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CRITERIA FOR MODEL SPECIFICATION OF DEMAND SYSTEMS

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ABSTRACT

Four general principles are formulated as criteria for model specification of demand systems. The concepts shed light on systems credibility, validation, strengths, and weaknesses. The principles are used to judge the empirical performance of the S_1-branch system.

CRITERIA FOR MODEL SPECIFICATION OF DEMAND SYSTEMS

Introduction

During the 1960s and the 1970s, theoretical work on consumer demand systems began to bear the fruit of empirical applications [Barten (1977), Brown and Deaton (1972), Phlips (1974), Powell (1974), and Theil (1975)]. Presently, a variety of complete systems of demand equations are available for econometric use. Some systems such as the Rotterdam system, the constant elasticity of demand system, and the Australian models are directly specified demand systems. Other systems such as the addilog models and the various expenditure systems are analytically derived from the constrained maximization of particular utility functions. Also, there is a growing literature on dynamic specifications of complete systems to provide a scope for adjustment, habit, and inventory features on consumer behavior [Houthakker and Taylor (1970) and Lluch and Williams (1975)].

Although all pay homage to classical and modern demand theory, on theoretical grounds, none of the models seems to dominate. The choice for model specification of demand systems then rests primarily on empirical grounds. However, no formal criteria exist to evaluate the empirical performance of complete demand models. The purpose of this paper is to formulate workable criteria for model specification of demand systems. The concepts shed light on systems credibility, validation, strengths, and weaknesses. The application of such criteria to judge the empirical performance of a generalized linear expenditure system follows in a later section.

Criteria for Model Specification

The criteria for model specification of demand systems rely on four fundamental principles: (1) use of parameter values, signs, magnitudes, and test statistics to provide the information about the validity of the results; (2) goodness-of-fit to sample data and predictive ability to independent data samples; (3) computational burdens and program costs; and (4) comparisons with related research.

Probably the most common criterion of model specification, complementary to examination of theoretical and statistical support of parameter estimates, involves goodness-of-fit and predictive ability in terms of the average budget shares over the sample period. The measure which describes the goodness-of-fit is the weighted R-squared statistic (R_w^2), and the measure which describes predictive ability of average budget shares is Theil and Mnookin's (1966) information inaccuracy statistic (IIS).

The R_w^2 statistic is merely an index of the predictive power of a system of equations. Mathematically,

$$R_w^2 = 1 - \sum_{i=1}^{n} SSE_i \Big/ \sum_{i=1}^{n} SST_{c_i}, \qquad (1)$$

where SSE_i is the residual sum of squares for the i^{th} commodity and SST_{c_i} is the corrected total sum of squares for the ith commodity, i=1,...,n. The IIS evaluates the overall fit to the data for the entire set of commodities. Mathematically,

$$IIS = \sum_{i=1}^{n} w_i \ln(w_i/\hat{w}_i), \qquad (2)$$

where w_i denotes the observed average budget share of the i^{th} commodity and $\hat{w}_i$ denotes the predicted average budget share of the i^{th} commodity. Since $\sum_{i=1}^{n} w_i = 1$ and $\sum_{i=1}^{n} \hat{w}_i = 1$ and since $0 < w_i, \hat{w}_i < 1$ for all i, each of the n shares, predicted as well as observed, is regarded as a complete set of probabilities. The forecasts $\hat{w}_i$ are the "prior" probabilities, and at some point in time, a message comes in depicting the actual average budget shares, henceforth changing the "prior" probabilities into "posterior" probabilities, w_i. (2) is always positive unless $w_i = \hat{w}_i$ for each i (perfect forecasts) in which case IIS = 0. The larger the differences between w_i and $\hat{w}_i$, the worse the forecasts and the larger the information content of the message. A weighted R-squared value close to one and an information inaccuracy value close to zero indicate an exceptional fit of the demand system to the sample data.

The acid test concerns the predictive performance on the basis of independent samples. With the availability of the independent samples, the evaluation of predictive ability rests on cross validation. One sample, called the "estimation data," is used to estimate the coefficients in the demand system. The other sample, called the "prediction data," is used to measure the predictive accuracy of the demand system based on the previously estimated coefficients. Pearson product-moment correlation coefficients, median absolute residuals, mean absolute residuals, and mean squared error of observed and predicted purchases typically serve as statistical measures of predictive ability.

