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by

Bryan W. Schurle and Jeffery R. Williams

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Department of Agricultural Economics
Kansas Agricultural Experiment Station
Kansas State University
Manhattan, Kansas
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# Application of Stochastic Dominance Criteria to Farm Data

Bryan W. Schurle and Jeffery R. Williams*

Determining farm characteristics that generate net incomes that are preferred by farmers is difficult. One of the difficulties involves the selection of farms which generate preferred incomes. Characteristics of the farms with the highest incomes can be investigated each year but income fluctuates from year to year, causing farms to move in and out of the "high income" group. Average income over a number of years can be used, but farms with high average incomes also have high variability of income (Zenger and Schurle). There are questions as to whether high income farms with high variability of income are preferred by individuals who are risk averse. Thus there are problems in determining which farms generate preferred incomes.

Stochastic dominance criteria provide one method for determining which income distributions are preferred even by individuals who are risk averse. The use of stochastic efficiency was first proposed by Quirk and Saposnik. This work was extended by Hadar and Russell and Whitmore. Porter and Robison and Barry have made comparisons of stochastic efficiency criteria and other methods of ordering strategies which produce different income distributions. Stochastic dominance has been applied to the selection of irrigation-scheduling strategies (Harris and Mapp), the selection of machinery systems (Hardaker and Tanago), analyzing

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*Associate Professor and Assistant Professor, Department of Economics, Kansas State University.

participation in Federal Crop Insurance (Kramer), selection of rental and cropping strategies (King and Robison), and participating in commodity programs (Kramer and Pope). (See Anderson for a discussion of stochastic dominance and a list of other references.)

Stochastic dominance is generally used to select efficient strategies from among a group of strategies. That use is normative in nature; that is, the judgement is made that stochastic efficient strategies should be preferred over any of the inefficient strategies given certain utility function characteristics. This reduces the decision-making burden by eliminating some alternative strategies. We propose to use stochastic dominance criteria to identify farms that are efficient and then to examine the characteristics of the farms thus selected. That will allow us to identify the farms whose incomes are preferred by individuals (even those who are averse to risk) and to study the characteristics of those farms.

## Stochastic Dominance Criteria

Using stochastic dominance to select efficient strategies relies on comparing cumulative distributions of the possible outcomes for each strategy. Strategy "A" will dominate strategy "B" by first-degree stochastic dominance (FSD) if the cumulative distribution of outcomes from "A" lies entirely or partly below (to the right of) the cumulative distribution for "B" (as long as the cumulative distribution of "A" never lies above that of "B"). Strategy "A" will dominate strategy "B" by second-degree stochastic dominance (SSD) if the area under the cumulative distribution "A" is equal to or less than (must be less than at one or more points) the area under the cumulative distribution of "B" at all points along the range of possible outcomes. These rules hold for both discrete and continuous cumulative distribution functions.

With the development of some notation, these criteria can be stated more formally. If x takes only a finite number of values $x_i$, $i = 1, 2, \ldots, n$ in the interval [a, b] then a cumulative density function $F_1(R) = P(x_i \leqq R) = \sum_{\substack{\text{all} \\ x_i \leqq R}} f(x_i)$ where $f(x_i)$ = the probability that $x_i$ occurs. The first-degree stochastic dominance rule for discrete cumulative distribution functions can be written as strategy F dominates strategy G if $F_1(R) \leqq G_1(R)$ for all possible R in the range [a,b] with at least one strong inequality (i.e., the < holds for at least one value of R).

The second-degree stochastic dominance rule needs notation for the area under the cumulative distribution. Let the outcomes of strategy F be arranged in ascending order so that $x_i > x_{i-1}$. Then define $\Delta x_i = x_i - x_{i-1}$. Then the area under the cumulative distribution is

$$F_2(x_r) = \sum_{i=2}^{r} F_1(x_{i-1}) \, \Delta x_i \qquad r = 2, \ldots, n$$

$$F_2(x_1) = 0.$$

The second-order efficiency rule is that strategy F dominates strategy G if $F_2(x_r) \leqq G_2(x_r)$ for all $r \leqq n$ with at least one strong inequality.

The rules of first-degree stochastic dominance identify those strategies preferred by an individual whose utility is a positive function of income. That is, the only requirement for a decision maker to choose the efficient strategy is to have $\frac{\partial \text{ utility}}{\partial \text{ income}} > 0$.

