L has been completely burned and is just beginning regrowth. The initial fuel load is zero. Denote the area beginning regrowth as y_{t-1}, where $y_{t-1} = L$. In the first year after regrowth begins, accidental burning affects ϕy_{t-1} hectares of the area regrowing and removes $\phi R_t = 0$ tonnes of fuel load. On the burned ϕy_{t-1} there is no opportunity for regrowth in the first year. On the remaining θy_{t-1} hectares, there is a fuel load build-up of $\theta s_1 y_{t-1}$ tonnes. Overall, $R_{t+1} - R_t = -\phi R_t + \theta s_1 y_{t-1} = \theta s_1 y_{t-1}$ for the first year after regrowth.

When considering the increase in fuel load in the second year after regrowth has begun, we define $y_{t-2} = L$ as the area initially beginning regrowth, $R_{t-1} = 0$, and $R_t = \theta s_1 y_{t-2}$ as the fuel load after 1 year. To calculate $R_{t+1} - R_t$, recognize that there will again be accidental burning of ϕy_{t-2} hectares, causing a fuel load loss of $\phi R_t = \phi\theta s_1 y_{t-2}$ tonnes. While there will be no fuel load growth on the newly burned land during that year, fuel load growth will occur on θy_{t-2} hectares, of which $\phi\theta y_{t-2}$ were burned in the previous year and $\theta^2 y_{t-2}$ have not been burned for 2 years. The fuel load increase on the θy_{t-2} hectares is $(\phi\theta s_1 + \theta^2 s_2)y_{t-2}$. Defining $S_2 = \phi\theta s_1 + \theta^2 s_2$, gives $R_{t+1} - R_t = -\phi R_t + S_2 y_{t-2}$.

A similar logic can be used for the third, fourth and subsequent years after regrowth. The details are provided in Appendix I. Defining $S_3 = \phi\theta s_1 + \phi\theta^2 s_2 + \theta^3 s_3$, gives $R_{t+1} - R_t = -\phi R_t + S_3 y_{t-3}$. For the fourth year after reburning, $s_4 = 0$, and fuel load change becomes $R_{t+1} - R_t = -\phi R_t + (\phi\theta s_1 + \phi\theta^2 s_2 + \phi\theta^3 s_3)y_{t-4}$. Defining the bracketed term as S_4, $R_{t+1} - R_t = -\phi R_t + S_4 y_{t-4}$. Because there is no growth in fuel load after the third year, the change in fuel load for all subsequent years after burning remains the same as for the fourth year. With all of the options included, the fuel load dynamics without deliberate burning are described as

$$R_{t+1} - R_t = -\phi R_t + \theta s_1 y_{t-1} + S_2 y_{t-2} + S_3 y_{t-3} + S_4 \sum\nolimits_{n=4}^{N} y_{t-n}. \qquad (3)$$

The steady state level of fuel load occurs when $R = \frac{S_4 y_{t-n}}{\phi}$. For illustration, using the parameter values $\phi = 0.2$, $\theta = 0.8$, $s_1 = 1$, $s_2 = 0.5$, $s_3 = 0.2$ and $y_{t-1} = y_{t-2} = y_{t-3} = y_{t-n} = L = 5$, the steady state fuel load is $R = 6.112$. This is the top line on Figure 2.

Now, consider deliberate burning. It will occur only on land not accidentally burned. Deliberate burning in year t can take place on land that was last burned n years ago ($x_{n,t}$), where $n \,\varepsilon\, \{1,2,3\ldots\}$ and $0 \le x_{n,t} \le L \ \forall\ n, t$. On land, which is deliberately burned, all of the accumulated fuel is removed. With deliberate burning, the general form for the fuel load dynamics is:

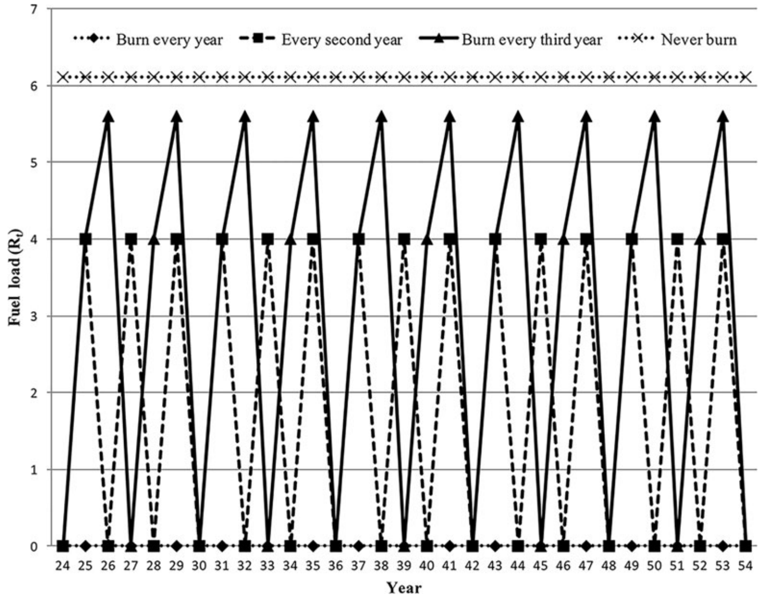

Figure 2 Equilibrium fuel load (R).

$$
\begin{aligned}
R_{t+1} - R_t = {} & -\phi R_t + \theta s_1 y_{t-1} + S_2(y_{t-2} - x_{1,t-1}) \\
& + S_3(y_{t-3} - x_{1,t-2} - x_{2,t-1}) + S_4 \sum_{n=4}^{N} \left(y_{t-n} - \sum_{j=1}^{n-1} x_{j,t-n+j} \right) \\
& - \theta s_1 x_{1,t} - \left(\theta s_1 + \theta^2 s_2\right) x_{2,t} - \left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) \sum\nolimits_{n=3}^{N} x_{n,t}. \qquad (4)
\end{aligned}
$$

The first RHS term is the loss of fuel load due to accidental burning. The next four terms represent fuel load growth, adjusted for any previous deliberate burning. The final three terms are to account for deliberate burning in year t.

Suppose that the land is deliberately burned every year, then only the x_1 terms are positive, with $x_{1,t-1} = y_{t-2}$, $x_{1,t-2} = y_{t-3}$, and $x_{1,t-n+1} = y_{t-n}$. The third, fourth and fifth terms on the RHS of (4) are equal to zero. This leaves $\theta s_1 y_{t-1}$ as the fuel load accumulation between t and $t+1$. However, deliberate burning, $\theta s_1 x_{1,t}$, removes this accumulation, and $R_{t+1} - R_t$ is simply equal to the loss from accidental burning, $-\phi R_t$. Using the same parameter values as above, the steady state fuel load is $R = 0$. This is the line along the horizontal axis in Figure 2.

