Machinery

1981

Modeling Optimal Replacement Decisions for Farm Machinery: Some Theoretical and Empirical Problems*

By

Garnett Bradford and
Donald Reid*

Dept. of Agricultural Economics
University of Kentucky
Lexington, Kentucky 40546

*Paper presented at the American Agricultural Economics Meetings, July 28, 1981, Clemson, S.C.

Garnett Bradford is Professor of Agricultural Economics and Donald Reid is Assistant Extension Professor of Agricultural Economics, both at the University of Kentucky, Lexington.

ABSTRACT

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Garnett L. Bradford and Donald Reid
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Machine replacement research has concentrated on delineating and comparing present value (PV) and derived marginal analysis models. With such, one encounters problems in generating reliable estimates of new and used machine prices, repairs and opportunity costs. Some empirical problems may be surmounted by altering the traditional PV framework, but theoretical considerations require more comprehensive models.

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Attention to the optimal replacement problem has centered on specifying the theoretically appropriate criterion. Today, the most commonly accepted replacement decision theory for machinery assumes the owner will replace each older machine, "defender", with an identical new machine, "challenger", in accordance with long-run profit maximizing or cost minimizing criteria (i.e. wealth maximization). Perrin (1972, p. 60) states the basic marginal principle: "A machine should be kept another period if the marginal costs of retaining it...are less than the 'average' periodic costs of a replacement machine". As Chisholm (1966) noted, this is a "deceptively simple" criterion. Support for Chisholm's contention can be found throughout the literature, for acceptance of an appropriate criterion has come about slowly.

Samuelson (1976) cited an extended list of writings in forestry and economics in which the optimal replacement criterion is partially or wholly incorrect. It includes among others: Boulding's (1966) microeconomic text and writings by Hotelling (1925) and Fisher (1930). To this list the agricultural economics profession can add a number of writings. This is the case since much of the research in agricultural economics is applied, and thus, draws heavily upon the correct or incorrect theoretical writings in economics. The JFE article by Faris (1960) was intended to demonstrate criteria for replacements occurring within a production period and for longer term point-input, point-output and point-input, continuous-output replacements. The problem with his criteria was the failure to account for the opportunity costs beyond the first replacement, i.e. he considered the case of only one replacement. Winder and Trant (1961) and Chisholm (1966) dealt almost entirely with how to correctly specify and interpret the marginal replacement criterion. The major contribution of Perrin's article was to show equivalence of the marginal criterion to the net present value criterion. If correctly stated and applied, notwithstanding Perrin's contention, the two criteria will yield identical replacement decisions.

The marginal criterion with its logical linkage to neoclassical theory probably has greater appeal to the theoretician. In contrast, the present value criterion, with its more explicit connection to the standard investment net present value criterion, may have greater appeal to the empirical researcher, especially to financial management analysts. Considering replacement as a special type of investment, the replacement problem can be viewed as a mutually exclusive investment decision. But unlike the standard investment decision, in which the projects of a specific time horizon are mutually exclusive, the replacement case is one in which the time periods of ownership for a specific project are mutually exclusive. Recently, the net present value (PV) criterion has been more common in the literature than the marginal criterion (e.g. Kay and Rister, 1976; Bates et al., 1979; Crane and Spreen, 1980). This paper follows the PV treatment.

Other than demonstrating the PV criterion, little research has been directed toward applying it to the problem of machinery replacement. In applying the criterion to a practical machinery replacement problem, two general problems are encountered. First, there is a problem with generating precise estimates for the parameters in the formula. The second problem involves how to analyze the problem of replacing with non-identical challengers. The purpose of this paper is to explicitly point out information needs in using the replacement criteria and to demonstrate how the identical challenger

PV model can be modified to consider the case of non-identical replacements. In addition the need for a more powerful analytical method is pointed out along with a potential solution. First, the basic, identical challenger PV criterion will be reviewed and related to the standard investment PV formulation.

Identical Challenger Criterion

The optimal replacement age for a machine to be successively replaced by an infinite series of identical challengers can be determined in the discrete case by finding the age which minimizes the absolute value of the expression:

$$PV(S) = (PERP)\left[-M(O) - \sum_{t=1}^{S} (1+r)^{-t} R(t) + (1+r)^{-S} M(S)\right] \qquad (1)$$

where

PV(S) = Present value for each value of s (units for t may be years or other appropriate time intervals),

PERP = $[1-(1+r)^{-S}]^{-1}$, formula for the present value of a \$1 perpetual annuity received (paid) at the beginning of each and every S years, e.g., S may vary from 5 to 20 years for a tractor replacement problem,

M(O) = New cost of the machine, assumed to constant for the identical challenger problem,

M(S) = Remaining (salvage) value of each machine when replaced, also assumed to be constant for the identical challengers problem,

R(t) = Costs attributable to the machine during each time period t, including opportunity costs associated with revenues foregone due to breakdown time,

r = the appropriate periodic discount rate.