Rules of second-degree stochastic dominance identify those strategies preferred by an individual who enjoys higher incomes but who is risk averse. His utility is a positive function of income, but the slope of the function is flatter at higher income levels. That is, $\frac{\partial \text{ utility}}{\partial \text{ income}} > 0$ and $\frac{\partial^2 \text{ utility}}{\partial \text{ income}^2} < 0$.

Applying Stochastic Dominance Criteria to Farm Data

Each of these dominance rules can be used to divide a set of strategies into efficient and inefficient subsets. Efficient strategies are not dominated by any other strategy. Inefficient strategies are dominated by at least one other strategy. We apply stochastic dominance rules to data from farms to determine which are efficient, and then analyze characteristics of these preferred farms.

Data from 128 farms in north-central Kansas were used for the analysis. Each farm has 7 years of data on net farm income calculated by using accrual accounting procedures. The net farm income for each farm was ordered from smallest to largest, then each net income was assigned a probability of 1/7th. From this information a cumulative probability density function was generated for each farm. The cumulative functions were then used to determine stochastic efficiency by first-order and second-order criteria.

To group farms by FSD criteria, we compared each farm's cumulative probability density function with that function for every other farm. If a farm was never dominated, it was efficient. This process resulted in several "efficient" farms which were not dominated by any other farm. We gave the group of farms determined to be efficient by this technique a group number: number 1 in this case. That left a large group of farms considered to be "inefficient." We further classified the "inefficient" farms by repeating the process. We used the FSD criteria again comparing each "inefficient" farm to all the other "inefficient" farms. This resulted in another group of farms which were not dominated by any other farm. We gave this group of farms group number 2. We continued this iterative process until all farms had been classified by FSD criteria into 9 groups. The same iterative procedure was used to group farms by SSD criteria. This process resulted in 17 groups.

## Results of Ordering by FSD Criteria

Table 1 illustrates the results of the first-degree stochastic dominance ordering. As a result of the process described earlier, the 128 farms were categorized into nine different groups. Each group contains a different number of farms. The farms within each individual group are not dominated by any other farm within the group because the cumulative probability density functions intersect at least once. Table 1 indicates that there are six farms in group 1. The remaining 122 farms in the sample were dominated by one or more of those 6 farms. Group 2 contained 14 farms; the remaining 108 farms being dominated by one or several of the 14 farms in group 2. At the other extreme of the categories, group 9 consisted of only one farm, which did not dominate any of the other 127 farms. The groups in the middle generally contained larger numbers of farms. Table 1 also reveals some of the characteristics of the farms in the groups. The mean number of acres, value of capital managed, gross farm income, net farm income, and age of the operator for each group are given. In addition, the average standard deviation of net farm income and two diversification indexes are given for each group.

Table 1. Results of Ordering By First-degree Stochastic Dominance (FSD) for 128 Farms in North-central Kansas.

| | Group number | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | All groups combined |
| Number of farms within each group | 6 | 14 | 13 | 18 | 33 | 22 | 16 | 5 | 1 | 128 |
| Average number of acres | 1,593 | 1,036 | 1,022 | 720 | 634 | 546 | 456 | 392 | 400 | 725 |
| Average value of capital managed ($1,000) | 1,318 | 910 | 765 | 644 | 490 | 429 | 415 | 320 | 539 | 598 |
| Average gross farm income ($1,000) | 423 | 204 | 132 | 129 | 86 | 79 | 70 | 75 | 53 | 121 |
| Average net farm income ($1,000) | 107 | 50 | 37 | 31 | 22 | 13 | 6 | -3 | -33 | 27 |
| Average Std. deviation of net income ($1,000) | 93.5 | 51.8 | 38.3 | 36.4 | 23.4 | 21.5 | 20.4 | 18.5 | 16.0 | 32.2 |
| Average $D_1$* | .60 | .42 | .42 | .50 | .42 | .38 | .40 | .56 | .23 | .43 |
| Average $D_2$* | 2.85 | 3.36 | 3.17 | 3.13 | 3.36 | 3.52 | 3.60 | 3.18 | 5.32 | 3.35 |
| Average age of farm operator | 41 | 50 | 46 | 46 | 47 | 45 | 44 | 44 | 47 | 46 |

*$D_1$ and $D_2$ = formulas (discussed under "Results of Ordering by FSD Criteria") used to measure specialization and diversification.

