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ESTIMATION OF A LONG RUN AVERAGE COST CURVE USING A FRONTIER FUNCTION: AN APPLICATION TO THE TENNESSEE LIVESTOCK AUCTION MARKET INDUSTRY

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Considerable research has been conducted to explore economies of size in the livestock auction market industry. Since cost functions for auction markets are expected to conform to microeconomic theory, conclusions are often drawn by estimating a long run average total cost (LRATC) function for the industry [French, Stoddard]. This function suggests a least cost firm size as well as the structure of size economies. The least squares method and the economic-engineering model have dominated economies of size research in the past [French, Bressler, Polishuk]. The use of a frontier function to estimate an LRATC function has been suggested as an alternative approach to these methods [Bressler, Miller and Nauhein, Seitz], although its application in economies of size studies has not been widespread [Bressler, French, Farrell and Fieldhouse, Lesser and Greene].

The purpose of this paper is to evaluate the limitations of the ordinary least squares (OLS) method in contrast to the frontier function method of estimating a long run average cost function. Both methods will be applied in an evaluation of economies of size in the Tennessee livestock auction market industry.

Methodology

The OLS method is commonly used in cost studies examining economies of size due to familiarity and low cost. This approach uses cross-section data in regression of total unit cost on some output measure to estimate an LRATC function. Recent studies utilizing OLS have confirmed that cost economies do exist in auction markets [Wootan and McNeely, Grinnell and Shuffett, Grimes and Cramer, and Wilson and Kuehn].

A frontier function may be estimated using several methods, including those developed by Farrell and Fieldhouse, Aigner and Chu, and Timmer. To obtain an envelope curve fitted to the bottom of the point scatter of unit cost plotted against volume, this study incorporated the minimum absolute deviations (MAD) method using linear programming techniques [Hazell]. Minimization of the absolute deviation of observed average total cost from estimated average total cost, subject to the constraint that estimated average total cost be less than or equal to observed average total cost, results in an envelope function fitted to the bottom of the scatter diagram.

Although the frontier function has not received widespread use by researchers in estimating industry cost functions, this approach may be a more appealing estimator of the LRATC function than the OLS method. The OLS method produces a function which indicates the mean unit cost incurred on the average by markets of a certain size. Useful information is provided concerning the "maximum likelihood" average cost for a range of market size. However, since the observations on cost actually represent firms operating at various points along numerous <u>short run</u> average cost curves, the OLS method actually produces a function showing expected short run average cost at a series of volumes. This function must be above the envelope LRATC assuming small errors of measurement in the observed data. A frontier function fitted to the lower extremes of the scatter of cost-volume observations would more closely approach the theoretical notion of an LRATC curve which is envelope to the short run average cost curves.

Although the frontier function more accurately resembles the theoretical LRATC curve, measurement error may have a greater influence on the results of this method than the OLS method. The slope of the frontier function is constrained by the location of a few points at the lower extreme of the scatter of average costs for any given volume. Thus, errors in measurement in these few observations may have a disproportionately large impact on the estimated function.

Using the MAD method, a long run average cost function of the form

$Y_i = b_o + b_1 X_i + E_i$

where: Y_i = Actual average total cost for market i

X_i = Volume (output) handled by market i

E_i = Random error term

may be estimated by constraining $b_o + b_1 X_i \leq Y_i$, that is $E_i \geq 0$. Only efficient firms will satisfy the equality, since the remainder of firms produce at a cost above the frontier function.

Because an infinite set of b_i will satisfy the above condition, an optimizing criterion must be imposed. Minimizing the linear sum of the errors (E_i) forces the estimated cost function to lie as near the center of the scatter as possible, subject to $b_o + b_1 X_i \leq Y_i$.

Expressing the term $\sum_{i=1}^{n} E_i$ as a linear function of $b_o + b_1 X_i$, the programming problem is:

