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DYNAMIC UTILITY FUNCTIONS FOR MEASURING ADVERTISING RESPONSE

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From a cursory reading of the economics of advertising literature one reaches a tentative conclusion that the most frequently made statement is one similar to the following: Advertising is one of the most controversial issues in the American economy. Some of the questions addressed in the literature are: Does advertising raise or lower the price of products? Does advertising persuade consumers to buy things they do not need? Does advertising make the price elasticity of demand more or less elastic?

Two schools of thought have emerged with respect to how economists describe the effects of advertising [1, 10]. One model views advertising as making consumers less responsive to price changes and giving firms more market power. The other school maintains that advertising provides additional information to consumers thereby making markets more competitive and consumers more sensitive to price changes. From these observations it appears that a need exists for developing a theoretical model that explicitly incorporates the effects of advertising on the behavior of economic agents, yet remains flexible enough to be able to test the theoretical implications without imposing the restrictions *a priori*.

The primary purpose of this paper is to develop a model that could be used empirically to measure the effects of advertising on consumer behavior using dynamic utility functions.¹ A theoretically plausible demand system, the almost ideal demand system (AIDS) of Deaton and Muellbauer [8, 9], will serve as the basic specification. Not only does it allow for consistent aggregation across consumers, but it is flexible enough to allow the researcher to test for the effects of advertising on price and income elasticities rather than having to impose these restrictions in a nonflexible manner. Thus, some of the issues involved with how advertising affects price and income elasticities of demand will be examined in a dynamic theoretically plausible framework. This objective also relates to one of the unresolved issues of advertising farm products mentioned by Morrison [14], namely, that agricultural economists need to acquire a better understanding of the

intercommodity effects of widespread generic advertising. By examining these issues in a flexible demand system context, the interrelationships existing among commodities are explicitly taken into account.

An alternative approach to the one taken in this paper would involve an extension of Stigler's and Becker's [18] work. Using a household production function approach, they assume that tastes are stable over time and allow advertising to affect the technological relationship between commodities and market goods, consumers' own time, their own skills, other human capital and other inputs. While this approach is attractive conceptually, the author of this paper prefers the more direct approach of allowing advertising variables to appear in the expenditure (cost) function generating a viable demand system.² Estimation problems, while not simple, can still be carried out using rather standard econometric techniques such as the full information maximum likelihood method.

The format of the paper is as follows. A brief description of the AIDS is given. Next, an explanation of how to incorporate dynamic elements into the model is provided. Various methods of introducing advertising effects into the model are presented together with their economic implications. The paper ends with some tentative conclusions and a discussion of future research ideas with regard to advertising effects on food commodities.

The Model

Which demand model should be chosen to evaluate the effects of advertising on consumer behavior? First, it should be a demand system derived from an optimization procedure. This allows the demand analyst to account for the interrelationships across commodities and work with a framework consistent with economic theory. Second, there are many demand systems to choose from—the translog, AIDS, linear expenditure system, the Fourier flexible form, etc. I decided to choose the AIDS because of the six desirable properties mentioned by Deaton and Muellbauer [8, p. 312]. In addition, since economists usually work with aggregate data, the AIDS permits individual demand function restrictions to hold for aggregate or market demand functions; see, e.g., Johnson *et al.*, [12] for a more thorough discussion of these points. The AIDS is flexible in that it allows the researcher to test for theoretical restrictions rather than automatically imposing them. This flexible feature is important when consideration is given to advertising impacts. Finally, the AIDS possesses desirable properties with respect to how income and price elasticities vary over time; see, e.g., Blanciforti and Green [4, 5]. These desirable properties also carry over for advertising effects on elasticities, as will be demonstrated in a later section.

The almost ideal demand system (AIDS) of Deaton and Muellbauer is generated from the following cost or expenditure function

$$\ln c(u, p) = \alpha_o + \sum_k \alpha_k \ln p_k + 1/2 \sum_k \sum_j \gamma_{kj}^* \ln p_k \ln p_j + \mu \beta_o \Pi p_k^{\beta_k} \quad (1)$$

where p_k 's are prices, u represents utility and α_k, γ_{kj}^* and β_k are parameters to be estimated. By applying Shepard's Lemma to (1), i.e., by differentiating equation (1) with respect to prices the compensated demand functions are obtained in budget share form. More specifically,

$$\frac{\partial \ln c}{\partial \ln p_i} = w_i \quad (2)$$

where $w_i = p_i q_i / x$, q_i represents the quantity of the i th commodity, p_i represents the price of the i th commodity and x denotes total expenditure (income). After appropriate substitutions are made, see, e.g., Deaton and Muellbauer [8, p. 313] the AIDS is given by

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln \{x/P\} \quad (3)$$

where P is a price index defined by

$$\ln P = \alpha_o + \sum_k \alpha_k \ln p_k + 1/2 \sum_j \sum_k \gamma_{kj} \ln p_k \ln p_j.$$

Each γ_{ij} represents the change in the i th budget share with respect to a percentage change in the j th price with real expenditures or income held constant. The β_i coefficients represent the change in the i th budget share with respect to a percentage change in real income or expenditures with prices held constant.