The mean net farm income for each group was consistently related to the group number. In other words, the lowered numbered groups, according to FSD, had higher mean net farm incomes than the higher numbered groups. The lower numbered groups also had higher gross farm incomes than did the higher numbered groups, group number 8 excepted. We also examined proxy variables for size, the mean values of capital managed, and number of acres, for each group. In general, except for the single farm in group 9, the lower numbered groups had larger farms which provided larger average net farm incomes. This is an expected result since a necessary, but not sufficient, condition for one farm to dominate another is that it have a higher average net farm income.

The variance of net farm income also seems to be consistently related to the group numbers. Previously we stated that the average net farm income appeared to be related to the group numbers. It can also be seen that the group average of the standard deviations of net farm income for each farm was related to the group numbers. The lower numbered groups had higher average net farm incomes, accompanied by higher standard deviations of the net farm incomes.

To measure the diversification characteristics of the groups, we used two formulas: $D_1$, as suggested by Pope and Prescott; and $D_2$, developed as an alternative measure of diversification. The Herfindahl index, $D_1$, is represented as $\sum_{i=1}^{n} P_i^2$, where $P_i$ = proportion of the business in an enterprise. A value approaching 1 with this measurement indicates specialization; the smaller values reflect diversification. The values of $D_1$ ranged from .23 to .88 for the 128 farms. In the second measure of diversification,

$D_2 = N - \frac{N}{2} \left\{ |P_1 - 1/N| + |P_2 - 1/N| + \ldots + |P_N - 1/N| \right\}$, where N = the maximum number of enterprises and $P_i$ = proportion of the business in enterprise i. This formula modifies a summation of linear differences from the perfectly diversified operation as a proxy for diversification on the farm. As this measurement approaches 1, it indicates specialization; values approaching N indicate the operation is highly diversified. The values for $D_2$ for the 128 farms fell within the range 1.63 to 5.32. The average values of $D_1$ and $D_2$ for each group did not appear to be related to the group numbers obtained by FSD criteria. (For more information on how $D_1$ and $D_2$ were calculated and what relationships have been found see Zenger and Schurle.)

The mean age of the operators of the farms for each group did not appear to indicate any particular trend. However, by this ordering the mean average age was lowest for operators of group number 1.

## Results of Ordering by SSD Criteria

Second-degree stochastic dominance criteria provide a more restrictive rule for ordering. SSD requires an important additional behavioral assumption: that the decision maker is averse to risk. That means that for second-degree dominance, the manager has positive but decreasing marginal utility of income. This additional assumption allowed us to reduce the number of farms in the groups, thus increasing the number of groups (see Table 2).

Using SSD criteria we distributed the 128 farms into 17 groups. There were two farms in group 1 since they were not dominated by any other farm according to SSD criteria. The remaining 126 farms were dominated by one or the other of these farms. Again, more farms fell into the middle groups than into either the higher or the lower groups.

The mean net farm income for each group was again consistently related to the group number. The higher the mean net farm income was for

Table 2. Results of Ordering By Second-degree Stochastic Dominance (SSD) for the 128 North-central Kansas farms.

| | Group number | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| Number of farms within each group | 2 | 5 | 5 | 7 | 8 | 13 | 18 | 16 | 13 | 10 | 8 | 8 | 5 | 4 | 3 | 2 | 1 |
| Average number of acres | 1,271 | 1,631 | 849 | 808 | 864 | 854 | 723 | 720 | 575 | 570 | 383 | 567 | 479 | 680 | 650 | 756 | 400 |
| Average value of capital managed ($1,000) | 1,135 | 1,264 | 775 | 691 | 701 | 617 | 579 | 668 | 456 | 443 | 371 | 446 | 408 | 527 | 546 | 636 | 539 |
| Average gross farm income ($1,000) | 399 | 309 | 206 | 152 | 126 | 99 | 109 | 142 | 77 | 71 | 86 | 78 | 78 | 128 | 97 | 161 | 53 |
| Average net farm income ($1,000) | 122 | 87 | 61 | 47 | 39 | 30 | 27 | 26 | 18 | 16 | 12 | 8 | 7 | 4 | -3 | -7 | -33 |
| Average Std. deviation of net income ($1,000) | 66.2 | 72.5 | 48.1 | 34.2 | 37.6 | 25.6 | 26.8 | 39.5 | 23.7 | 23.3 | 28.2 | 22.0 | 24.2 | 32.0 | 36.5 | 39.6 | 16.0 |
| Average $D_1$* | .62 | .47 | .43 | .48 | .43 | .41 | .47 | .43 | .36 | .41 | .52 | .37 | .36 | .55 | .43 | .40 | .23 |
| Average $D_2$* | 3.08 | 3.17 | 3.24 | 3.18 | 3.17 | 3.31 | 3.24 | 3.26 | 3.48 | 3.26 | 3.20 | 3.85 | 4.08 | 2.98 | 3.36 | 3.59 | 5.32 |
| Average age of farm operator | 51 | 43 | 51 | 48 | 47 | 45 | 45 | 48 | 49 | 42 | 46 | 46 | 43 | 41 | 42 | 43 | 47 |