Minimize: $\sum_{i=1}^{n} E_i = \sum_{i=1}^{n} [Y_i - (b_o + b_1 X_i)]$

Subject to: $b_o + b_1 X_1 \leq Y_1$

$$\vdots \qquad \vdots \qquad \vdots$$

$b_o + b_1 X_n \leq Y_n$

Because Y_i is constant for any specific observation, minimizing $\sum_{i=1}^{n} E_i$ maximizes $b_o + b_1X_i$. Thus Y_i may be eliminated and the programming problems becomes:[1]

$$\text{Maximize:} \quad b_o\sum_{i=1}^{n}(1_i) + b_1\sum_{i=1}^{n}(-1_i) + b_2\sum_{i=1}^{n}(X_i) + b_3\sum_{i=1}^{n}(-X_i)$$

$$\text{Subject to:} \quad b_o(1) + b_1(-1) + b_2(X_1) + b_3(-X_1) \leq Y_1$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$b_o(1) + b_1(-1) + b_2(X_n) + b_3(-X_n) \leq Y_n$$

$$(b_o, b_1, b_2, b_3 \geq 0)$$

Application to the Tennessee Livestock Auction Market Industry

An LRATC function for the Tennessee livestock auction market industry was estimated using both OLS and frontier function methods. Accounting records were obtained from the Packers and Stockyards Administration Form 130 for 1978 and 1980. One-hundred and one observations were available (Table 1). Market output was measured according to volume of livestock handled gauged in "Animal Marketing Units" (AMU). An AMU has been defined by USDA as one cow, one calf, three hogs, four sheep or goats, or one horse or mule [Stoddard].

Data from both 1978 and 1980 were combined to estimate the LRATC function. Costs for 1980 were deflated to 1978 dollars by the Index of Prices Paid by Farmers.

Economic theory suggests a cost function which would decrease at a decreasing rate over the small firm sizes and may or may not turn up for large sizes. Four functional forms were hypothesized as potential appropriate forms for the Tennessee auction market LRATC:

Table 1. Number and Average Volume of Livestock Auction Markets by State Group, Tennessee, 1978 and 1980

Volume handled per year (AMU)	1978 Number of markets	1978 Average volume handled (AMU)	1980 Number of markets	1980 Average volume handled (AMU)
Less than 9,000	7	5,814	14	5,840
9,000 - 17,999	15	13,810	14	13,115
18,000 - 26,999	13	21,648	5	20,892
27,000 - 35,999	4	31,244	6	32,276
36,000 - 44,999	5	39,455	2	40,711
45,000 - 53,999	7	51,453	3	46,046
54,000 or more	4	75,412	2	69,702

(1) $Y = b_o + b_1X + b_2X^2$.

(2) $Y = b_o + b_1 \frac{1}{X}$

(3) $\text{Ln } Y = b_o + b_1 \text{ Ln } X$

(4) $Y = b_o + b_1 \frac{1}{X} + b_2 \frac{1}{X^2}$

where: Y = Total cost per animal marketing unit (AMU)

X = Number of AMU's handled per year.

Each of these models was used in an OLS regression of average total cost against AMU's. Model (4) was considered to be the best OLS estimate of the LRATC function based upon R^2.

A frontier function was estimated using the same function form. The linear programming problem was:

Maximize: $$b_o n(1) + b_1 n(-1) + b_2 \sum_{i=1}^{n} \frac{1}{X_i} + b_3 \sum_{i=1}^{n} \frac{-1}{X_i} + b_4 \sum_{i=1}^{n} \frac{1}{X_i^2} + b_5 \sum_{i=1}^{n} \frac{-1}{X_i^2}$$

Subject to: $$b_o (1) + b_1 (-1) + b_2 \frac{1}{X_1} + b_3 \frac{-1}{X_1} + b_4 \frac{1}{X_1^2} + b_5 \frac{-1}{X_1^2} \le Y_1$$

$$\vdots$$

$$b_o (1) + b_1 (-1) + b_2 \frac{1}{X_n} + b_3 \frac{-1}{X_n} + b_4 \frac{1}{X_n^2} + b_5 \frac{-1}{X_n^2} \le Y_n$$

Results

Estimates of the Tennessee auction market industry LRATC function are given in Table 2 and Figure 1. Results from both the OLS and frontier function estimates indicate that some economies of size existed in the industry. The OLS estimate lies above the frontier function, with substantial economies realized at volumes up to 30,000 AMU's. The frontier function estimate showed the largest cost reductions occurring below 15,000 AMU's.

Table 2. Long Run Average Total Cost Functions for Tennessee Livestock Auction Markets, 1978 and 1980

Model	Parameter Estimates			
	b_o	b_1	b_2	R^2
Ordinary Least Squares Estimates[a]				
(1) $Y = b_o + b_1X + b_2X^2$	6.998	-0.0001203 (0.0000347)	0.000000001105 (0.000000000453)	.164
(2) $Y = b_o + b_1 \frac{1}{X}$	4.215	10859.756 (2084.052)	---	.215
(3) $\ln Y = b_o + b_1 \ln X$	3.426	-0.19062 (0.03919)	---	.193
(4) $Y = b_o + b_1 \frac{1}{X} + b_2 \frac{1}{X^2}$	3.419	26964.497 (5547.660)	-34433902.026 (11065375.041)	.286
MAD Estimate of Frontier Function				
(4) $Y = b_o + b_1 \frac{1}{X} + b_2 \frac{1}{X^2}$	2.05	7929.66	-5749146.8	

[a]Numbers in parentheses are standard errors of estimate.