The adding up, homogeneity and Slutsky symmetry conditions hold, respectively if

$$\sum_i \alpha_i = 1, \sum_i \gamma_{ij} = 0 \text{ and } \sum_i \beta_i = 0, \quad (4)$$

$$\sum_j \gamma_{ij} = 0, \text{ and} \quad (5)$$

$$\gamma_{ij} = \gamma_{ji}. \quad (6)$$

Expressions for income, own-price and cross-price elasticities from the AIDS in equation (3) are given, respectively, by (see, e.g., Blanciforti and Green [4])

$$\eta = 1 + \beta_i / w_i, \quad (7)$$

$$\epsilon_{ii} = -1 + [\gamma_{ii} - \beta_i(\alpha_i + \sum_k \gamma_{ik} \ln p_k)] / w_i, \text{ and} \quad (8)$$

$$\epsilon_{ij} = [\gamma_{ij} - \beta_i(\alpha_i + \sum_k \gamma_{ik} \ln p_k)] / w_i. \quad (9)$$

Determining the effects of advertising on these elasticity expressions is an important objective of this paper.

Advertising Effects

How can advertising effects be incorporated into a flexible demand system such as the AIDS? First, the expenditure function can be generalized to include these effects. But since advertising exhibits carryover effects [2, 7, 13] the cost function must be made dynamic. All of these extensions must be incorporated in such a manner that the cost function still possesses the five theoretical properties discussed, e.g., by Deaton and Muellbauer [9, pp. 39, 40]. That is, the cost function should be homogeneous of degree one in prices, concave in prices, etc. Second, the derived demand functions must still permit the testing of the conditions of adding-up, homogeneity, and Slutsky symmetry restrictions. By using the method of translation [15] these generalizations can be carried out.

Following Pollak and Wales [15], the parameters of the expenditures and demand functions are assumed to depend upon previous advertising levels.³ Three alternative formulations are considered even though there exist an infinite number of possible combinations for incorporating advertising effects that are consistent with demand theory (see, e.g., Blanciforti, *et al.*, [5]).⁴

Ray [17] proposes the following dynamic AIDS cost function

$$\begin{aligned} \ln c(u, p) = & \alpha_o + \sum_i \delta_i e_{it-1} + \sum_i \alpha_i \ln p_{it} \\ & + 1/2 \sum_{i,j} (\gamma_{ij} + \theta_{ij} \mu_{t-1}) \ln p_{it} \ln p_{jt} \\ & + u \beta_o \prod_i p_{it}^{\beta_i + \eta_{it-1}} \end{aligned} \quad (10)$$

where e_{it-1} denotes lagged purchases of item i and $\mu_{t-1} = \sum e_{it-1}$ is lagged aggregate purchase. Even though Ray is concerned with habits or persistences in consumption patterns, the same principle carries over to advertising effects. In addition, both habits and advertising effects could be simultaneously incorporated into the model. Ray shows that the cost function in (10) is a valid representation of consumer preferences, nevertheless his approach has a rather severe limitation if it is to be used for incorporating advertising effects. This can be easily seen by considering the demand system that is derived from equation (10). The dynamic AIDS system is given by

$$w_{it} = \alpha_i + \sum_j (\gamma_{ij} + \theta_{ij} \mu_{t-1}) \ln p_{jt} + (\beta_i + \eta_i \mu_{t-1}) \ln(\mu_t / P_t^*) \quad (11)$$

where

$$\begin{aligned} \ln P_t^* = & \alpha_o + \sum_i \delta_i e_{it-1} + \sum_i \alpha_i \ln p_{it} + \\ & 1/2 \sum_{i,j} (\gamma_{ij} + \theta_{ij} \mu_{t-1}) \ln p_{it} \ln p_{jt} . \end{aligned}$$

From observing equation (11) it can be seen that individual (advertising or commodity) expenditures in previous time periods only affect demand through the income term $[\ln(\mu_t/P_t^*)]$. This appears to be an overly restrictive formulation. By observing the expressions for elasticities which are not given here, however, there exists a great deal of flexibility in the way advertising can affect price and income elasticities. Another attractive feature of Ray's specification is that brand as well as generic advertising can be treated simultaneously. For example, aggregate or generic advertising expenditures could be represented by μ_{t-1} and brand or individual advertising expenditures can be represented by e_{it-1} . If the demand analyst is concentrating on a subset of commodities, say different brands of the same commodity, he or she could include generic advertising effects (e.g., purchase more oranges) as well as brand advertising effects (e.g., buy more Sunkist oranges).