*See footnote, Table 1.

a group of farms, the lower was the group number. Again, this is an expected result since a necessary, but not sufficient condition for one farm to dominate another by SSD is that it have a higher average net farm income. The indicators of size, mean number of acres, mean of capital managed and mean gross farm income, for the farms also tended to illustrate that the larger the farm, the lower the group number. However, the relationships were not nearly as consistent as those exhibited in Table 1. For example, the two farms in group 1 averaged fewer acres and smaller amounts of capital managed but higher gross and net incomes than did the second group.

The relationship between standard deviation of net farm income and the group numbers should be interesting since SSD criteria are based on the assumption that decision makers are averse to risk. Although the relationship was not without exceptions, the lowest numbered groups (according to SSD) tended to have higher average standard deviations of net farm incomes. The middle groups tended to have the lowest average standard deviations and the highest numbered groups again had slightly higher average standard deviations. The higher average net farm incomes associated with lower numbered groups are accompanied by higher standard deviations. These results indicate that individuals--even those considered to be averse to risk--would prefer the large farms with the higher variability.

Average diversification indexes were also compared to group numbers. The tabulated results did not indicate any obvious relationship. Furthermore, there appeared not to be any obvious relationship between the average diversification indexes and the average standard deviation of the net farm income for the 17 groups.

Again, as was the case with FSD, the mean age of the operators for the farms within the group did not appear to indicate any obvious trend or relationship.

## Summary

First-order stochastic dominance (FSD) criteria were used to classify 128 farms into 9 groups. Characteristics of the groups were then investigated and comparisons between characteristics and group numbers were made. Groups with higher net farm incomes, higher standard deviations of net farm incomes, higher gross farm incomes, more capital, and more acres generally had lower FSD group numbers.

Second-order stochastic dominance (SSD) criteria were then used to classify the 128 farms into 17 groups. Groups with higher mean net farm incomes had lower SSD group numbers. Groups with larger farms as measured by number of acres, capital, or gross farm income, also tended to have lower SSD numbers although this relationship was not as consistent as for FSD. Groups with higher average standard deviations of net farm incomes also tended to have lower group numbers although this relationship had exceptions.

Neither diversification nor age of the operator, two other characteristics reported here, showed a very consistent relationship with the group numbers developed by FSD or SSD criteria.

While stochastic dominance criteria appear to have uses in analyzing farm data, there are some concerns that need to be addressed more fully. There are some questions concerning what data should be used and how it should be manipulated. For example, some farms change over time by growing or changing the size of some enterprises. This would affect the net incomes they produce. Inflation over time also needs to be accounted for. In this study, equal probabilities were assumed for each of the incomes. It may be more reasonable to put less weight on extremely high or low incomes.

There are also some questions regarding the way in which stochastic dominance groups farms. There are some practical situations where the preference between two income distributions cannot be determined by stochastic dominance criteria, yet an ordering is obvious from casual observation. In other situations, a "bad" income distribution can be included in a group with much better distributions. This "bad" income distribution can be much worse than many other income distributions that would be placed in higher numbered groups. This results in some farms being placed in lower numbered groups than they should be. As a result, the similarity of farms within a group should be analyzed as well as differences between groups.

A last major concern of applying stochastic dominance criteria to net farm incomes is that not all incomes are generated by the same set of resources. Not all of the opportunity costs for the resources used are deducted from the net farm income. This can be partially corrected by applying stochastic dominance criteria to similar sized farms to determine which generate the preferred incomes.

While there are some concerns that need to be resolved, stochastic dominance criteria still appear to have some useful applications in this area. The potential for future research in this area appears to have great potential.

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