Frequently, only costs are considered as in the case above. When the cost minimizing criterion is used, the opportunity costs of revenues foregone due to untimely breakdowns must be considered. However, expression (1) can also be used in a profit maximizing sense, by selecting the age, S, for which the value of PV is a maximum. In this case the R(t) includes revenues and thus represents periodic net cash flows. It should be noted that, in the identical challenger case, revenue streams and costs for all successive challengers are assumed to be identical to those associated with the current machine (i.e. constant expectations are assumed).

Notice that the terms inside the brackets of expression (1) are arranged like a standard capital investment problem for a fixed planning horizon of S years, viz., (1) M(O) signifies the value of the machine at t = 0, i.e. the initial value, (2) $\sum_t (1+r)^{-t} R(t)$ denotes discounted revenues and/or costs during each machine's life, and (3) $(1+r)^{-S} M(S)$ denotes the discounted value of the machine at t = S, i.e. the discounted remaining or salvage value. The perpetuity factor (PERP) converts the standard capital investment criterion to one which allows determination of the optimal replacement timing. Expressing R(t), M(0), and M(S) all at t=0 evaluates the standard investment at the beginning of each investment period of length S. This present value is then treated as the amount of payment of an annuity paid every S years. PERP can then be used to find the PV of an infinite stream of such payments.

In summary, the replacement criterion is simply one that evaluates a infinite stream of standard investments of length S. The objective is to determine the value of S for which the present value is optimized.

Research Needs for Parameter Information

Even if one accepts model (1) as valid, a number of empirical problems must be confronted in conducting research on optimal replacement of farm machines. Some of the major problems are:

(1) Realistically estimating the cost of machine maintenance and repairs (M & R) over time,

(2) Accurately estimating remaining values of the machines [M(S) in expression (1) or sometimes denoted as RV],

(3) Determining opportunity costs of untimely breakdowns.

Perhaps the most serious empirical problem is due to the lack of data on the time incidence of actual maintenance and repair (M & R) costs. Researchers continue to rely almost wholly on formulas from the Agricultural Engineers Yearbook or other similar sources. Prior to 1979 the formula for tractors was:

$$\text{TAR}\ \% = .0012\ X^{1.5} \qquad (2)$$

where TAR denotes total cummulative annual repairs and X equals the percentage of accumulated hourly use relative to total estimated lifetime use. Formulas for other machines are similar power functions, all having exponents around 1.5. But any exponent less than 2.0 means that annual M & R will increase smoothly at a decreasing rate. With such a function it is probable that major overhauls, usually necessary in later years, would be grossly underestimated. Hunt (pp. 69-71) takes note of this problem and presents study results for two other formulas. However, neither covers machine use beyond 4,000 hours for tractors or comparable lives for other machines.

Starting in 1979, the Agricultural Engineering Handbook presents TAR percentages for midwest conditions with exponents around 2.1. Still the appropriate M & R function should be capable of capturing ups and downs in yearly expenditures. Thus, at least a third-degree polynomial or some transcendental function would seem plausible. Perhaps a spline function approach would be more practical. Again, the basic problem is a lack of data on M & R for tractors and other major farm machines over an extended number of years.

This implies a need for a long-term commitment to somewhat basic, tedious research.

Formulas for estimating RV's for tractors and other machines are available in the Agricultural Engineers Yearbook, from research by Peacock and Brake, and from recent research in Canada by McNeill. The respective tractor formulas are given as follows:

$$RV(1) = 68(0.92)^{(\text{Age in years})} \quad (3a)$$
[Agricultural Engineering Yearbook, 1976, p. 324]

$$RV(2) = 65.6 - 4.1\ (\text{Age in years}) \quad (3b)$$
[Peacock and Brake, 1970]

$$RV(3) = e^{-0.4299 - .0436\ \text{Age} + .0691\ \text{Condition}} \quad (3c)$$
[McNeill, 1979]

where, RV = percentage of the original cost.

The predictive accuracy of each formula may be criticized. Original research underlying the engineering formula is not documented. The engineering formula shows RV at the end of year 1 to be only 63% of a tractor's original cost. But, during the 1970's, one-year-old tractors frequently resold for more than the original list price. The other two formulas exhibit similar deficiencies. The formula from the Canadian study was developed from limited cross-section data, so certain explanatory variables may be absent. As Peacock and Brake recommended, an extensive study of RV's is needed for tractors and for other major farm machines. Resultant formulas seemingly should account not only for the machine's age but also the machine size (e.g., tractor horsepower), shift's in farmer demand due to changes in their cash flow, differences in demand for different machine makes, inflationary effects, etc.

Determining opportunity costs of breakdowns actually consists of two problems. First , the amount of downtime that will occur must be estimated.

Then, in order to place a value on the downtime, the amount of revenue foregone must be determined. Accumulated downtime functions for tractors are available in the Agricultural Engineering Handbook (1979, p. 254) and are given as follows:

$$B = 0.0000021\ X^{1.9946} \quad \text{(Spark Ignition)} \tag{4a}$$

$$B = 0.0003234\ X^{1.4173} \quad \text{(Diesel)} \tag{4b}$$

where B is the accumulated amount of downtime in hours, and X is the accumulated usage in hours. Again, the data or methods of estimating the parameters are not documented. No downtime formulas are available for other machinery and equipment, although some rules regarding downtime are stated for selected machinery and equipment. Like the M & R case, a long-term committment to basic research is needed for good breakdown formulas to be developed.