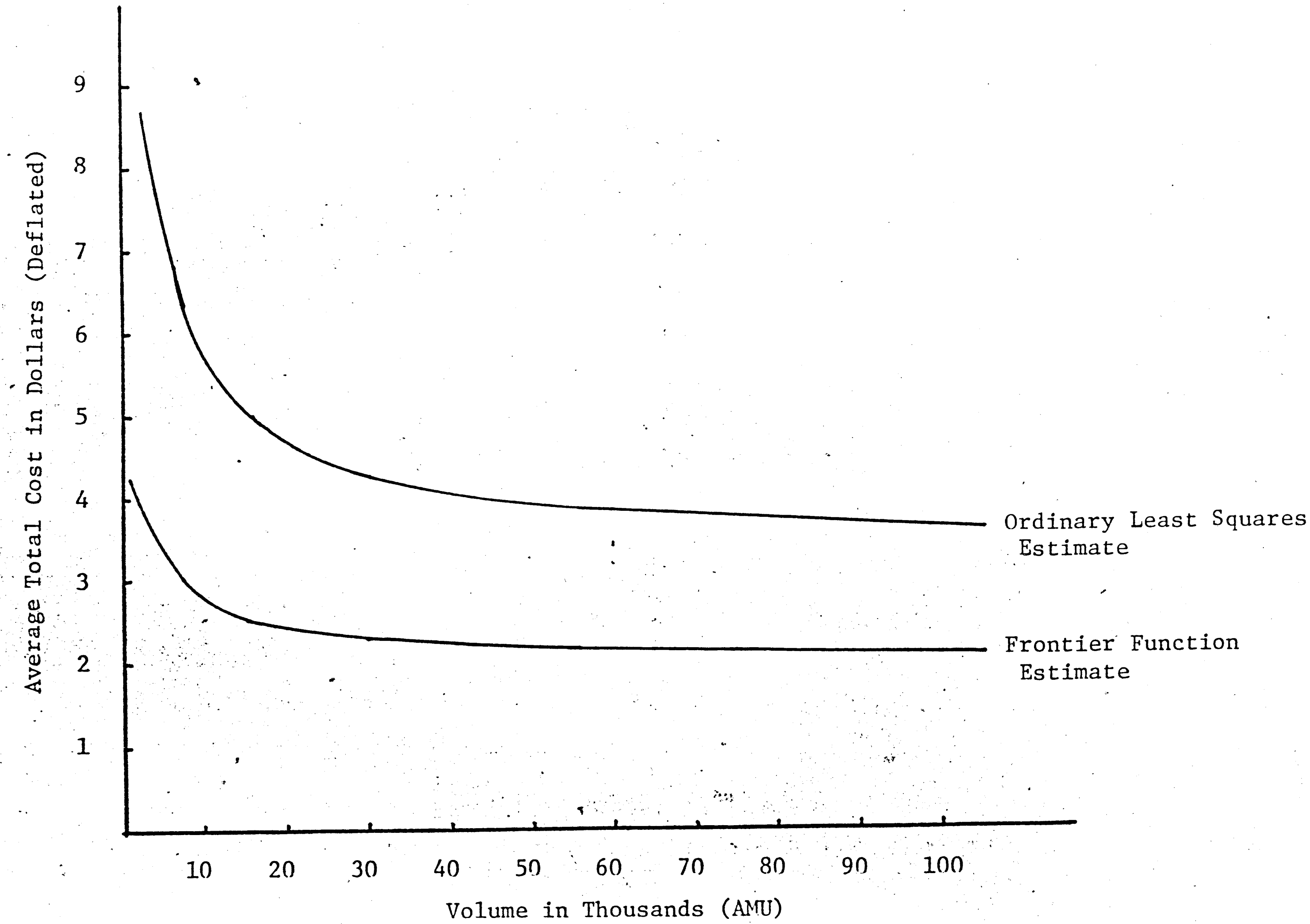

Figure 1. Long Run Average Total Cost Functions for the Tennessee Livestock Auction Market Industry Using Ordinary Least Squares and Frontier Function Methods (1978 and 1980 Data Combined).

