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EXTERNAL DISECONOMIES AND UNCERTAINTY

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Introduction

Since Sandmo's path-breaking article incorporating uncertainty and utility maximization into the theory of the competitive firm, considerable effort has been expended by many authors in an attempt to expand standard micro-theory to include uncertainty. One of the most important results of this work is that the pattern of optimal resource allocation under stochastic utility maximization differs from the pattern derived from the principles of profit maximization in the absence of random disturbances. Another important, and perhaps not totally unexpected, result is that unambiguous results are not always forthcoming when aleatory variables are included in the models. However, little work has been done to incorporate uncertainty into the theory of externalities. The objective of this paper is to meld the concept of utility maximization under stochastic production with that of technological external diseconomies. The paper is organized as follows. In the next section the basic principles involved in technological externalities are discussed and a simple two-farm model is presented. The third section will present a model similar to the one in the previous section but the externality producing farm will be assumed to maximize the expected utility of profits rather than just profits, as in the first model. Following a comparison of the results obtained from the two models, will be a concluding section.

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A Simple Model of Agricultural Externalities

In a two-firm, two-good economy, a technological externality is said to exist when one firm produces, as a by-product of the production of its saleable output, another good which in some way affects the production process of the second firm but for which there exists no market price. The absence of a market price for the externality means that the producing firm does not consider the level of the externality which it produces. By the same token the firm receiving the externality is unable to affect the externality output level, at least not through the market mechanism. If the externality enhances the production process of the affected firm, then it is called an external economy. If it is detrimental, then it is an external diseconomy. An example of an external diseconomy, the case that will be considered in this paper, would be a pesticide which is aerially applied but which reduces the output of the neighboring farm either because it is phytotoxic (damages the crops directly) or because it kills beneficial insects. In a situation such as this, the first farmer (Farmer A) will spray his crop at a rate which maximizes his profits. He will not take into account any damage imposed by the wind-borne pesticide upon his neighbor's crop. His neighbor (Farmer B), on the other hand, must accept whatever level of pesticide, and the concomitant damage it causes, that happens to come his way.

Assuming that each farmer is a profit maximizer operating in a certain world, the two production functions can be written:[1/]

(1a) $q_A = q_A(x_1, x_2)$

(1b) $q_B = q_B(x_3, e)$

(1c) $e = e(x_1)$

where q_A and q_B are the outputs of the two farms; x_1, x_2 and x_3 are the inputs; and e is the level of externality produced, in this case the quantity of farm A's pesticide reaching farm B. If the production functions are well behaved, the marginal products of x_1, x_2 and x_3 are positive but decreasing. By the definition of an external diseconomy, $\frac{\partial q_B}{\partial e} < 0$. Assume also that $e' > 0$. As long as the externality e has no price, the profits of each farm are:

$$\text{(2a)} \quad \Pi_A = P_A q_A - r_1 x_1 - r_2 x_2$$

$$\text{(2b)} \quad \Pi_B = P_B q_B - r_3 x_3$$

where P_A and P_B are output prices and r_1, r_2 and r_3 are input prices. Individual farm profits are maximized when the first order conditions are satisfied:

$$\text{(3a)} \quad P_A \frac{\partial q_A}{\partial x_1} = r_1$$

$$\text{(3b)} \quad P_A \frac{\partial q_A}{\partial x_2} = r_2$$

$$\text{(3c)} \quad P_B \frac{\partial q_B}{\partial x_3} = r_3$$

There is no corresponding first order condition for the externality e in B's production function because e is not a choice variable for him.