For burning every second year, $y_{t-1} = 0$ because the last deliberate burning must have been 2 years ago. Only the x_2 terms are positive, with $y_{t-3} = x_{2,t-1}$ and $y_{t-n} = x_{2,t-n+2}$. The second, fourth and fifth terms on the RHS of (4) are equal to zero. This leaves $R_{t+1} - R_t = -\phi R_t + S_2 y_{t-2} - (\theta s_1 + \theta^2 s_2) x_{2,t}$, where $S_2 y_{t-2} - (\theta s_1 + \theta^2 s_2) x_{2,t}$ is the fuel load growth net of deliberate burning. Although $x_{2,t} = y_{t-2}$, the amount deliberately burned will exceed the fuel load growth between t and $t+1$ because some of what is burned will have been accumulated in the previous year.[5]

Following a similar approach, gives $R_{t+1} - R_t = -\phi R_t + S_3 y_{t-3} - (\theta s_1 + \theta^2 s_2 + \theta^2 s_3)x_{3,t}$ for burning every third year, and $R_{t+1} - R_t = -\phi R_t + S_4 y_{t-4} - (\theta s_1 + \theta^2 s_2 + \theta^2 s_3)x_{4,t}$ for burning every fourth year. Because $s_4 = 0$, the dynamics for all subsequent years will be the same as those for the fourth year.

With the same parameter values as used for the extreme cases of annual burning and no burning, the equilibrium fuel loads for the intermediate burning regimes are shown in Figure 2. For burning every second year, initial regrowth began 2 years ago in $t-2$, with y_{t-2} hectares. $R_t - R_{t-1} = -\phi R_{t-1} - \theta s_1 y_{t-2}$ and $R_{t+1} - R_t = -\phi R_t + S_2 y_{t-2} - (\theta s_1 + \theta^2 s_2)x_{2,t} = -\phi R_t - \theta^2 s_1 x_{2,t}$. If the initial condition is $R_{t-1} = 0$, $R_t - R_{t-1} = \theta s_1 y_{t-2} = 4$, and $R_{t+1} - R_t = -\phi R_t - \theta^2 s_1 x_{2,t} = -0.8 - 3.2 = -4$. The equilibrium is cyclical, with the fuel load moving between $R = 0$ and $R = 4$, as is shown by the dashed line in Figure 2. For burning every third year, initial regrowth began 3 years ago in $t-3$, with y_{t-3} hectares. $R_{t-1} - R_{t-2} = -\phi R_{t-2} + \theta s_1 y_{t-3}$, $R_t - R_{t-1} = -\phi R_{t-1} + S_2 y_{t-3}$, and $R_{t+1} - R_t = -\phi R_t + S_3 y_{t-3} - (\theta s_1 + \theta^2 s_2 + \theta^3 s_3)x_{3,t}$. With $R_{t-2} = 0$, $R_{t-1} - R_{t-2} = \theta s_1 y_{t-3} = 4$, $R_t - R_{t-1} = -0.8+2.4 = 1.6$, and $R_{t+1} - R_t = -1.12+1.632 - 6.112 = -5.6$. The fuel load cycles between $R = 0$, $R = 4$ and $R = 5.6$, as is shown by the solid line in Figure 2.

While (4) incorporates regrowth after accidental burning, within a burning cycle, it does not accommodate regrowth between cycles (after deliberate burning). There must be an additional constraint that requires regrowth between burning cycles or after deliberate burning. At any year t in which deliberate burning occurs, the area accidently or deliberately burned must immediately begin regrowth. The regrowth constraint is expressed as:

$$\sum_{n=1}^{N} x_{n,t} - y_t = 0. \qquad (5)$$

In the maximizing problem, the shadow price of this constraint represents the marginal benefit/cost associated with the fact that there will be fuel load regrowth on deliberately burned land.

Deliberate burning on L, whenever it occurs, has a marginal cost per hectare of c, and produces some food benefits. Let p be the marginal value product of food per hectare burnt on L. Deliberate burning on L also reduces its fuel load, and the risk of an uncontrolled wildfire. The cost of an uncontrolled wildfire is that it will spread to a patch of rainforest vegetation.

[5] $S_2 y_{t-2} = \phi\theta s_1 y_{t-2} + \theta^2 s_2 y_{t-2}$ is the fuel load growth between t and t+1. However, the growth implied by the second RHS term could not have occurred if there had not been $\theta^2 s_1 y_{t-2}$ remaining from growth in the previous year. Deliberate burning in t will destroy both $\theta^2 s_1 y_{t-2}$ and $\phi\theta s_1 y_{t-2} + \theta^2 s_2 y_{t-2}$, for a total of $(\theta s_1 + \theta^2 s_2)y_{t-2} = (\theta s_1 + \theta^2 s_2)x_{2,t}$ tonnes removed by burning.

A healthy rainforest patch produces food with a value q, and its complete destruction imposes a loss of q. From Section 3, we know that the probability of an uncontrolled fire on L, conditional on accidental ignition, is a function of available fuel load. With R_t as the fuel load that would be burned by an accidental fire on L, let that function be a linear one: $\Pi_t = \pi R_t$.[6] Assuming that deliberate fires are always controlled, uncontrolled fires on L are the result of accidental ignitions when fuel loads are high. The unconditional probability of an uncontrolled fire is then $\phi\Pi_t = \phi\pi R_t$. If a fire is accidentally ignited, and becomes uncontrolled, it will cause damage to the rainforest patch. The expected rainforest damage is the product of the unconditional probability of an uncontrolled fire and the damage that fire will cause to the rainforest, or $D_t = q\phi\pi R_t$.

Finally, there are cultural benefits of burning: the cleaning-up benefits. We treat the cleaning up motive as creating an additional benefit, P, per unit of food made available on L, and an additional cost, Q, for the loss of the rainforest patch.

