OPTIMAL GROUNDWATER MINING IN THE OGALLALA AQUIFER: ESTIMATION OF ECONOMIC LOSSES AND EXCESSIVE DEPLETION DUE TO COMMONALITY

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ABSTRACT

The optimal rates of intertemporal and within-group groundwater mining in the Ogallala Aquifer are estimated for the year 1985 to 2005. The gains realized by the optimal policy are measured and compared with gains from the free market policy and life of the aquifer is estimated.

OPTIMAL GROUNDWATER MINING IN THE OGALLALA AQUIFER: ESTIMATION OF ECONOMIC LOSSES AND EXCESSIVE DEPLETION DUE TO COMMONALITY

The chief aim of this paper is to derive optimal rates of groundwater mining over time in the southern portion of the Ogallala Aquifer, and to measure the gains realized by the optimal policy. The Ogallala is a groundwater aquifer extending from north of the Nebraska and South Dakota border to the southern edge of the Texas Panhandle. The study area consists of Bailey, Castro, Crosby, Floyd, Hale, Hockely, Lamb, Lubbock, Lynn, and Parmer counties in Texas and Curry and Roosevelt counties in New Mexico. This group of counties has been identified as a single watershed of the Brazos River Basin. In this area the water table has been falling because the amount of water that recharges the aquifer is small relative to the withdrawals. Falling water stocks and rising energy prices are threatening the agricultural economy. How to control and conserve the limited groundwater stock is crucial in the region. The fact that groundwater is a common property resource like ocean fisheries complicates management. Without an appropriate agreement or regulation by all users of the resources, market forces lead to the over-exploitation of the resources and this results in welfare loss.

The objective of this paper is to estimate optimal rates of groundwater mining over time with deposits of different grades under alternative levels of energy and crop prices, and to measure economic losses and excessive depletion due to commonality. In the second section a model for optimal groundwater mining allocation over time is presented. It is compared with the allocation in a free market model in the third section. In the fourth section numerical solutions are given and evaluated. Summary and conclusions are presented in the final section.

Optimal Groundwater Mining Model

The exploitation of common property resources with reference to groundwater has been discussed by Milliman. The paper by Burt, Cummings, and McFarland estimated the steady state stock in the Estancia Valley of New Mexico but ignored the commonality problem governing allocations of groundwater. Also, the model does not include the impact of rising energy price on irrigation productions.

In this paper a control model is developed to provide the optimal rates of groundwater mining over time under alternative levels of energy and crop prices and to measure economic losses and excessive depletion due to commonality. We assume that there are n different resource situations which form a common pool, and each resource situation has different pumping lifts and land fertilities. We also assume that the market is atomistic so that each producer is a price-taker.

The problem is to find an optimal path $[U_i, i = 1, 2, \ldots, n]$ such that the time-discounted net present joint-profit of all producers is maximized subject to an equation of motion. The problem can be formulated as follows:

$$\max_{U_i} \int_o^T \sum_{i=1}^{n} NRI_i \, [U_i\,(t),\, D_i(t),\, CP,\, EP,\, t] e^{-rt} dt$$

(A) Subject to

$$(1)\ \dot{D} = \alpha_0 \sum_{i=1}^{n} U_i(t) - \alpha_1,$$

$$(2)\ U_i(t) \geq 0, \text{ and}$$

$$(3)\ D_i(t) \geq 0.$$

$U_i(t)$ is the control variable which represent the amount of water applied in the ith resource situation. $D_i(t)$ is the state variable which describes the depth to water in the ith resource situation. CP and EP are the crop and energy price index, respectively. t is time (o, 1,...,T). NRI_i is the net return for irrigation farming in the ith water situation, and defined as:

$$(4)\ NRI_i \equiv P(cp) \cdot f_i(U_i,t) - C_i(U_i, D_i, EP)$$

where P is the price of crops, f_i is the production function in the ith resource situation and C_i is the pumping and production cost in the ith resource situation. $\dot{D}^1$ represents the time rate of change of the depth to water. α_0 and α_1 are the nonnegative coefficients of the equation (1). Equation (1) is an equation of motion which expresses the physical relation of intertemporal and within-group water uses and stocks. Equations (2) and (3) represent the nonnegativity constraint for the amount of water used and the depth to water, respectively.