Estimation methods for several of the demand systems should take into account essential parameter nonlinearity, cross-equation correlation, variance-covariance singularity of the disturbance terms, and various parameter restrictions. Consequently, the estimation of the parameters of demand systems may be burdensome and expensive.

Since estimates of demand parameters are reported in the applied economic literature, the estimated coefficients of demand systems may be put into proper perspective via comparisons with previous and related studies. Caution is in order, however, with respect to such comparisons due to differences in data bases, estimation methods, commodity groups, and demand systems.

Application of Principles

To illustrate the use of the four principles, a generalized linear expenditure system, the S_1-branch system, is employed. The model permits a fine classification of commodities where other systems allow only broad categories of commodities.

The set of demand functions for this system is given by:

$$q_{si} = \gamma_{si} + \left(\frac{\beta_{si}}{p_{si}}\right)^{\sigma_s} \left(\sum_{j \varepsilon G_s} \left(\frac{\beta_{si}}{p_{si}}\right)^{\sigma_s} p_{sj} \right)^{-1} w_s \; (y - \sum_{r=1}^{S} \sum_{j \varepsilon G_r} p_{rj} \gamma_{rj}) \qquad (3)$$

The subscripts si refer to the i^{th} commodity within group (branch) s; s=1,...,S; i=1,...,n. Brown and Heien (1972) offer the interpretation of the parameters γ_{si}, β_{si}, σ_s, and w_s. The following parameter restrictions apply: $\gamma_{si} > 0$, $w_s > 0$, $\beta_{si} > 0$, $(q_{si} - \gamma_{si}) > 0$, and $\sigma_s > 0$ for all i,s. The marginal rate of substitution between any two goods in different branches is independent of any goods from other branches. Thus, the S_1-branch utility function is strongly separable with respect to the partitions $G_1,\ldots,G_S$.

The enumeration of the branches and the respective commodities for this application are as follows: branch 1--ground beef, steaks, roasts, poultry, pork, other meats, and seafood; branch 2--food away from home and other foods; and branch 3--fuels for home heating and gasoline. Other foods is an aggregate commodity which consists of cereal and bakery products, dairy products, prepared food, fruits, and vegetables. Although these eleven nondurable commodities constitute about 30 percent of the representative consumer's budget, due to the fact that the sum of the expenditures on the individual goods equals the total expenditure under consideration, this set of consumption categories constitutes a complete demand system. Emphasis is on the meat and seafood commodities due to their relative importance in food consumption. The technique that assigns commodities into separable groups is primarily intuitive in the absence of a priori knowledge of actual relationships among marginal utilities.

The source of data is the 1972-74 U.S. Bureau of Labor Statistics Consumer Expenditure Survey. Implicit prices for meat and seafood products and other foods are derived from expenditure and quantity data compiled in the survey. Regional price indices are employed for gasoline, fuels for home heating, and food away from home due to the unavailability of quantity data for these commodities. To explore national and regional differences in consumer behavior, purchase patterns are analyzed in the Northeast, the North Central, the South, the West, and the U.S. The number of observations in the sample for each region is as follows: 4041 observations for the U.S., 855 observations for the Northeast, 1293 observations for the North Central, 1180 observations for the South, and 713 observations for the West. For validation, the respective samples are split into two random samples of almost equal size for each region.

The estimates of systems parameters and elasticities are reasonable on a priori grounds in the S_1-branch system. Due to space limitations, however, only the own-price and expenditure elasticities are shown in Table 1. All the direct-price coefficients are negative for the respective commodities, in conjunction with theoretical expectations. In particular, the own-price elasticities for the meat and seafood commodities and for food away from home exceed unity. Such elastic responses to own-price changes may be attributable in part to the level of disaggregation of the commodities and to the type of data. Estimates based on household survey data typically represent longer-run behavior than estimates based on time-series data. The level of disaggregation of the meat and seafood products increases the number of substitutable products. All the expenditure coefficients are positive which indicates that the various commodities are normal goods, again in conjunction with theoretical expectations.