With a discount rate of r, a discount factor of $\rho = \frac{1}{1+r}$, λ_{t+1} as the shadow price of the fuel load dynamics constraint, and β as the shadow price for the regrowth constraint, the current value Hamiltonian for this problem, incorporating the constraints from both (4) and (5), is:

$$
\begin{aligned}
H = {} & (p + P - c)\sum_{n=1}^{N} \theta x_{n,t} - (q + Q)\phi\pi R_t \\
& + \rho\lambda_{t+1}\left[-\phi R_t + \theta s_1 y_{t-1} + S_2\left(y_{t-2} - x_{1,t-1}\right) + S_3\left(y_{t-3} - x_{1,t-2} - x_{2,t-1}\right)\right] \\
& + \rho\lambda_{t+1}\left[S_4\left(\sum_{n=4}^{N} y_{t-n} - \sum_{j=1}^{n-1} x_{j,t-n+j}\right)\right] \\
& - \rho\lambda_{t+1}\left[\theta s_1 x_{1,t} + \left(\theta s_1 + \theta^2 s_2\right)x_{2,t}\right] \\
& - \rho\lambda_{t+1}\left[\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right)\sum_{n=3}^{N} x_{n,t}\right] + \beta\left[\sum_{n=1}^{N} x_{n,t} - y_t\right].
\end{aligned}
\tag{6}
$$

The control variables are the $x_{n,t}$ and y_t. Since the Hamiltonian is linear in the control variables, the solution is of the bang–bang type. Any $x_{n,t}$ will be either zero or L in a given year. At most, one of the $x_{n,t}$ can be positive. The area regrowing will be zero if $x_{n,t} = 0$, or L if $x_{n,t} = L$. The optimal controls are determined by the following switching functions. Because either $x_{3,t}$ or never burning dominate any intermediate burning regimes, switching functions for $x_{n,t}$, $n \geq 4$ are not presented.[7]

[6] There are non-linear functions that could have been used. For example, a probit or logit function. The linear function is used here because it is the simplest.

[7] Appendix II provides the proof that $x_{3,t}$ or never burning dominate any intermediate burning regimes.

$$x_{1,t} = \begin{cases} 0 & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\theta s_1 - \sum_{m=2}^{3}\rho^m\lambda_{t+m}S_m - S_4\sum_{n=4}^{N-1}\rho^n\lambda_{t+n} + \beta < 0 \\ \text{switch} & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\theta s_1 - \sum_{m=2}^{3}\rho^m\lambda_{t+m}S_m \\ & \quad - S_4\sum_{n=4}^{N-1}\rho^n\lambda_{t+n} + \beta = 0. \\ L & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\theta s_1 - \sum_{m=2}^{3}\rho^m\lambda_{t+m}S_m - S_4\sum_{n=4}^{N-1}\rho^n\lambda_{t+n} + \beta > 0 \end{cases} \tag{7a}$$

$$x_{2,t} = \begin{cases} 0 & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2\right) - \rho^2\lambda_{t+2}S_3 - S_4\sum_{n=3}^{N-2}\rho^n\lambda_{t+n} + \beta < 0 \\ \text{switch} & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2\right) - \rho^2\lambda_{t+2}S_3 \\ & \quad - S_4\sum_{n=3}^{N-2}\rho^n\lambda_{t+n} + \beta = 0. \\ L & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2\right) - \rho^2\lambda_{t+2}S_3 - S_4\sum_{n=3}^{N-2}\rho^n\lambda_{t+n} + \beta > 0 \end{cases} \tag{7b}$$

$$x_{3,t} = \begin{cases} 0 & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) - S_4\sum_{n=2}^{N-3}\rho^n\lambda_{t+n} + \beta < 0 \\ \text{switch} & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) \\ & \quad - S_4\sum_{n=2}^{N-3}\rho^n\lambda_{t+n} + \beta = 0. \\ L & \text{if } (p+P-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) - S_4\sum_{n=2}^{N-3}\rho^n\lambda_{t+n} + \beta > 0 \end{cases} \tag{7c}$$

$$y_t = \begin{cases} 0 & \text{if } \rho^2\lambda_{t+2}\theta s_1 + \rho^3\lambda_{t+3}S_2 + \rho^4\lambda_{t+4}S_3 + S_4\sum_{n=5}^{N}\rho^n\lambda_{t+n} - \beta > 0 \\ \text{switch} & \text{if } \rho^2\lambda_{t+2}\theta s_1 + \rho^3\lambda_{t+3}S_2 + \rho^4\lambda_{t+4}S_3 + S_4\sum_{n=5}^{N}\rho^n\lambda_{t+n} - \beta = 0. \\ L & \text{if } \rho^2\lambda_{t+2}\theta s_1 + \rho^3\lambda_{t+3}S_2 + \rho^4\lambda_{t+4}S_3 + S_4\sum_{n=5}^{N}\rho^n\lambda_{t+n} - \beta < 0 \end{cases} \tag{7d}$$

For deliberate burning to occur, one of the (7a), (7b) or (7c) must be non-negative, and (7d) must be non-positive. Only one burning interval will be chosen, and it will be the one which generates the highest value for β.[8] If all of

[8] We assume that the switch to burning occurs instantaneously when the switching condition holds.

(7a) through (7c) are negative and (7d) is positive, there will never be deliberate burning.

The dynamics of the shadow price for the fuel load is described by the adjoint equation:

$$(q+Q)\phi\pi = \rho\theta\lambda_{t+1} - \lambda_t. \tag{8}$$

The $t+1$ shadow price is discounted by $\rho\theta$, the product of the discount factor and the probability of no accidental burning. The discounted shadow price of the fuel load, λ_t, is negative and represents the marginal present value of the cost of fuel load build-up. With annual rainforest loss, $(q + Q)\phi\pi$, positive and constant through time, $\rho\theta\lambda_{t+1}$ will be less negative than λ_t and the difference will remain constant though time. As time passes, the discounted shadow price increases by the expected annual rainforest loss, which includes both food loss, q, and cultural loss, Q.

The next step is to find the burning interval which maximizes the present value of the net benefits generated. We consider the three burning intervals, $x_{1,t}$, $x_{2,t}$ and $x_{3,t}$, and the option to never burn. First consider burning every second year, or $x_{2,t} = L$, defining $\beta = \beta_2$ for this interval. Recognizing that the burning actually takes place 2 years after regrowth from the last burning, condition (7b) can be adjusted forward to $t+2$ and written as:

$$(p+P-c)\theta\rho^2 - \rho^3\lambda_{t+3}(\theta s_1 + \theta^2 s_2) - \rho^4\lambda_{t+4}S_3 - S_4\sum_{n=5}^{N}\rho^n\lambda_{t+n} + \beta_2\rho^2 \geq 0. \tag{9}$$

The first LHS term represents the annual food and clean-up benefits from burning on L. The second and third, and fourth are the present value of benefits from removing fuel load. The fifth is the value of the newly burned land that will be burned again every 2 years. When there is burning, it is also implied that (7d) gives:

$$\beta_2 \geq \rho^2\lambda_{t+2}\theta s_1 + \rho^3\lambda_{t+3}S_2 + \rho^4\lambda_{t+4}S_3 + S_4\sum_{n=5}^{N}\rho^n\lambda_{t+n}. \tag{10}$$

It is as least as beneficial (or no more costly) to regrow when there will be deliberate burning every second year, than if the regrowth would never be deliberately burned. We can use (9) and (10) to calculate the implied value of β_2. Solving (9) for $\rho^4\lambda_{t+4}S_3 + S_4\sum_{n=5}^{N}\rho^4\lambda_{t+n}$, and substituting into (10), gives:

$$\beta_2 \geq \frac{\rho^2\theta(p+P-c) + \rho^2\theta s_1(\lambda_{t+2} - \rho\theta\lambda_{t+3})}{1-\rho^2}. \tag{11}$$

The RHS of (11) is the present value of a hectare of land in a 2 year burning cycle repeated indefinitely. Because this is the definition of β_2, the equality in (11) must hold. Finally, substituting $\lambda_{t+2} - \rho\theta\lambda_{t+3} = -(q + Q)\phi\pi$ from (8), gives:

$$\beta_2 \geq \frac{\rho^2\theta(p+P-c)+\rho^2\theta s_1(q+Q)\phi\pi}{1-\rho^2}. \tag{12b}$$

In the numerator we have the direct net benefits on L of burning after 2 years, minus the expected rainforest damage accruing within the 2 year cycle. Both are discounted back to t. In the denominator, we have the factor that adjusts for repeated cycles.

