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REGIONAL BEEF CATTLE INVENTORY FUNCTIONS WITH AGGREGATION CONSTRAINTS

by

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Beef cattle producers, like most farmers, make use of locally available resources but sell in nationally oriented markets. Climatic, soil and spatial differences cause differences in regional beef cattle supply, but this disparity is seldom taken into account in supply function estimation. The usual approach is to concentrate on national-level supply. Regional analysis, when it is done, either has used a mathematical programming approach or has dealt with a single region and assumed away interdependencies among variables that would be considered jointly determined at the national level. An example of the latter is Tryfos' study of Canadian livestock supply in which Canadian prices were assumed to be exogenous because they were determined by the U.S. market. When all regions are considered, however, interdependency can not be assumed away, and consistent estimation requires that interdependency be preserved in the regional functions and that they aggregate to the national level. The objective here is to estimate regional beef cattle inventory functions which have these properties.

Such functions would be useful to have, in order to sharpen the focus of national-level projections and to evaluate the regional consequences of policy changes. The "top down" approach of regional economic analysis is used, in which the macrolevel functions dominate and provide the controlling constraints in regional breakdowns (Milne, Adams and Glickman)[1]. Six regions are used - the Great Plains, Midwest, Northeast, Southeast, Southern Plains, and West. The Northeast and Southern Plains regions are identical to USDA's farming regions of the same name (USDA, 1980, p. 476). Except for the Great Plains, the other regions are combinations of adjacent USDA regions. The Great Plains is composed of Colorado, Kansas and Nebraska and is delineated separately because of its importance in beef supply.

Analytical Procedure

The "top down" approach begins with the estimation of a macrolevel model in which some or all of the endogenous variables may be jointly dependent and require simultaneous equations methods for consistent estimation of their parameters. Next, regional functions are estimated with macrolevel functions used as constraints in some fashion. Milne, Adams and Glickman simply use national aggregates as control quantities to make proportional corrections in regional estimates, but a more rigorous procedure is used here. Regional functions are estimated with constraints imposed on their parameters rather than their predicted values. The estimation method used is an extension of least-squares regression under linear constraints (Theil, p. 42), the basic equations for which are as follows:

(1) $Y = Xb + E$

subject to constraints

(2) $Wb = B$

which are imposed as Lagrangean functions in the estimation of b, where

$Y = MT \cdot 1$ observation matrix for variable Y in M regions for T periods,

$X = MT \cdot KM$ observation matrix for K explantory variables in M regions for T periods,

$b = KM \cdot 1$ regional function parameter matrix,

$W = M \cdot KM$ matrix of parameter weights, and

$B = K \cdot 1$ matrix of national-level parameters for Y.

The constraints in equation (2) cause regional parameters in b to aggregate to their macrolevel counterparts in B, and b is estimated on this basis. If B_k is a parameter between jointly dependent variables and is consistently estimated, this consistency is passed to the regional parameter estimates through the aggregation constraint. The matrix X is large but not dense, since it is composed of a series of T · K submatrices arranged as a super-diagonal, and zeros elsewhere. The procedure can be applied to all functions in the macrolevel model, but for reasons of space the exposition here concentrates on one function - the beef cow inventory function.

Macrolevel Beef Cow Inventory Function

Beef cow inventories are fundamentally important to beef cattle supply for obvious reasons. Also, since beef cow herds tend to be fixed spatially, their regional functions should be interesting. The macrolevel beef cow inventory function is actually a submodel of 3 equations which transform in the usual fashion to a single estimating equation. The 3 basic structural equations in the submodel are:

(3) $$BCI_t^* = A_0 + A_1\hat{P}_t + A_2X1_t + A_3X2_t + A_4X3_t + V_t,$$

$$V_t = \rho V_{t-1} + \varepsilon_t$$

(4) $$BCI_t = \alpha BCI_t^* + (1 - \alpha)(BCI_{t-1} + BH_{t-1})$$

(5) $$\hat{P}_t = \gamma P_t + (1 - \gamma)P_{t-1}$$

where BCI_t^* = desired beef cow inventories as of Dec. 31 in year t,

BCI_t = observed beef cow inventories as of Dec. 31, year t,

$\hat{P}_t$, P_t = expected and observed U.S. prices received for beef cattle,

BH_{t-1} = beef heifer inventory, Dec. 31, year t - 1,

$X1_t$ = U.S. index of prices paid for feed, 1967 = 100,

$X2_t$ = harvested acreage of cropland, millions of acreas, year t,

$X3_t$ = dummy variable to account for inventory definition change Jan. 1, 1970 (Dec. 31, 1969): X3 = 1 if t < 1969, 0 otherwise.