The OLS function indicates that economies of size exist, but it may over-estimate the range over which these economies occur in the long run. The OLS function indicates that the average market handling a volume below 30,000 AMU's would experience the greatest decline in unit total costs by increasing volume (Table 3). At this level of output, auctions achieve 80 percent of the economies of size which may be realized along the OLS function.[2] The elasticity of cost with respect to volume declines from -0.37 at a volume of 7,500 AMU's to -.19 at 30,000 AMU's.[3] That is, the function declines substantially at volume levels up to 30,000 AMU's, but is relatively flat past 30,000 AMU's.

The frontier function estimate indicates that if auction markets increase volume, those markets with output less than 15,000 AMU's will realize the greatest economies. At this volume, 80 percent of the cost economies which are possible along the frontier function have been achieved. The elasticity of cost with respect to volume of the function at 15,000 AMU's is -0.19, compared to -0.30 along the OLS estimate at the same volume. The frontier function estimate indicates that economies of size are realized at a lower volume than indicated by the OLS estimate.

Conclusions

The appropriateness of the OLS or the frontier function as an estimator of LRATC depends on the goals of the researcher. The OLS approach estimates the expected short run average cost conditions and yields statistical estimates of significance. The frontier function approach is more appealing theoretically because it is analogous to the envelope concept, although measurement error may result in misestimation of the LRATC.

Table 3. Comparison of Long Run Average Total Cost Functions Derived From Minimum Absolute Deviation Estimation of a Frontier Function and From Ordinary Least Squares

	OLS			Frontier Function		
Volume (AMU)	Average total cost ($)	Cost function elasticity[a]	% cost economies realized[b] (%)	Average total cost ($)	Cost function elasticity[a]	% cost economies realized[b] (%)
5,000	7.44	-.36	46	3.41	-.33	60
7,500	6.40	-.37	53	3.01	-.28	68
10,000	5.77	-.35	59	2.78	-.24	74
15,000	5.06	-.30	68	2.55	-.19	80
25,000	4.44	-.22	77	2.36	-.13	87
30,000	4.28	-.19	80	2.30	-.11	89
40,000	4.07	-.16	84	2.25	-.09	91
50,000	3.95	-.13	87	2.19	-.07	93
70,000	3.80	-.10	90	2.16	-.05	95

[a]Elasticity of average total cost with respect to volume was determined according to:

$$\text{Elasticity} = \frac{dATC}{dVol} \cdot \frac{Vol}{ATC} .$$

[b]Percent cost economies realized was defined as the difference between estimated average total cost (ATC) at the minimum observed volume and estimated ATC at the volume under consideration, divided by the difference between estimated ATC at the minimum observed volume and estimated ATC at the asymptotic minimum of the function

$$\left[\frac{ATC_{min\ vol} - ATC_{vol\ i}}{ATC_{min\ vol} - ATC_{asympt\ min}} \right] .$$

The OLS estimate indicates that auctions in Tennessee may experience substantial cost economies by increasing volume up to 30,000 AMU's. Along the frontier function estimate, cost economies are important only up to an output level of 15,000 AMU's. Thus, the OLS estimate indicates that economies are possible over a wider range of volume in the long run than with the frontier function. Fifty percent of the livestock auctions in Tennessee operated at volumes less than 18,000 AMU's. These data indicate that a large portion of auction market firms operated at volumes which leave substantial cost economies uncaptured assuming the OLS function to be the appropriate estimate of LRATC. This would suggest that the frontier function estimate more accurately reflects observed industry behavior and thus, is the more appropriate estimator of LRATC.

Footnotes

[1]Positive and negative values of the intercept and X_i terms are included to allow the coefficients b_o and b_1 to be negative.

[2]Percent cost economies realized was defined as the difference between estimated average total cost (ATC) at the minimum observed volume and estimated ATC at the volume under consideration, divided by the difference between estimated ATC at the minimum observed volume and estimated ATC at the asymptotic minimum of the function

$$\left[\frac{ATC_{min\ vol} - ATC_{vol\ i}}{ATC_{min\ vol} - ATC_{asympt\ min}}\right].$$

[3]Elasticity of average total cost with respect to volume was determined according to: Elasticity $= \frac{dATC}{dVol} \cdot \frac{Vol}{ATC}$.

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