Using the same approach, but assuming $x_{1,t} = L$ or $x_{3,t} = L$, instead of $x_{2,t} = L$, yields (12a) and (12c), respectively.

$$\beta_1 = \frac{\rho\theta(p+P-c)}{1+\rho}. \tag{12a}$$

$$\beta_3 = \frac{\rho^3\theta(p+P-c)-(\rho^2\theta s_1+\rho^3\theta S_1+\rho^3\theta^2 s_2)(q+Q)\phi\pi}{1-\rho^3}. \tag{12c}$$

In (12a), because there is no fuel accumulation under an annual burning regime, there are no rainforest damages. On the other hand, (12c) exhibits a greater expected rainforest damage than (12b) because of the longer period for fuel accumulation.

In addition, we consider the option of never burning. Condition (7d) alone is used to derive β_∞. The LHS of (7d) will be positive, yielding $y_t = 0$ (no burning implies no regrowth) and giving β_∞ as:

$$\beta_\infty < \frac{-(\rho^2\theta s_1+\rho^3\theta^2 s_2+\rho^4\theta^3 s_3)(q+Q)\phi\pi}{1-\rho}. \tag{12d}$$

The RHS is the boundary of β values below which burning would never occur.

Society's goal is to maximize the present value of its L hectares. Because unburned fuel loads reach their maximum within 4 years ($s_4 = 0$), either β_3 or β_∞ will be at least as great as the β for any intermediate burning frequency. Appendix II provides the proof. Hence, society will choose $\beta_{\max} = \max(\beta_1, \beta_2, \beta_3, \beta_\infty)$.

We parameterize the model to mimic burning regimes in Northern Australia, where, under traditional management, burning by Aborigines living on the land took place annually. With $s_1 = 1$, $s_2 = 0.5$, $s_3 = 0.2$, $r = 0.03$, $\rho = 0.97087$, $\theta = 0.8$, $p = P = 1.5$, $c = 1$, and $p + P - c = 2$, we can plot β_1, β_2, β_3 and β_∞ as functions of $(Q + q)\phi\pi$ using (12a) through (12d). Three of the four β functions are negatively sloped: as $(Q + q)\phi\pi$ (the annual marginal cost of an extra unit of fuel load) increases, β (the marginal present value of newly burned land) decreases. The exception is (12a), the function for annual deliberate burning, which has a zero slope because there is never any fuel load build-up. Each of (12a), (12b) and (12c) have positive intercept terms, (12d) has a zero intercept term. With $\rho < 1$ and $\theta < 1$, the intercept increases with the frequency of burning. Figure 3a shows the four β functions. Regardless of the value of $(Q + q)\phi\pi$, the optimal strategy is to burn every year. The net benefits of burning ($p + P - c = 2$) are obtained more frequently, and there is no cost associated with fuel load build-up. The other options provide less frequent net benefits and greater costs from fuel load build-up.

5. Traditional aboriginal regimes in contemporary management

Traditional Aboriginal burning practices are derived from an understanding of fires and their influence on the ecological system. The ethic of cleaning-up

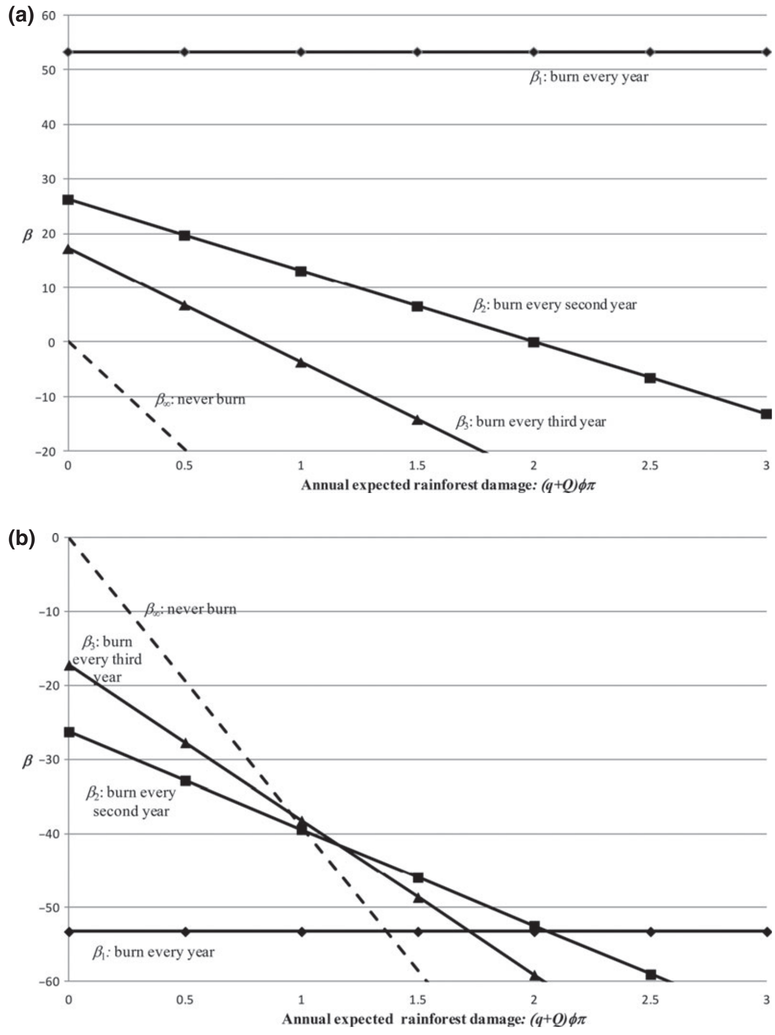

Figure 3 (a) Land value (β): positive net benefits from burning. (b) Land value (β): negative net benefits from burning.

the country to remove fuel accumulation arose from the desire to avoid intense uncontrolled fires and acquire food. When Aboriginals no longer live on the land, or have easy access to it, the understanding of fire may not have changed, and the ethic of cleaning-up the country may remain, but practicalities will have changed. The cost of burning will be higher, due to lack of access. The food supply benefits will be less, due to the availability of cheap alternative food sources. When these parameters change, the outcome will change also. For example, if c is increased to $c = 3.5$, p is decreased to $p = 0$, and P remains unchanged, $p + P - c = -2$. Figure 3b shows the influence of these parameter changes on the optimal-burning intervals. Compared to Figure 3a, the intercepts of the three β functions involving burning have shifted down, with the magnitude of the shift directly related to the burning frequency. The β_∞ function for never burning does not change. The food supply benefits from the rainforest patch, q, will also be smaller, so $(Q + q)\phi\pi$ will be smaller. With a sufficiently small $(Q + q)\phi\pi$, never burning is optimal. Burning is made too costly by Aborigines being absent from the land.