Although this complete system yields plausible estimates of elasticities for all commodities, since the variances of the sampling distributions of these measures have not been explicitly derived, the investigation of their statistical reliability through formal tests of significance is precluded. This reservation regarding sampling properties is quite important and can only be lessened by the consistency of results within the analysis and among related and additional studies.

The S_1-branch system accounts for a substantial amount of variation in the purchases of the respective commodities and accurately predicts the average budget shares over the range of the sample. The weighted R-squared statistics range from .5722 to .7913, and the information inaccuracy statistics vary from .0015 to .0071 (Table 2).

Table 1. Own-Price Elasticities and Expenditure Elasticities.[a]

Commodity	Own-Price Elasticities										Expenditure Elasticities									
	U.S.		Northeast		No. Central		South		West		U.S.		Northeast		No. Central		South		West	
	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]
Ground Beef	-1.81	-1.49	-1.79	-1.76	-1.52	-1.38	-1.63	-1.60	-1.37	-1.31	2.09	1.14	1.40	1.27	1.49	1.11	1.21	1.32	1.45	.90
Steaks	-1.78	-1.78	-1.44	-1.74	-1.82	-1.87	-1.88	-1.63	-1.48	-1.77	2.03	1.40	1.14	1.29	1.80	1.54	1.44	1.36	1.56	1.26
Roasts	-1.99	-1.78	-1.68	-1.87	-2.01	-1.81	-1.60	-1.82	-2.06	-1.89	2.29	1.39	1.34	1.40	1.99	1.47	1.20	1.52	2.21	1.30
Poultry	-1.13	-1.29	-1.26	-1.25	-1.17	-1.28	-1.32	-1.26	-1.28	-1.38	1.27	1.00	.99	.92	1.13	1.04	1.00	1.05	1.34	.97
Pork	- .82	-1.46	-1.72	-1.47	-1.14	-1.45	-1.57	-1.51	- .67	-1.73	.92	1.14	1.38	1.09	1.11	1.19	1.20	.79	.70	1.22
Other Meats	-1.27	-1.36	-1.55	-1.42	-1.42	-1.37	-1.50	- .97	-1.39	-1.64	1.47	1.04	1.21	1.03	1.39	1.09	1.11	1.27	1.48	1.11
Seafood	-1.82	-2.52	-2.06	-1.77	-2.54	-1.59	-2.59	-2.80	-1.37	-1.81	2.14	1.92	1.60	1.26	2.52	1.26	1.92	2.32	1.47	1.21
Food Away From Home	-1.29	-1.21	-1.19	-1.36	-1.36	-1.11	-1.18	-1.21	-1.10	-1.21	1.03	1.14	1.14	1.21	1.15	1.11	1.12	1.09	1.02	1.27
Other Foods	- .95	- .95	- .92	-1.02	- .95	- .91	- .93	- .96	- .90	- .88	.71	.84	.83	.86	.75	.87	.85	.82	.80	.86
Fuels for Home Heating	- .83	- .85	- .93	- .94	- .93	- .90	- .89	- .84	- .90	-1.01	.77	.90	.88	.91	.81	.93	.90	.85	.85	.89
Gasoline	- .70	- .76	- .93	- .87	- .83	- .70	- .80	- .88	- .86	- .97	.80	.92	.95	.92	.77	.82	.93	.98	.86	.84

[a]The estimates hold at the sample mean quantities, prices, and average budget shares. Substantial movement away from such coordinates may involve dramatic changes in the economic indices.

[b]S1 and S2 refer to Sample 1 and Sample 2, respectively.

Table 2. Weighted R-Squared Statistic, Theil's Information Inaccuracy Statistic, Program Cost, and Time of Execution for the S_1-Branch System by Sample and Region.