A second method of introducing advertising effects, following Blanciforti and Green [5] is to allow the α 's to depend on previous advertising expenditures, i.e., let

$$\alpha_k = \alpha_k^* + \alpha_k A_{kt-1} \quad (12)$$

where A_{kt-1} denotes advertising expenditures on the k th commodity in the previous time period.⁵ A limitation of this approach is that the theoretical properties of the cost function and derived demand system only hold locally, i.e., at specified points for A_k 's. This consideration leads to the other method which will be discussed at some length in this paper.

Let

$$\alpha_k = \alpha_k^* + \alpha w_{kt-1}^A \quad (13)$$

where α is assumed to be the same across commodities and w_{kt-1}^A is the k th budget share of advertising on that commodity. With this specification substituted in equation (1) it can be shown that a valid cost function exists if

$$\sum_k \beta_k = 0, \sum_j \gamma_{kj}^* = \sum_k \gamma_{kj}^* = 0 \text{ and} \quad (14)$$

$$\sum_k \alpha_k^* = (1 - \alpha) . \quad (15)$$

(Refer to equation (1) together with equation (13) for an interpretation of the parameters.)⁶ Thus a normalization exists so that the resultant demand function can be estimated.

In addition, with the advertising scheme given in equation (13) the adding-up condition for the demand function is satisfied if

$$\sum_i \alpha_i^* = (1 - \alpha), \sum_i \gamma_{ij} = 0, \text{ and } \sum_i \beta_i = 0. \quad (16)$$

Similar conditions can be shown to exist in order for the demand functions to be homogeneous of degree zero in all prices and income.

With the advertising model given by equation (13) substituted into equation (3), expressions for the income, own-price, cross-price, and advertising elasticities are given, respectively, by

$$\eta = 1 + \beta_i/w_i, \quad (17)$$

$$\epsilon_{ii} = -1 + \{\gamma_{ii} - \beta_i[(\alpha_i^* + \alpha w_{it-1}^A) + \sum_j \gamma_{ij} \ln p_j]\}/w_i, \quad (18)$$

$$\epsilon_{ij} = \{\gamma_{ij} - \beta_i[(\alpha_j^* + \alpha w_{jt-1}^A) + \sum_k \gamma_{jk} \ln p_k]\}/w_i \text{ and} \quad (19)$$

$$\epsilon_{iA} = \alpha w_{it-1}^A [1 - \beta_i \ln p_i]/w_i. \quad (20)$$

Consider the advertising elasticity given in equation (20). It represents the percentage change in the quantity demanded of the i th commodity with respect to a percentage change in the quantity of advertising devoted to that commodity.⁷ *A priori* the advertising elasticity would be expected to be positive; otherwise, additional amounts of advertising directed toward a particular commodity would result in a reduction of the quantity demanded. For necessities, i.e., commodities whose income elasticities are less than one, the AIDS implies that $\beta_i < 0$ (refer to equation (17)). Consequently, with $\alpha > 0$, the advertising elasticity is always positive. However, for luxuries ($\eta > 1$) the AIDS implies that $\beta_i > 0$. Hence, with $\alpha > 0$, the advertising elasticity is positive only if $1 > \beta_i \ln p_i$.

Now what are the effects of advertising on income and own-price elasticities of demand? First, with respect to the income elasticity

$$\partial \eta / \partial w_{it-1}^A = \beta_i \alpha (\beta_i \ln p_i - 1) / w_i. \quad (21)$$

Thus, for necessities ($\beta_i < 0$) an increase in the advertising budget share of the i th commodity in the previous time period results in the income elasticity becoming more elastic, assuming in addition that $\alpha > 0$. That is, $\partial \eta / \partial w_{it-1}^A > 0$ under these conditions. It would be preferable that the sign of the partial in equation (21) not be determined *a priori*. If this were the case the demand analyst would let the data determine if advertising results in the income elasticity becoming more or less elastic.

Second, with respect to the own-price elasticity

$$\partial \epsilon_{ii} / \partial w_{it-1}^A = -\beta_i \alpha / w_i + \beta_i \alpha w_{it-1}^A (\alpha - \beta_i \ln p_i) / w_i^2. \quad (22)$$

In case of necessities, for example, certain food products, the sign of the partial derivative in equation (22), can be shown to depend on the condition

$$w_{it-1}^A / w_i [\alpha (1 - \beta_i \ln p_i)] \begin{matrix} > \\ < \end{matrix} 1. \quad (23)$$

A priori it is difficult to determine if advertising results in the own-

price elasticity becoming more or less elastic. However, there exists sufficient flexibility in the model to allow the empirical results to answer this important question.