A first-order autoregressive process in the disturbances is postulated to account for possible distributed lags in the diffuse effects represented by the disturbances. Lagging inventories one day to make them year-end observations is done to emphasize that inventory adjustment is a year-long process that interacts with current prices through the formulation of price expectations. Transforming the sub-model into a single equation on observeable variables yields

(6) $$BCI_t = B_0 + B_1P_t + B_2(BCI_{t-1} + BH_{t-1}) + B_3P_{t-1} + B_4X1_t + B_5X2_t + B_6X3_t + U_t, \quad U_t = U_{t-1} + E_t$$

which specifies beef cow inventories as jointly determined with price. The U.S. price received for beef cattle series is used in preference to price at some specific market or for some specific class to facilitate regional disaggregation.

Equation (6) is only one equation in a structural model which, besides beef cow inventories and prices, specifies inventory functions for other cattle classes, beef cattle slaughter and beef supply, and domestic beef demand and imports. The overall model is specified as a dynamic, simultaneous system of equations subject to first-order autocorrelation within equations over time and contemporaneous correlation among equations. Only recently has econometric methodology caught up with the demands of such a model. The estimation method used was developed by Dhrymes and Taylor in the U.S. and independently by Hatanaka in Japan. Estimates for macrolevel parameters were obtained on annual data for the 1953-79 period[2]. Data are from USDA's Livestock and Meat Statistics, Agricultural Prices, and Agricultural Statistics. All prices and incomes in the model were deflated by the CPI (1967 = 100).

Parameter estimates for the beef cow inventory function (standard errors in parentheses) are as follows:

$$BCI_t = 1874.9 + \underset{(39.46)}{235.95}\, P_t + \underset{(0.0232)}{0.80407}\, (BCI_{t-1} + BH_{t-1}) + \underset{(42.21)}{199.59}\, P_{t-1} \tag{7}$$

$$- \underset{(909.4)}{1655.9}\, X1_t - \underset{(4.564)}{37.314}\, X2_t + \underset{(369.6)}{1561.4}\, X3_t - \underset{(0.1626)}{0.34521}\, U_{t-1}$$

This function fits the national data fairly well. The performance of the function in a full-model Gauss-Seidel simulation over 1955-80 is summarized in the following statistics:

(a) Percentage of variation explained by predictions ("R^2") = .9834,

(b) Mean absolute percentage error = 1.81%, and

(c) Root mean square percentage error = 2.34%.

Estimation of Regional Functions

Regional function estimation requires valuation of the weights W in equation (2) in addition to the macrolevel parameter estimates above. Values of W were selected according to the nature of their corresponding variables. Variables for which the average of the regional observations equals the national-level observation, e.g. price, are weighted as 1s so that

$$\sum^{M} b_{mk} = B_k \tag{8}$$

for the kth variable. Variables for which the sum of the regional observations equal the national-level value, e.g. crop acreage harvested, are weighted by the average proportion of beef cows in each region so that

$$(9) \qquad \sum^{M} W_m b_{mk} = B_k \, , \quad \sum^{M} W_m = 1$$

for the kth variable. This weighting system can not be extended to the lagged disturbances, U_{t-1}, because no observations or independent estimates of the disturbances on a regional basis are available. Accordingly, the macrolevel autoregression coefficient has to be applied to all regions using proportionally allocated estimates of disturbances obtained from simulations using the full macrolevel model.

Data interpolations are necessary for X1, the index of prices paid for feed, because this index is not published for individual states or regions. Surrogate indexes are calculated from corn and soybean meal (cottonseed meal in the West) price weighted according to standard cattle feeding rations.

Estimates of the regional beef cow inventory function are shown in Table 1. The results show not only a statistical conformity to the macrolevel model, which is forced upon them, but a structural conformity as well in 5 of the 6 regions. Performance statistics indicate fits well in line with the macrolevel model for all except the Northeast. These statistics suggest that the Northeast's beef cow inventory function differs from the macrolevel structural specification. However, the macrolevel model does seem to apply to the other regions and hence to the great bulk of beef cows in the country.