Sample and Region	R-Squared[a]	IIS[b]	Program Cost[c]	Time of Execution[d]
U.S./Sample 1	.6909	.0030	161.33	5802
U.S./Sample 2	.6911	.0026	73.94	2640
Northeast Region/Sample 1	.7418	.0056	60.48	2138
Northeast Region/Sample 2	.7703	.0027	75.39	2712
North Central Region/Sample 1	.6600	.0015	103.63	3770
North Central Region/Sample 2	.7913	.0058	72.54	2571
South/Sample 1	.7646	.0036	63.35	2227
South/Sample 2	.5783	.0042	87.97	3174
West/Sample 1	.5722	.0071	41.76	1425
West/Sample 2	.5804	.0024	69.41	2493

Source: Computation by the authors.

[a]The R-squared that corresponds to the approximate F-test on all non-intercept parameters in the system.

[b]Information inaccuracy statistic.

[c]In dollars.

[d]In seconds.

Since the S_1-branch system is a complete set of nonlinear demand functions the coefficients are estimated using a full information maximum likelihood (FIML) algorithm [Bard (1967)]. As exhibited in Table 2, the convergence of the iterative estimation technique is not at all rapid, and the computations are costly. The program costs range from 60.48 to 161.33 dollars, and the times of execution vary from 1425 to 5802 seconds. The slow convergence of the FIML method is attributable to the extreme flatness of the likelihood functions and/or poor initial parameter guesses. Attempts were not made to check on whether or not relative maxima had been reached by varying the starting values due to the costliness of the procedure.

The Pearson product-moment correlation coefficients and the median absolute residuals of the observed and predicted quantities of commodities are shown in Table 3. The correlation coefficients are comparatively large for ground beef, steaks, roasts, pork, poultry, food away from home, other foods, and fuels for home heating. In all cases, actual and predicted purchases of the commodities are positively associated. The median absolute residuals are relatively small for meat and seafood commodities.

The direct-price elasticities of the S_1-branch system for meat and seafood products are consistent with the direct price elasticities of Christensen and Manser (direct translog system) (1977) (Table 4). The expenditure elasticities for the meat and seafood commodities are similar to the income elasticities published by Blackorby, Boyce, and Russell (1978) and Christensen and Manser (direct translog system) (1977).

Table 3. Median Absolute Residuals and Pearson Product-Moment Correlation Coefficients of Observed and Predicted Quantities.

Commodity	Median Absolute Residuals										Pearson Product-Moment Correlation Coefficients[a]									
	U.S.		Northeast		No. Central		South		West		U.S.		Northeast		No. Central		South		West	
	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]	S1[b]	S2[b]
Ground Beef	.86	.78	.64	.64	.84	.99	.74	.76	.85	1.15	.76	.71	.78	.72	.77	.68	.77	.76	.82	.72
Steaks	.61	.43	.75	.52	.48	.48	.40	.47	.70	.53	.82	.79	.80	.77	.83	.80	.77	.84	.88	.74
Roasts	.68	.54	1.02	.82	.66	.50	.56	.47	.68	.43	.79	.79	.83	.74	.79	.84	.78	.76	.77	.79
Poultry	.85	.60	.98	.77	.76	.60	.81	.78	.81	.64	.70	.71	.65	.65	.69	.69	.70	.72	.71	.75
Pork	1.78	1.35	1.28	1.28	1.66	1.23	1.56	1.55	1.54	1.27	.74	.32	.73	.80	.78	.63	.76	.71	.70	.39
Other Meats	1.75	1.79	1.97	2.04	1.67	1.67	1.38	1.47	1.63	1.50	.57	.53	.64	.60	.57	.56	.39	.48	.64	.43
Seafood	.73	.76	1.10	.93	.69	.56	.75	1.13	.76	.84	.36	.42	.40	.43	.29	.38	.54	.47	.33	.35
Other Foods	13.47	12.97	15.48	13.75	12.04	12.34	12.81	13.07	15.42	14.22	.85	.88	.83	.90	.85	.88	.83	.88	.87	.83
Food Away From Home	4.56	4.45	5.42	4.69	4.12	4.21	4.34	4.44	5.29	4.79	.74	.71	.67	.67	.78	.70	.71	.77	.78	.68
Fuels for Home Heating	27.01	27.56	28.41	27.37	26.21	26.02	26.03	31.06	23.29	23.89	.85	.85	.90	.88	.91	.83	.77	.89	.76	.78
Gasoline	15.79	15.83	16.70	15.97	15.01	15.13	15.20	17.42	13.79	14.74	.61	.63	.52	.55	.58	.66	.70	.63	.69	.71

[a]All statistically significant at the .10 level.