The problem of predicting revenues is, of course, a general problem in agricultural economics which already receives considerable attention. The concern of revenues within the context of agricultural machinery replacement is related to the weakness of directly applying the identical challenger specification. The direct application of the criterion has two basic weaknesses. First, fluctuating prices of both inputs and outputs, fluctuating yields, and technological changes causes the assumption of identical net revenues for each challenger to be too unrealistic. The second problem is one of simultaneity -- that is, arbitrarily selecting the production system from which the revenue values are taken without considering the appropriate interaction between replacement investment and production. These two problems are addressed in the remaining two sections.

Non-Identical Challengers

The assumption of identical challengers may be reasonable for many forestry problems or for the aging of wines -- the sort of problem most often studied by those interested primarily in theoretical properties of the basic marginal analysis replacement model. However, this assumption is rarely met in real world machinery replacement situations. Improved fuel usage, lower repairs or some other technical improvements or market changes may well be the primary reason for considering replacement. In these cases, consistent with what has been reported in the literature (e.g., Perrin, pp. 62-63), relatively simple modifications to the framework of the identical-challenger model, expression (1) above, may suffice.

Suppose, for example, that at $t = 0$ the decision maker expects that at some $t > 0$ technical improvements will be embodied in all successive challengers. Specifically, suppose $t = 0$ is at the beginning of the defender's life and the expectation set consists of the following:

(A) Challengers which require 20% lower repairs for each period, i.e., for each period after purchase of the initial challenger (this implies technical innovations which are fully embodied in the machine at the time of replacement), and

(B) Consistent with the expected reality of (A), a 15% higher real purchase price for each challenger and a 18% higher resale price.

Model (1), accordingly, can be altered to

$$PV(2S) = PV(D) + (1+r)^{-S1^*}[PV(C)] \qquad (5)$$

where

r = the appropriate discount rate

$$PV(D) = -M(0) - \sum_{t=1}^{S1^*} (1+r)^{-t} R(t) + (1+r)^{-S1^*} M(S1^*),$$

$$PV(C) = PERP[-M(0)(1.15) - \sum_{t=1}^{S2*} (1+r)^{-t}(.8)R(t) + (1+r)^{-S2*}(1.18)M(S2*)]$$

S1* = optimal replacement age of the current defender,

S2* = optimal replacement age of the infinite stream of identical challengers,

$S1* \lesseqgtr S2* \lesseqgtr S.$

Note that due to the specific change in expectations, one would expect that the optimal replacement strategy for the infinite replacements (S2*) at t = 0 could differ from the optimal strategy (S) for the identical challenger model, expression (1). Also note that the changed expectation implies that the defender's optimal life (S1*) may be different from the optimal life found via the identical challenger model (S). Model (5), just as model (1), assumes certainty of all expectations; a true dynamic model must, of course, allow for changing expectations as the decision maker changes the planning horizon and over time implements actual replacement decisions. Model (5), however, does incorporate expectation parameters which allow for introduction of expected technical and market changes at t = 0. In general, the logic of model (5) can be expanded to account for expected technical and market changes which would be embodied at the time of investment in several successive challengers. For each challenger which is different from the previous series, one must add another term to expression (5), e.g., 3 terms for two technical changes etc.

Toward a More Complete Model of Replacement

Even with good M & R, RV, and breakdown formulas and with the specification of non-identical challengers, a basic weakness in replacement decision modeling still remains for application to farm machinery and equipment. Generally speaking, this weakness is due to the lack of simultaneous considerations. For example, replacement of farm machinery depends on the production projects available, but the production projects selected also depend on the machinery available. One consideration within this production-investment decision is that of opportunity costs of breakdown time. The appropriate breakdown costs can only be determined if the appropriate production decision is considered, since the cost of breakdowns is simply the cost of production revenues foregone.

Another need for simultaneous evaluation occurs in the case of funds rationing. When this occurs the discount rate must be determined simultaneously with the capital budget. But since the PV replacement models consider only one asset and the discount rate must be known, determination of optimal replacement decisions is difficult. In addition to the examples cited, several other situations exist for which simultaneous analysis is needed.

Therefore, how does one model replacement decisions under these circumstances? One answer may be a mathematical programming model--more specifically a multiperiod mixed integer programming model. The probem of simultaneity has been dealt with in the context of the standard production-investment problem (Boehlje and White). However, programming methods have not been extended to analyze the problem of replacement. Two problems with replacement have probably precluded its use: (1) replacement of machinery can be analyzed only if integer activities are possible, and (2) an infinite horizon cannot be explicitly modeled with programming methods.

The integer activity requirement can be overcome with the improved mixed integer algorithms that have been developed over the past decade. But the problem of the infinite horizon remains. Thus, solving the infinite horizon problem in such a way that the simultaneous aspects of mathematical programming can be used, at least for time periods near the decision period, would represent a significant breakthrough in decision models for farm machinery and equipment replacement.