A standard result of welfare economics is that in the absence of externalities, Pareto optimality is achieved when the joint profits of all firms are maximized. However, in the presence of an external diseconomy, the producer of the externality must be taxed and the recipient subsidized at a rate equal to the marginal cost that the externality imposes on the other farm if a Pareto optimal allocation of resources is to result.[2/] In our example, joint profits are:

$$(4)\quad \Pi = \Pi_A + \Pi_B = (P_A q_A - r_1 x_2 - r_2 x_2) + (P_B q_B - r_3 x_3)$$

Joint profits are maximized when input levels are chosen to satisfy the first order conditions:

$$(5a)\quad \frac{\partial \Pi}{\partial x_1} = P_A \frac{\partial q_A}{\partial x_1} - r_1 - P_B \frac{\partial q_B}{\partial e} e' = 0$$

$$(5b)\quad \frac{\partial \Pi}{\partial x_2} = P_A \frac{\partial q_A}{\partial x_2} - r_2 = 0$$

$$(5c)\quad \frac{\partial \Pi}{\partial x_3} = P_A \frac{\partial q_B}{\partial x_3} - r_3 = 0$$

Rearranging and recalling that $e' > 0$ and $\frac{\partial q_B}{\partial e} < 0$.

$$(6a)\quad P_A \frac{\partial q_A}{\partial x_1} = r_1 + P_B \frac{\partial q_B}{\partial e} e'$$

$$(6b)\quad P_A \frac{\partial q_A}{\partial x_2} = r_2$$

$$(6c)\quad P_B \frac{\partial q_B}{\partial x_3} = r_3.$$

Comparing equations (3a) and (6a) it is evident that joint profits will not be maximized unless farm A faces the full "social" cost of utilizing x_1. That is to say the market marginal factor cost of x_1 must be increased by $P_B \frac{\partial q_B}{\partial e} e'$, the marginal cost imposed upon farm B by farmer A's use of x_1. Furthermore, it is evident that if the tax is imposed from an initial position of profit maximization, farmer A will reduce, but not necessarily eliminate, his use of x_1. Output of the externality will fall as will the output of q_A.[3/] The lower level of e combined with the proceeds from the tax on x_1 will bring forth an increase in q_B, farm B's output. In this case, where administrative costs are ignored, an equal tax and subsidy will bring the marginal rates of technical substitution for each farm into equality with the ratio of factor costs, in a social sense.

Externalities With Uncertainty in Production

As in the previous section, assume that farmer A sprays pesticides which adversely affect farm B. However, now we will consider the case where the output of farm A is subject to random disturbances and the objective function of the farmer is to maximize the expected utility of profits. Profits for each farm are as follows:

Farm A:

(7a) $\Pi_A = P_A q_A - r_1 x_1 - r_2 x_2$, and

(7b) $q_A = F(x_1, x_2\ \Sigma) = f(x_1, x_2) + h(x_1, x_2)\ \Sigma.$

where Σ is a random variable, $E(\Sigma) = 0$ and $\mathrm{var}(\Sigma) = \sigma^2$

Farm B:

(8a) $\Pi_B = P_B q_B - r_3 x_3$, and

(8b) $q_B = Z(x_3\ e)$ as before.

Equation (7b) and the following analysis draw extensively from the model developed in Pope and Kramer. The nonstochastic component of F, $f(x_1, x_2)$, exhibits positive and diminishing marginal products for each input. The second component, $h(x_1, x_2)\Sigma$, is positive so that high "draws" of the random variable Σ increase output and low "draws" decrease output. The partials $h_1 = \partial h/\partial x_1$ and $h_2 = \partial h/\partial x_2$ will be positive if the input is marginally risk increasing and negative if it is marginally risk decreasing. In the present example, where x_1 is a pesticide, $h_1 < 0$. As the focus of this paper is on the externality producing input, we need not sign h_2 at this point.

In the absence of a price on the externality produced by farm A, each farm will set input levels to maximize their respective objective functions. The first order conditions for farm A are:[4/]

$$(9a) \quad E[u'(P_A F_1 - r_1)] = P_A f_1 + P_A h_1 t - r_1 = 0$$

$$(9b) \quad E[u'(P_A F_2 - r_2)] = P_A f_2 + P_A h_2 t - r_2 = 0$$

where $t = \dfrac{\mathrm{cov}(u'\Sigma)}{E(u')}$.