In Figure 3b, occasional burning is also a possibility. If Q is large because of a strong cleaning-up ethic, $(Q + q)\phi\pi$ will be in the intermediate range even if q is small. Burning every 2 or 3 years could still occur. Finally, if $\underline{Q}$ is very large, $(Q + q)\phi\pi$ will be large enough that annual burning could still occur. However, the evidence on modern aboriginal burning suggests that Q is at best large enough to justify occasional burning (Head 1994).

Whitehead *et al*. (2003) recommend the reestablishment of traditional Aboriginal burning regimes to limit uncontrolled wildfires and achieve the broader social objectives of biodiversity preservation and reduced greenhouse gas emissions. The above analyses confirm that this will work best if at least some of the Aboriginal burning motives are maintained, there can be additional incentives provided for the societal objectives, and/or the cost of burning is not too high.

An example of providing incentives for reducing greenhouse gases comes from the West Arnhem Land Fire Abatement (WALFA) project. WALFA aims to reduce greenhouse gas emissions by 100,000 tonnes of CO_2-equivalents (carbon dioxide) per year in non-CO_2 gases (methane and nitrous oxides) through better fire management on 28,000 km^2 of Western Arnhem Land, involving the reintroduction of traditional Aboriginal fire management regimes.[9] ConocoPhillips has agreed to pay \$1 million per annum for these offsets over 17 years, and the offsets also will be recognised under the proposed Australian Carbon Pollution Reduction Scheme (Whitehead *et al*. 2008; Australian Government 2011).

[9] At present only methane and nitrous oxide emissions can be counted as net greenhouse emissions under the accounting guidelines of the Intergovernmental Panel on Climate Change (IPCC). Work such as that by Murphy *et al*. (2009) showing that wildfires are also net CO_2 emitters, may eventually lead to reduced CO_2 emissions also counting as offsets.

We introduce the payment for emissions offsets into our model, assuming clean-up values are the only other incentives remaining and burning cost are high. This is the case shown in Figure 3b, with $p + P - c = -2$. Without any offset payments, $(Q + q)\phi\pi$ would be sufficiently small that deliberate burning would not occur, or would occur infrequently. Since expected greenhouse gas emissions increase as a function of fuel load (or expected fire intensity), we introduce the offset payment as a replacement for q. This means higher values for $(Q + q)\phi\pi$. As can be seen from Figure 3b, a sufficiently large increase in $(Q + q)\phi\pi$ will result in a switch to the annual burning choice.

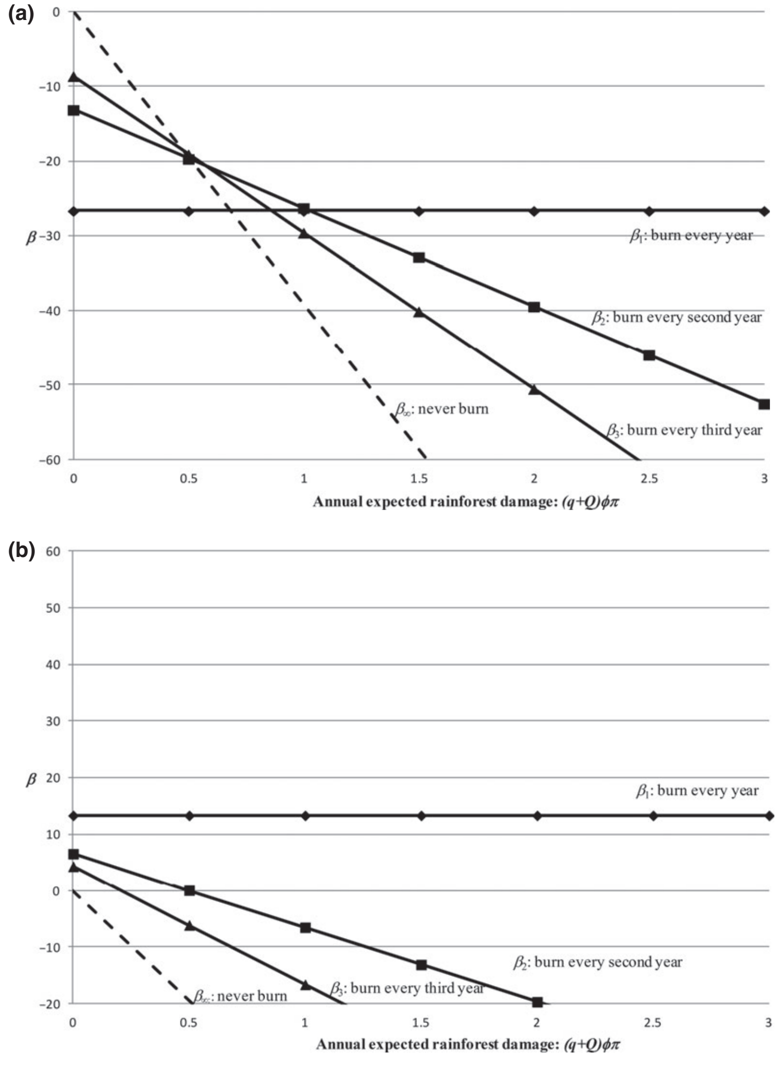

Figure 4 (a) Land value (β): small burning subsidy. (b) Land value (β): large burning subsidy.

A subsidy for burning is also useful. It can take the form of providing equipment, transportation or funding. A burning subsidy decreases c for all three burning intervals, increasing the vertical intercepts of their β functions. Assume a small burning subsidy, reducing the burning cost to $c' = 2.5$ for all burning frequencies, while the availability of alternative food sources causes p to remain at $p = 0$, giving $p + P - c' == 0 + 1.5 - 2.5 = -1$. Figure 4a shows the β functions with the small burning subsidy. The annual burning option will now be chosen, even if $(Q + q)\phi\pi$ is smaller. In Figure 3b, annual burning is chosen only if $(Q + q)\phi\theta > 2$. In Figure 4a it is chosen if $(Q + q)\phi\theta > 1$. A sufficiently large burning subsidy will cause annual burning to be chosen for all $(Q + q)\phi\theta \geq 0$. Figure 4b shows the β functions with a large burning subsidy ($c' = 1$). Now, no offset payments are necessary to induce annual burning. In general, burning subsidies and offset payments tend to be substitutes for one another. They can be used separately or in tandem.