What effects does generic advertising have on intercommodity relationships? This question can be answered, in part, by examining the impact of advertising on the cross-price elasticities. Mathematically,

$$\partial \epsilon_{ij} / \partial w_{j,t-1} = - \beta_i \alpha [1 + (w_{j,t-1} \beta_i \alpha \ln p_j) / w_i] / w_i. \quad (24)$$

Expression (24) gives the effect on the cross-price elasticity of a change in the advertising on the j th commodity. For example, if the budget share of advertising for soft drinks increases, what effect will this have on the cross-price elasticity between, say, milk and soft drinks? Assuming that milk and soft drinks are substitutes and ignoring the difference between compensated and uncompensated elasticities for the moment, an increase in the price of soft drinks would be expected to result in an increase in the quantity demanded of milk. If, however, there was an increase in the advertising devoted to soft drinks, one might expect this to mitigate somewhat the impact of the associated price increase of soft drinks on the quantity demanded of milk. That is, *a priori* the expression in equation (24) would be expected to be negative. For necessities ($\beta_i < 0$), the sign of the expression in (24)

depends on whether $(w_{j,t-1}^A \beta_i \alpha \ln p_j) / w_i \stackrel{>}{\leq} 1$. Similar questions could be addressed to further enlighten the demand analyst in regard to how advertising impacts intercommodity relationships.

Conclusions

What conclusions can be drawn from this exercise? Given that advertising effects carry over, depending upon the commodity, for four to five quarters, the method of translation can be employed to explicitly incorporate these effects into a theoretically plausible dynamic demand system. The advertising impacts on income and price elasticities can be obtained mathematically. In addition, both persistencies in consumption patterns and advertising effects can be incorporated simultaneously into a demand system. Although not done in this paper, possibilities exist for testing both types of effects on demand elasticities.

Since advertising attracts new consumers as well as causing increased consumption from existing consumers, an alternative approach to take these phenomena into account would be a discrete/continuous model similar to the one developed by Hanemann [11]. No attempt was made in the presented conceptual framework to take these considerations into account.

Finally, much more could be done with the present approach. However, since the AIDS requires rather large data sets, weekly, monthly,

or quarterly information would be needed in order to obtain reasonable estimates of the demand parameters.

FOOTNOTES

¹The term "dynamic" in this paper refers to using the method of translation to allow certain parameters in the expenditure function and demand system to depend upon, for example, previous consumption or advertising expenditures. No attempt is made to consider a completely dynamic model by using control-theoretic techniques. In addition, since a demand system is used where income is treated as an exogenous variable, i.e., we are considering an allocation model, the methodology chosen is incapable of addressing the question of whether or not advertising increases the aggregate demand for commodities.

²There exist four equivalent ways of representing consumer preferences assuming certain regularity conditions hold: (1) direct utility functions, (2) indirect utility functions, (3) cost or expenditure functions and (4) distance functions; see, e.g., Blackorby, *et al.*, [3].

³Pollak and Wales introduced habit effects into static demand systems. The approach in this paper replaces habit effects with advertising effects. The conceptual framework holds for both. A complete generalization of the model would incorporate both persistences in consumption behavior and advertising effects. This development can be thought of as an extension of the work of Ward [19].

⁴If generic advertising has the greatest impact during the time period (quarter) funds are spent as Ward [19] has found, then current advertising expenditures need to be included in the proposed advertising scheme. This can be easily accomplished. If, however, current consumption also affects advertising as Ashley, *et al.*, [2] have found, then additional "advertising" equations need to be added to the demand system. Otherwise, simultaneous-equations bias problems would exist in the estimations.

⁵All of these translation methods can be generalized to include all past advertising levels of a good. For example, let

$$\alpha_{it} = \alpha_i^* + \alpha_i^{**} y_{it-1} \quad \text{where}$$

$$y_{it-1} = (1-\delta) \sum_{j=0}^{\infty} \delta^j x_{it-1-j}, \quad 0 \leq \delta < 1,$$

δ represents the "memory" coefficient which is assumed to be the same for all goods and the x 's represent previous advertising levels of a good (Pollak [16]).

In the above formulation, the advertising effects decay geometrically over time.

⁶Note that if a logarithmic function $\ln f(x)$ is linearly homogeneous, then $\ln f(\theta x) = \ln \theta + \ln f(x)$ where θ is an arbitrary real number.

⁷From a measurement viewpoint it might be preferable to express the advertising elasticity in terms of the percentage change in the i th budget share of the commodity with respect to a percentage change in the i th advertising budget share for that particular commodity.

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