Parameter signs in the 5 regions are, with only one exception, the same as for the macrolevel model. That exception is feed costs in the Southeast, which is approximately zero rather than negative as in the

TABLE 1. Beef Cow Inventory Function Parameter Estimates by Region, 1955-80

	Great Plains	Midwest	Northeast	Southeast	Southern Plains	West
b_0, Constant	1268	648.3	-34.0	-295.7	1475	-1186
b_1, Price	41.97	46.42	27.88	23.32	21.93	74.43
	(10.55)	(20.85)	(1.82)	(22.39)	(17.50)	(15.05)
b_2, Lagged Inventory	.7271	.8132	.8406	.8073	.7345	.9219
	(.0503)	(.0473)	(.0933)	(.0572)	(.0542)	(.0599)
b_3, Lagged Price	21.55	53.19	-1.616	71.10	48.05	7.33
	(10.39)	(20.48)	(1.77)	(21.32)	(16.77)	(14.87)
b_4, Feed Price	-664.0	-123.4	39.79	41.09	-405.8	-543.6
	(226.4)	(462.2)	(36.89)	(555.8)	(467.0)	(285.9)
b_5, Crop Acreage	-43.85	-25.31	-43.39	-33.62	-58.59	-28.23
	(7.48)	(3.09)	(7.90)	(13.62)	(15.30)	(10.69)
b_6, Inventory Definition	293.8	234.1	77.6	55.4	407.4	493.2
	(119.5)	(229.5)	(26.1)	(277.2)	(229.0)	(144.7)
R, Autoregression	-.3452	-.3452	-.3452	-.3452	-.3452	-.3452
"R^2"	.953	.976	.617	.963	.957	.918
Mean Absolute % Error	2.38	2.53	18.39	3.22	3.07	2.44
Root Mean Square % Error	3.25	3.36	22.86	4.47	3.79	3.12

other 4 regions. The near-zero coefficient in the Southeast may be explained by the small average size of enterprise and less supplemental feeding there. However, it should be noted that the standard errors of estimates for feed price parameters in all regions seem to be quite high, indicating inadequacies in the surrogate index calculation.

Relationships between current and lagged price parameters seem to indicate differences in price expectations functions among regions. The larger the current price parameter is relative to the lagged price parameter, the greater the implied weighting of current prices in the price expectations functions. Thus, results for the West indicate a high weighting for current price, the Southeast and Southern Plains a low weighting, while the Great Plains and Midwest are intermediate in their weightings. It may be that these differences are just due to chance, but they can be interpreted in terms of known regional characteristics. Western ranches contend with highly variable range conditions and winter feed availabilities, so the West may be less able than other regions to distinguish last year's price effects on operations from weather effects and so focus on current prices. Climatic conditions may also cause the West to be more ruthless in culling barren and less desirable cows than other regions and thus have less capacity for year to year changes in herd sizes. This would explain the large coefficient on lagged inventory for the West. On the other hand, the mild winters of the Southeast and Southern Plains and the availability of winter grazing encourage earlier spring calving and a higher incidence of fall calving than in other regions (Fowler, p. 531). Earlier calving requires earlier breeding and thus earlier establishment of "desired" beef cow inventories, so lagged price would receive greater weighting in the formation of price expectations in these regions.

Crop acreage is inversely related to beef cow numbers in all regions and is relatively significant everywhere, indicating the importance of the competition for land between crops and the pastures and forages necessary for beef cow herds.

Conclusions

Regional inventory and supply functions which are consistent with their macrolevel counterparts can provide a good deal of additional information on the way a commodity's market works that should be useful in a variety of ways. Some of these are suggested by the beef cow inventory example given here. Analysts interest in the effects of an increase in beef cattle prices should be interested to know that their greatest effect on end-of-year beef cow inventories will be in the West and the least in the Southeast and Southern Plains. The main effect on inventories in those regions will be delayed a year. Other things being equal, the regional distributions of future calf crops will be affected. Similarly, Great Plains and Western beef cow numbers seem to be most sensitive to feed prices, while Southeastern inventories are indifferent to them. Since inventories belong to constituents, these differences might well impact Congressional votes on legislation impacting feed prices.

Crop acreages affect beef cow numbers everywhere, but changes in specific crops will affect regions differently. For example, a major increase in the set-aside program in wheat would tend to have concentrated effects on the Great Plains regions, but virtually none on the Southeast. The regional functions also allow one to project the effects of secular increases in crop acreage by regions.

From a methodological point of view, the use of restricted regression to disaggregate functions is belived to be a distinct improvement over earlier methods which did not ensure consistency between macrolevel and regional parameters. This consistency also means that joint dependency effects which are captured by the use of appropriate estimation techniques at the macrolevel are retained in the disaggregated parameters. Put another way, it is a shame to go to the trobule of estimating consistent models at the national level and then have to abandon the results when disaggregating to regions. Use the restricted regression procedure and you will not have to.

FOOTNOTES

[1] The alternative approach is the "bottom up" one in which regional functions are estimated and then aggregated to the national level. Building up from microunits may be a more realistic picture of the way supply works in the real work, but it is difficult to build interdependencies into such models.

[2] Results for the rest of the model are in process of publication.

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