6. Conclusions

The traditional burning regimes of Australian Aborigines controlled fire and increased the productivity of their environment with respect to their own needs. We use a dynamic, delayed-response, economic model to describe traditional Aboriginal burning behavior. The model predicts different burning regimes, depending on the food and cultural net benefits from the land that is deliberately burned, and on the cost of fuel load accumulation. The Aboriginal ethic of cleaning-up, combined with knowledge, access and incentives to maintain food sources and limit wildfire, led to a pattern of frequent deliberate burning in which uncontrolled fires were rare. But, the outcome is sensitive to changes in the basic parameters. Aborigines currently have less ready access to the lands they once burned and less dependence on food from them. In this constrained situation, it is not surprising that Aboriginal burning practices differ from those that would be observed in a more traditional setting. Deliberate burning is either prevented or occurs only occasionally, and the resultant fuel load build-up results in more uncontrolled fires.

If the goal is to maintain elements of the Australian landscape in something close to the condition prevailing prior to the changes brought about by Europeans, Aboriginal fire management practices can be an important tool. However, incentives are required to replace those which have disappeared. The WALFA project which generates greenhouse gas offsets through indigenous fire management is a good example. Australia is also a leader in payments for other ecosystem services (Stoneham *et al.* 2003). If explicit environmental management goals can be set and outcomes monitored, fire management could be another form of eco-tender. These, along with burning subsidies, can be used to influence Aboriginal burning regime choices in ways that are deemed socially desirable.

References

Altman, J.C. (2004). Economic development and Indigenous Australia: contestations over property, institutions and ideology, *Australian Journal of Agricultural and Resource Economics* 48, 513–534.

Andrews, P.L. (1988). The use of the rothermal fire spread model for forest danger rating and fire behavior prediction in the United States, in: Cheney, N.P. and Gill, A.M. (eds), *Conference on Bushfire Modeling and Fire Danger Rating Systems: Proceedings 11–12 July 1988*. CSIRO Division of Forestry, Canberra, ACT, pp. 1–8.

Australian Government, Department of Climate Change and Energy Efficiency (2011). Carbon Farming Initiative. Available from URL: http://www.climatechange.gov.au/cfi [accessed 11 Nov 2012].

Bird, R.B., Bird, D.W., Codding, B.F., Parker, C.H. and Jones, J.H. (2008). The "fire stick farming" hypothesis: Australian Aboriginal foraging strategies, biodiversity, and anthropogenic fire mosaics, *Proceedings of the National Academy of Sciences of the USA* 105, 14796–14801.

Chapman, D. (1999). *Natural Hazards*. Oxford University Press, Melbourne, Vic.

Clark, C.W. (1976). A delayed recruitment model of population dynamics, with application to baleen whale populations, *Journal of Mathematical Biology* 3, 381–391.

De Bondt, R.R. (1976). Limit pricing, uncertain entry and entry lag, *Econometrica* 44, 939–946.

Flannery, T. (1994). *The Future Eaters: An Ecological History of the Australasian Lands and People*. Reed New Holland, Sydney, NSW.

Gammage, B. (2011). *The Biggest Estate on Earth: How Aborigines Made Australia*. Allen and Unwin, Sydney, NSW.

Gill, A.M., Hoare, J.R.L. and Cheney, N.P. (1990). Fires and their effects in the wet-dry tropics of Australia, in: Goldammer, J.G. (ed.), *Fire in the Tropical Biota: Ecosystem Processes and Global Challenges*. Springer Verlag, Berlin, pp. 159–178.

Haynes, C.D. (1985). The patterns and ecology of Munwag: traditional Aboriginal fire regimes in north-central Arnhemland, *Proceedings of the Ecological Society of Australia* 13, 203–214.

Head, L. (1994). Landscapes socialized by fire: post-contact changes in Aboriginal fire use in northern Australia, and Implications for prehistory, *Archaeology Oceania* 29, 172–181.

Heckbert, S., Russell-Smith, J., Reeson, A. and James, G. (2011). Indigenous Australians fight climate change with fire, *Solutions* 2, 50–56. Available from URL: http://www.thesolutionsjournal.com/node/1006 [accessed 10 Nov 2012].

Hesseln, H., Rideout, D.R. and Omni, P. (1998). Using catastrophe theory to model wildfire behavior and control, *Canadian Journal of Forest Research* 28, 852–862.

Jones, R. (1969). Fire-stick farming, *Australian Natural History* 16, 224–226.

Jones, R. (1975). The Neolithic, Paleolithic and the hunting gardeners: man and land in the Antipodes, in: Suggate, R.P. and Creswell, M.M. (eds), *Quaternary Studies*. The Royal Society of New Zealand, Wellington, pp. 21–34.

Kamien, M.L. and Schwartz, P.N. (1981). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. North Holland, Amsterdam.

Kohn, J.L. (1996). Aboriginal use of fire in southeastern Australia, *Proceedings of the Linnean Society of New South Wales* 116, 19–26.

Lewis, H.T. (1989). Ecological and technical knowledge of fire: Aborigines versus park rangers in Northern Australia, *American Anthropologist* 91, 940–961.

Ludwig, D., Jones, D.D. and Holling, C.S. (1978). Qualitative analysis of insect outbreak systems: the spruce budworm and forest, *Journal of Animal Ecology* 47, 315–332.

Luke, R.H. and McArthur, A.G. (1978). *Bushfires in Australia*. Australian Government Publishing Services, Canberra, ACT.

Murphy, B.P., Russell-Smith, J., Watt, F.A. and Cook, J.D. (2009). Fire management and woody biomass carbon stocks in mesic savannas, in: Russell-Smith, J., Whitehead, P. and

Cook, P. (eds), *Culture, Ecology and Fire Management in North Australian Savannas: Rekindling the Wurrk Tradition*. CSIRO, Collingwood, Vic., pp. 361–378.

Murray, J.D. (1989). *Mathematical Biology*. Springer-Verlag, Berlin.

O'Brian, P. (1987). *Joseph Banks: A Life*. Collins Harville, London.

Pyne, S.J. (1991). *Burning Bush-A Fire History of Australia*. Henry Holt and Company, New York, NY.

Rose, D.B. (1992). *Dingo Makes us Human: Life and land in an Australian Aboriginal Culture*. Cambridge University Press, Cambridge.