[b]S1 and S2 refer to Sample 1 and Sample 2, respectively.

Table 4. Direct Price and Income Elasticities of Meat and Seafood Commodities from Related Research.

Researcher	Data	Model	Direct Price Elasticities	Income Elasticities	
George and	TS[a] (1962-1966)	Complete	Beef	- .64	.29
		System	Veal	-1.72	.59
			Pork	- .41	.13
		Sequential	Lamb and Mutton	-2.63	.57
		Estimation	Chicken	- .78	.18
			Turkey	-1.56	.77
			Fish	- .23	c
Huang	TS[a]	Complete	Beef	- .68	.57
		System	Veal	-5.46	1.27
	U.S.	with	Pork	- .72	.60
		Linear	Lamb	- .43	.59
		Constraints	Chicken	- .72	.43
			Turkey	- .35	.65
			Fresh Fish	- .67	.54
			Processed Fish	- .47	.49
Christensen,	TS[a] (1947-1971)	Direct	Fish	-2.48 to -3.12	2.40
Manser		Translog	Beef	-1.24 to -1.78	2.13
	U.S.	System	Poultry	-1.01 to -1.06	.98
			Pork	-1.03 to -1.59	.91
Christensen,	TS[a] (1947-1971)	Indirect	Fish	- .10 to - .61	.28
Manser		Translog	Beef	- .96 to -1.31	1.14
	U.S.	System	Poultry	- .71 to - .98	.16
			Pork	- .38 to - .76	.05
Blackorby,	TS[a] (1946-1968)	Generalized	Fish[b]	- .64	.81 to .87
Boyce and		S-Branch	Beef[b]	- .27	1.04 to 1.05
Russell	U.S.	System	Poultry[b]	- .63	1.01
			Pork[b]	- .69	1.13 to 1.20
Brandow	TS[a] (1955-1957)	Complete	Beef	- .95	.47
		System	Veal	-1.60	.58
	U.S.		Pork	- .75	.32
		Sequential	Lamb and Mutton	-2.35	.65
		Estimation	Chicken	-1.16	.37
			Turkey	-1.40	.49
			Fish	- .65	.42
Capps and	(1972-1974)	S_1-Branch	Ground Beef	-1.31 to -1.81	.89 to 2.09
Havlicek		System	Steaks	-1.44 to -1.88	1.14 to 2.03
	U.S.		Roasts	-1.60 to -2.06	1.20 to 2.29
			Poultry	-1.13 to -1.38	.92 to 1.34
	Household		Pork	- .67 to -1.73	.70 to 1.38
	Budget Data		Other Meats	- .97 to -1.64	1.03 to 1.48
			Seafood	-1.37 to -2.80	1.21 to 2.52

[a]Time-series data.

[b]Compensated elasticities.

[c]Less than .01.

The direct price and expenditure elasticities of the S_1-branch system are in disagreement to some extent with the direct price and income elasticities of the George and King (1971), Huang (1978), Brandow (1961), and Christensen and Manser (indirect translog system) (1977) studies.

Concluding Comments

General principles which shed light on systems credibility, validation, strengths, and weaknesses are formulated as criteria for model specification of demand systems. The principles are used to judge the empirical performance of the S_1-branch system. Estimates of the parameters and elasticities in the system are reasonable on *a priori* grounds. This system accounts for a substantial amount of variation in the purchases of the respective commodities and accurately predicts the average budget shares over the range of the samples. However, the computations for the estimation of the S_1-branch system are costly. The direct-price elasticities and expenditure elasticities from the S_1-branch system are consistent with the elasticities from several similar studies and are in disagreement to some extent with the elasticities of other related research.

A logical generalization is to provide a comparison of several models in terms of the empirical criteria on the basis of the same set of data [Parks (1969), Green, Hassan, and Johnson (1978)]. This effort may provide additional information in regard to the issue for model specification of demand systems.

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