The problem (A) can be solved by the Maximum Principle (Intriligator and Kirk). Let $\lambda(t)$ be a co-state variable or a shadow price associated with the equation of motion, (1). Let $\mu_i(t)$ be the shadow price associated with the constraint (2). One of the necessary conditions for maximization is as follows:

$$[p \cdot f_{i}u_{i} - C_{i}u_{i}]e^{-rt} + \alpha_0 \cdot \lambda + \mu_i = 0 \text{ for all } i,$$ [2]

so that: (8) $p \cdot f_{i}u_{i} = C_{i}u_{i} - \alpha_0 \cdot \lambda \cdot e^{rt} - \mu_i \cdot e^{rt}$ for all i.

The left-hand side of the equation (8) is the marginal value product of water (flow). The right-hand side of equation (8) is divided into three parts: $C_{i}u_{i}$ is the marginal cost of water, $- \alpha_0 \cdot \lambda \cdot e^{rt}$ is a user cost or an opportunity cost which implies the value of profits foregone in future periods due to an increase in water utilization, and $- \mu_i \cdot e^{rt}$ is the boundary cost associated with the nonnegativity constraint of u_i. Ignoring the boundary cost, the optimal rate of groundwater use is the rate at which the marginal value product equals the sum of the marginal cost and the user cost (U_s in Figure 1) for all n water situations. The result is identical with that in the literature (Milliman and Cummings).

Social Optimal Versus Free Market Model

The model described in the previous section is the centralized, controlled or social optimal model. It implies the control of the entire stock is concentrated in a single decision maker or alternatively there is no horizontal movement of water between adjacent properties. The profit of all users who share the common aquifer is maximized by recognizing the finite nature of stock being exploited and the intertemporal effects of decisions.

Without regulations by the central authority or the agreement by all users, each user extracts water so as to maximize his own profit. This is the free market or unregulated model. In this case each user exploits resources at the point where the marginal value product equals the marginal exploitation cost (at U_f in Figure 1), and if the entry to industry (pumping groundwater) is free then each user exploits more resources until the point is reached where the marginal value product equals the average exploitation cost.

Application

Groundwater mining model described in the second section is applied to the southern portion of the Ogallala Aquifer for the years 1985 to 2005. The study area is projected to include about eight million acres of total land and four million acres of cropland in 1990 (Short et al.). The major crops currently produced are grain sorghum, cotton, wheat, and corn. This area depends heavily on groundwater, and irrigation farming has been developed intensively and the problem of water table decline is serious.

The area using water from the Ogallala Aquifer is broken into four water situations defined according to depth to water. The land is also divided into two classes according to different management possibilities and yield potentials. The total irrigation land is classified into eight classes by the combination of the water situation and the land class, and each classification is called a "resource situation."

Net returns for irrigation and dryland farming are obtained by using a linear programming model[3] with a parametric technique. They are used to estimate the net return functions. The estimated functions (by least squares fitting of a quadratic) are summarized in Table 1. In addition, they are modified by using the development of crop yield coefficients[4], since they do not include the time trend of net returns due to technical change. The equation of motion (the water relation equation) is derived from the technical relation of groundwater inflow and outflow:

$$(9) \quad D_{t+1}^{j} - D_{t}^{j} = 2.5 * 10^{-6} * \sum_{j=1}^{8} [U_{t}^{j} * A_{t}^{j}] - 0.4278$$

where D_t^j is the depth to water in the jth resource situation in time t, feet, U_t^j is the amount of water applied per acre in the jth resource situation in time t, feet, and A_t^j is the irrigated acreage in the jth resource situation, acre.

Since we do not have the continuous data, the model described in the second section, (A), is transformed to the discrete-time finite-horizon model. In getting the results, five scenarios are employed. Scenarios A, B, and C represent the normal, low, and high price levels of both energy and crops, respectively. Scenario D incorporates the low energy price and the high crop price, and finally Scenario E incorporates the high energy price and the low crop price.