Russell-Smith, J., Lucas, D., Gapindi, M., Gunbunuka, B., Kapirigi, N., Namingum, G., Lucas, K., Giulian, P. and Chaloupka, G. (1997). Aboriginal resource utilization and fire management in western Arnhem Land, monsoonal northern Australia: notes for prehistory, lessons for the future, *Human Ecology* 25, 159–195.

Russell-Smith, J., Edwards, A.C. and Cook, G.D. (2003). Reliability of biomass burning estimates from savanna fires: biomass burning in northern Australia during the 1999 Biomass Burning and Lightning Experiment B field campaign, *Journal of Geophysical Research* 108, BIB 9-1–BIB 9-12.

Russell-Smith, J., Murphy, P.B., Meyer, C.P., Cook, G.D., Maier, S., Edwards, A.C., Schatz, J. and Brocklehurst, P. (2009). Improving estimates of savanna burning emissions for greenhouse accounting in northern Australia: limitations, challenges, applications, *International Journal of Wildland Fire* 18, 1–18.

Singh, G., Kershaw, A.P. and Clark, R. (1981). Quaternary vegetation and fire history in Australia, in: Gill, A.M., Groves, R.H. and Noble, I.R. (eds), *Fire and the Australian Biota*. Australian Academy of Science, Canberra, ACT, pp. 23–54.

Stoneham, G., Chaudhri, V., Ha, A. and Strappazzoni, L. (2003). Auctions for conservation contracts: an empirical examination of Victoria's bush tender trial, *Australian Journal of Agricultural and Resource Economics* 47, 477–500.

Trevitt, A.C.F. (1991). Weather parameters and fire moisture content: standards for fire model inputs, in: Cheney, N.P. and Gill, A.M. (eds), *Conference on Bushfire Modelling and Fire Danger Rating Systems: Proceedings July 1988*. CSIRO, Division of Forestry, Canberra, ACT, pp. 157–166.

Vigilante, T., Murphy, B.P. and Bowman, D.M.J.S. (2009). Aboriginal fire use in Australian tropical savannas: ecological effects and management lessons, in: Cochrane, M.A. (ed.), *Tropical Fire Ecology: Climate Change*. Springer Praxis Books Chichester, Land Use and Ecosystem Dynamics, pp. 143–167.

Whitehead, P.J., Bowman, D.M.J.S., Preece, N., Fraser, F. and Cooke, P. (2003). Customary use of fire by indigenous peoples in northern Australia: its contemporary role in savanna management, *International Journal of Wildland Fire* 12, 414–425.

Whitehead, P.J., Purdon, P., Russell-Smith, J., Cooke, P.M. and Sutton, S. (2008). The management of climate change through prescribed savanna burning: emerging contributions of indigenous people in northern Australia, *Public Administration and Development* 28, 374–385.

Williams, R.J., Cook, G.D., Gill, A.M. and Moore, P.H.R. (1999). Fire regime, fire intensity and tree survival in a tropical savanna in northern Australia, *Austral Ecology* 24, 50–59.

Williams, R.J., Griffiths, A.D. and Allan, G.E. (2002). Fire regimes and biodiversity in the savannas of northern Australia, in: Bradstock, R.A., Williams, J.E. and Gill, A.M. (eds), *Flammable Australia*. Cambridge University Press, Cambridge, pp. 281–304.

Yibarbuk, D., Whitehead, P.J., Russell-Smith, J., Jackson, D., Godjua, C., Fisher, A., Brooke, P., Chaqyenot, D. and Bowman, D.M.J.S. (2001). Fire ecology and land management in central Arnhem Land, northern Australia, *Journal of Biogeography* 28, 325–343.

Appendix I: Fuel load dynamics without deliberate burning

For the first and second years after regrowth, the fuel load dynamics are described by (I-1) and (I-2) respectively, where S_2 = ($\phi\theta$s1 + $\theta^2 s2$):

$$R_{t+1} - R_t = -\phi R_t + \theta s_1 y_{t-1} \quad \text{(I-1)}$$

$$R_{t+1} - R_t = -\phi R_t + S_2 y_{t-2}. \quad \text{(I-2)}$$

With an initial fuel load of zero, R_t = 0 in (I-1). Assigning parameter values of $y_{t-1} = L = 5$, $\phi = 0.2$, $\theta = 0.8$, $s_1 = 1$, and $s_2 = 0.5$, (I-1) gives $R_{t+1} - R_t = 4$ and $R_{t+1} = 4$. This means $R_t = 4$ in (I-2), giving $R_{t+1} - R_t = -0.8+(0.16 + 0.32)5 = 1.6$ and $R_{t+1} = 5.6$.

The same logic can be extended to include the case in which regrowth began at the end of t–3 on $y_t - 3 = \text{L}$ hectares of land. We know from (I-2) that $R_t - R_{t-1} = -\phi R_{t-1} + (\phi\theta s_1 + \theta^2 s_2)y_{t-3}$. At the beginning of t, we have ϕy_{t-3} hectares burned last year, $\theta\phi y_{t-3}$ hectares unburned since $t-2$, and $\theta^2 y_{t-3}$ hectares unburned since t–3. In year t, ϕy_{t-3} hectares will burn, with a fuel load loss of ϕR_t and no immediate contribution to fuel load growth. The remaining θy_{t-3} hectares do contribute to fuel load growth: $\phi\theta y_{t-3}$ hectares contribute $s_1\phi \theta y_{t-3}$, $\phi\theta^2 y_{t-3}$ hectares contribute $s_2\phi\theta^2 y_{t-3}$, and $\theta^3 y_{t-3}$ contribute $s_3\theta^3 y_{t-3}$ tonnes. Overall the change in fuel load between t and $t+1$ is:

$$R_{t+1} - R_t = -\phi R_t + (\phi\theta s_1 + \phi\theta^2 s_2 + \theta^3 s_3)y_{t-3}. \quad \text{(I-3)}$$

Defining $S_3 = (\phi\theta s_1 + \phi\theta^2 s_2 + \theta^3 s_3)$, (I-3) can be written as:

$$R_{t+1} - R_t = -\phi R_t + S_3 y_{t-2}. \quad \text{(I-4)}$$

With $R_t = 5.6$, $R_{t+1} - R_t = -1.12 + (0.3264)5 = 0.512$, and $R_{t+1} = 6.112$.

The final case has regrowth beginning in t–4. We know from (I-3) that $R_t - R_{t-1} = -\phi R_{t-1} + (\phi\theta s_1 + \phi\theta^2 s_2 + \theta^3 s_3)y_{t-4}$. At the beginning of t, we have ϕy_{t-4} burned last year, $\phi\theta y_{t-4}$ unburned since t–2, $\phi\theta^2 y_{t-4}$ hectares unburned since $t-3$ and $\theta^3 y_{t-4}$ hectares unburned since t–4. In year t, ϕy_{t-4}, hectares will burn, with a fuel load loss of ϕR_t and no immediate contribution to fuel load growth. The remaining θy_{t-4} hectares do contribute to fuel load growth: $\phi\theta y_{t-4}$ hectares contributing $s_1\phi\theta y_{t-4}$ tonnes, $\phi\theta^2 y_{t-4}$ hectares contributing $s_2\phi\theta^2 y_{t-4}$ tonnes, $\phi\theta^3 y_{t-4}$ contributing $s_3\phi\, \theta^3 y_{t-4}$ tonnes and $\theta^4 y_{t-4}$ hectares contribute $s_4\theta^4 y_{t-4}$ tonnes. With $s_4 = 0$, the overall the change in fuel load between t and $t+1$ is:

$$R_{t+1} - R_t = -\phi R_t + (\phi\theta s_1 + \phi\theta^2 s_2 + \phi\theta^3 s_3)y_{t-4}. \quad \text{(I-5)}$$

Defining $S_4 = (\phi\theta s_1 + \phi\theta^2 s_2 + \phi\theta^3 s_3)$, (I-5) can be written as:

$$R_{t+1} - R_t = -\phi R_t + S_4 y_{t-2}. \quad \text{(I-6)}$$

With R_t = 6.112, $R_{t+1} - R_t = -1.2224+(0.24448)5 = 0$, and R_{t+1} = 6.112. Since $s_n = 0$ for $n \geq 4$, (I-6) will describe fuel load dynamics for regrowth beginning in $t-4$ or earlier. The fuel load reaches its steady state after 3 years.

Appendix II

The maximum land value that can be generated by burning every 3 years is the linear function:

$$\beta_3 = \frac{\rho^3\theta(p+P-c) - (\rho^2\theta s_1 + \rho^3\theta S_1 + \rho^3\theta^2 s_2)(q+Q)\phi\pi}{1-\rho^3}. \tag{II-1}$$

The boundary value that can be generated by never burning is the linear function:

$$\beta_\infty = \frac{-(\rho^2\theta s_1 + \rho^3\theta^2 s_2 + \rho^4\theta^3 s_3)(q+Q)\phi\pi}{1-\rho}. \tag{II-2}$$

Recalling, that $s_4 = 0$, the switching condition for $x_{4,t}$ is:

$$x_{4,t} = \begin{cases} 0 & \text{if } (p+p-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) - S_4 \sum\limits_{n=2}^{N-4} \rho^n \lambda_{t+n} + \beta < 0 \\ \text{switch} & \text{if } (p+p-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) - S_4 \sum\limits_{n=2}^{N-4} \rho^n \lambda_{t+n} + \beta < 0 \\ L & \text{if } (p+p-c)\theta - \rho\lambda_{t+1}\left(\theta s_1 + \theta^2 s_2 + \theta^3 s_3\right) - S_4 \sum\limits_{n=2}^{N-4} \rho^n \lambda_{t+n} + \beta < 0. \end{cases} \tag{II-3}$$

Following the same approach as used to derive β_1, β_2 and β_3,

$$\beta_4 = \frac{\rho^4\theta(p+P-c) - (\rho^2\theta s_1 + \rho^3\theta S_1 + \rho^3\theta^2 s_2 + \rho^4\theta s_1 + \rho^4\theta^2 s_2 + \rho^4\theta^3 s_3)(q+Q)\phi\pi}{1-\rho^4}. \tag{II-4}$$

If $\beta_4 \leq \max(\beta_3, \beta_\infty) \forall (q+Q)\phi\pi \geq 0$, then the 4 year burning interval is dominated by either the 3 year interval or never burning.

Compare β_3 and β_∞. From (II-1) and (II-2), with $\Psi = \rho s_1 + (1+\rho)\rho\theta s_2 + (1+\rho+\rho^2)\rho\theta^2 s_3$:

$$\begin{aligned} \beta_3 &> \beta_\infty \text{ if } (p+P-c) > -(q+Q)\phi\pi\psi \\ \beta_3 &= \beta_\infty \text{ if } (p+P-c) = -(q+Q)\phi\pi\psi \\ \beta_3 &< \beta_\infty \text{ if } (p+P-c) < -(q+Q)\phi\pi\psi. \end{aligned} \tag{II-5}$$

Now compare β_4 and β_∞ using (II-4) and (II-2). With the same definition for Ψ, again we get (II-5).

Since the conditions in (II-5) apply to both comparisons, $\beta_4 > \beta_\infty$ whenever $\beta_3 > \beta_\infty$, $\beta_4 = \beta_\infty$ whenever $\beta_3 = \beta_\infty$ and, $\beta_4 < \beta_\infty$ whenever $\beta_3 < \beta_\infty$. If we were to plot the β_3, β_4, and β_∞ as functions of $(q + Q)\phi\pi$, β_4 would intersect β_∞ at the same point at which β_3 intersects β_∞. However, when $\beta_4 > \beta_\infty$ and $\beta_3 > \beta_\infty$, the relative magnitudes of β_4 and β_3 depend on the vertical intercept.

For β_4, the vertical intercept is $\frac{\rho^4\theta(p+P-c)}{1-\rho^4}$; for β3, it is $\frac{\rho^3\theta(p+P-c)}{1-\rho^3}$; for β_∞, it is zero. The absolute value of the vertical intercept is largest for β_3. If the vertical intercepts are both positive ($p + P - c > 0$), the value of $(q + Q)\phi\pi$ at which $\beta_3 = \beta_4$ must be negative. It follows that $\beta_3 > \beta_4$ for all values of $(q + Q)\phi\pi$ greater than the intersection value. Since our concern is with non-negative values of $(q + Q)\phi\pi$, we can say $\beta_3 > \beta_4$ over that range. This is the case shown in Figure 3a. For that case the intersection point occurs at $(q + Q)\phi\pi = -0.953$. For non-negative values for $(q + Q)\phi\pi$, the β_4 line would lie in between the β_3 and β_∞ lines, and β_3 dominates.

If the vertical intercepts are both negative, the vertical intercept is largest (zero) for β_∞ and second largest (smaller negative) for β_4. The intersection point occurs where $(q + Q)\phi\pi > 0$. So β_∞ dominates between $(q + Q)\phi\pi = 0$ and the intersection point. At values of $(q + Q)\phi\pi$ greater than the intersection, point β_∞ dominates. This case is shown in Figure 3b. The intersection point is at $(q + Q)\phi\pi = 0.953$. β_∞ dominates between $(q + Q)\phi$ $\pi = 0$ and 0.953. When $(q + Q)\phi\pi > 0.953$, β_3 dominates.

The same comparison applies for $x_{5,t}$, $x_{6,t}$ and all other intermediate burning regimes. They are all dominated by either $x_{3,t}$ or the regime in which burning never occurs.