The free market solution has a shorter life of the aquifer than the social optimal solution (Table 2) since the former has no regulation on water use and utilizes more water for all periods. The aquifer has a shorter life with the high discount rate or the high crop price. Under Scenario E the dryland production is more profitable than irrigation and hence no groundwater is pumped. Table 3 reports the optimal level of the annual per-acre water use by resource situations for the five scenarios. For the social optimal solution the better resource situations such as resource situation one (less than 50 feet of the depth to water in land class 1) and five (less than 50 feet of the depth to water in land class 2) have the higher level of per-acre water use while worse resource situations such as four (more than 200 feet of the depth to water in land class 1) and eight (more than 200 feet of the depth to water in land class 2) have no water use or lower per-acre water use. The better the resource situation the higher the marginal productivity and the lower the marginal cost. The comparison of the social optimal to the free market solution tells us that the latter always has a higher rate of water use for both total and per-acre use than the former solution. The over-utilization and early-depletion of the water resource is obvious under the free market model.

Groundwater belongs to the common property resource and therefore an inefficient water allocation results without regulations by the central authority or agreements by all users of groundwater. Only if all users of water try to maximize their joint net return subject to water relation constraints, will the optimal intertemporal and within-group allocation be realized. The social optimal solution in this paper implies this joint benefit maximization problem, while the free market solution represents the outcome from each user's own benefit maximization problem. The gains realized from the social optimal policy and the free market policy, which include both the net farm income and the conservation of groundwater resources, are presented in Table 4. With the free market policy the region will make losses of a 50 million to 1.11 billion dollars in net farm income and a 3.8 to 19.5 foot decline of the depth to water under Scenarios A, C, and D. There is a loss in the conservation of the depth to water under Scenario B. No changes occur under Scenario E since all lands are used as drylands. The size of losses depends on the level of energy and crop prices and the level of the discount rate. The free market policy, however, provides clear losses to the agricultural economy of the region.

Summary and Conclusions

Making use of the model developed in this paper the optimal rates of intertemporal and within-group groundwater mining in a portion of the Ogallala Aquifer are estimated for the years 1985 to 2005, and losses due to commonality are measured.

It has been shown that: (1) the better the resource situation the higher the optimal rate of water use, (2) the higher the discount rate the greater the optimal rate of water use, (3) the higher the energy price the longer the economic life of the groundwater aquifer and the smaller the farm income, and (4) the higher the crop price the shorter the economic life of the aquifer and the greater the farm income. The free market policy evidently furnishes losses to the agricultural economy of the region in both net income and groundwater conservation. At normal levels of the energy and the crop price, commonality provides a 60 million dollars of the net farm income and a 10.4 foot of the groundwater depletion.

Footnotes

1/A dot above the variable indicates the time rate of change of that variable, e.g., $\dot{D} \equiv dx/dt$.

2/Partial derivatives of function of several variables are expressed by the function with a subscript, e.g., $f_u \equiv \partial f/\partial_u$ and $f_{uu} \equiv \partial^2 f/\partial u^2$.

3/Short et al. have completed the economic study on the Ogallala Aquifer using a regional, recursive, linear programming model. The model allows us to estimate the net return functions.

4/The total yield functions for irrigation and dryland cropping are estimated, respectively: 0.9663073 + 0.0067385t and 0.9723593 + 0.0055286t where t is years after 1985.

Table 1. Estimated Net Return Functions for Irrigation and Dryland Farming[a]

	Irrigation: NRI		Dry: NRD	
Intercept	-372.0722		-215.4881	
D	-0.4673	(0.025)[b]		
CP	340.7572	(27.896)	241.2301	(39.067)
EP	-73.4388	(8.324)		
D*EP	-0.0397	(0.014)		
U*CP	35.3584	(8.568)		
U*EP	13.7574	(2.810)		
U*U	-27.3139	(4.399)		
CP*CP	-29.31.00	(6.767)	-29.4853	(13.637)
EP*EP	3.1228	(1.202)	-1.7426	(0.532)
X1	245.6387	(3.452)	97.6060	(4.527)
R^2[c]	0.991		0.968	
MSE[d]	155.185		216.073	

[a]Variable definition: NRI = net return per acre for irrigation farming, dollars; NRD = the net return per acre for dryland farming, dollars; D = the depth to water, feet; U = the amount of water applied per acre in a given year, feet; cp = the crop price index; EP = the energy price index; X1 = the dummy variable for land class one.

[b]Figures in the parentheses are standard errors.

[c]R^2 is the coefficient of determination.

[d]MSE is the mean square error.

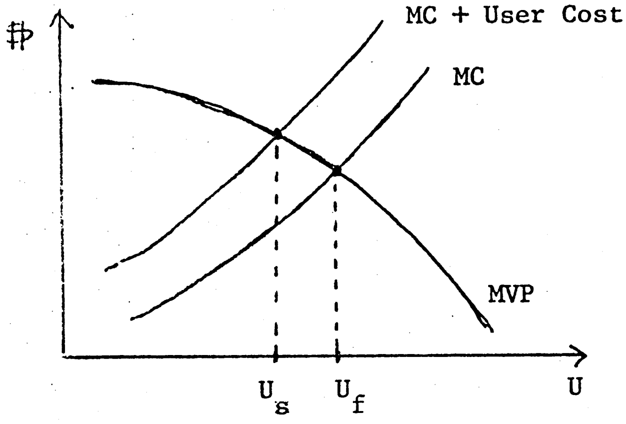

Figure 1. The equilibrium for the social optimal and the free market model

Table 2. Life of the groundwater aquifer for the five scenarios

Scenario	Years Before Depletion	
	Social Optimal Solution	Free Market Solution
	5% Discount Rate	
A	24	20
B	99	82
C	16	14
D	10	7
E	0	0
	15% Discount Rate	
A	21	20
B	91	82
C	15	14
D	8	7
E	0	0

Table 3. Annual per-acre water use for the five scenarios with 5% discount rate[a]

		Annual Per-acre Water Use (feet)															
		Social Optimal Solution								Free Market Solution							
Scenario	Resource Situation	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
		1985								1985							
A		1.26	1.19	1.03	-	-	-	-	-	1.32	1.32	1.32	-	-	-	-	-
B		0.54	0.48	-	-	-	-	-	-	0.60	0.60	0.60	-	-	-	-	-
C		1.82	1.75	1.56	-	-	-	-	-	1.88	1.88	1.88	-	-	-	-	-
D		1.32	1.26	1.12	0.76	1.34	1.32	-	-	1.38	1.38	1.38	1.38	1.38	1.38	-	-
E		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		2005								2005							
A		1.39	1.34	1.19	-	-	-	-	-	1.44	1.44	1.44	-	-	-	-	-
B		0.61	0.56	-	-	-	-	-	-	0.66	0.66	0.66	-	-	-	-	-
		2000								1998							
C		1.95	1.89	1.73	-	-	-	-	-	1.99	1.99	1.99	-	-	-	-	-
		1994								1991							
D		1.40	1.35	1.20	0.85	1.42	1.40	-	-	1.43	1.43	1.43	1.43	1.43	1.43	-	-

[a]For simplicity, levels of annual per-acre water use are listed only for the initial year (1985) and terminal year (1991, 1994, 1998, 2000, and 2005).

Table 4. A Comparison of the Social Optimal Solution to the Free Market Solution

Scenario	Social Optimal (S) Total Net Farm Income	Social Optimal (S) Total Net Present Value of Farm Income	Free Market (F) Total Net Farm Income	Free Market (F) Total Net Present Value of Farm Income	Economic Losses (S-F) Total Net Farm Income	Economic Losses (S-F) Total Net Present Value of Farm Income	Excessive Depletion of Water Stock Due to Commonality[a]
	(billions of year 1985 dollars)						(feet)
	5% Discount Rate						
A	11.24	7.10	11.18	7.08	0.06	0.02	10.4
B	2.75	1.71	2.75	1.71	-	-	1.8
C	16.02	10.21	15.88	10.15	0.14	0.06	7.9[b]
D	18.61	12.09	17.50	11.48	1.11	0.61	19.5[c]
E	2.08	1.28	2.08	1,28	-	-	-
	15% Discount Rate						
A	11.23	3.74	11.18	3.73	0.05	0.01	3.8
B	2.75	0.88	2.75	0.88	-	-	0.7
C	15.95	5.44	15.88	5.43	0.07	0.01	2.9[b]
D	17.92	6.58	17.50	6.47	0.42	0.11	7.0[c]
E	2.08	0.65	2.08	0.65	-	-	-

[a]The total decline of the water table for the free market solution minus that for the social solution.

[b]The difference is calculated for 14 years (1985-1998).

[c]The difference is calculated for 7 years (1985-1991).

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