Rearranging:

$$(9c) \quad \frac{r_1}{r_2} = \frac{(f_1 + h_1 t)}{(f_2 + h_2 t)} .$$

The first order conditions for farm B are the same as equation (3c), repeated here for convenience:

$$(3c) \quad P_B Z_3 - r_3 = 0.$$

A difficulty in the analysis arises at this point. By assuming that farmer A maximizes the expected utility of profits, we no longer have recourse to the simple Pareto optimality condition that we used in the previous section to determine the optimal tax on the externality. In fact, no measure of Pareto optimality in this sort of situation has been rigorously developed. Instead, we shall assume that the marginal cost imposed by the externality is the same in the present case as in the previous case where both farms were profit maximizers. From equation (6a) the tax rate would be $P_B \dfrac{\partial q_B}{\partial e} e' = P_B Z_e e'$. We then include this into the profit equation for farm A.

(10a) $\Pi_A = P_A q_A - r_1 x_1 - r_2 x_2 - (P_B Z_e\, e')x_1$

(10b) $\Pi_A = P_A q_A - (r_1 + T)x_1 - r_2 x_2$

where $T = P_B Z_e\, e'$. The subscripts on Π, P, and q will be dropped from hereon whenever it is obvious that we are considering only farm A.

The first order conditions resulting from the adjusted profit equation (10b) are:[5/]

(11a) $Pf_1 + Ph_1 t - (r_1 + T) = 0$

(11b) $Pf_2 + Ph_2 t - r_2 = 0.$

Rearranging:

(12) $$\frac{r_1 + T}{r_2} = \frac{(f_1 + h_1 t)}{(f_2 + h_2 t)}\ .$$

This specification makes clear that the essential feature of an external diseconomy is that the private cost of the externality producing input, r_1, is less than its social cost, $r_1 + T$.

What does equation (12) tell us about the manner in which the imposition of a tax on an externality producing input changes the optimal input and output levels of farm A? Consider first the effects on factor demands. It is rather obvious that this is a pivotal issue in the sense that unless the factor demand curve for the externality producing input is downward-sloping, a tax will not effect a reduction in its use. Unfortunately, it appears that it is not always possible, at least with the present state of the "arts", to determine a priori whether or not the demand for x_1 will be downward-sloping. Pope and Kramer (p. 496) have shown that the only cases in which the slope of factor demands can be determined in a model such as we are employing are when: (a) both inputs are marginally risk reducing and are substitutes in production, (b) both inputs are marginally risk increasing and are complementary, and (c) the input of interest is marginally risk increasing, the other is risk decreasing, and the inputs are substitutes. Why would this be so? Recall that the farm's production function can be decomposed into two parts; one nonstochastic and the other involving a random distrubance term. The nonstochastic component has previously been assumed to be concave. Thus, in the neighborhood of the optimum output level, the factor demand curves for this component will be downward-sloping.[6/] For lack of a better term, call this the "price" effect. The second component, $h(x_1, x_2)\Sigma$, introduces, speaking very loosely, an "uncertainty" effect into the production function by way of its interaction with the utility function. This is the ultimate source of the ambiguity on the sign of $\partial x_1/\partial r_1$.

Rewriting Pope and Kramer's equation (28), using the present notation:

$$(13)\quad \frac{\partial x_1}{\partial r_1} = \frac{1}{|H|}\{E[\delta^2 u''\,\Pi_1^2 + u'PF_{22}]E(u') + Px_1E(u''\,\Pi_1)\,[E(u'F_{22}) - \delta E(u'F_{12})]\},$$

where $\delta = h_1/h_2$, $\Pi_1 = \delta\Pi_2 = \delta(\partial\Pi/\partial X_2)$.

$|H|$ is positive by the satisfaction of sufficiency conditions. The first square-bracketed term is negative by our assumption of the concavity of F. $E(u''\,\Pi_1)$ and $E(u'F_{22})$ are both negative by the assumptions of risk aversion, concavity, and $h_1 < 0$. The ability to a priori sign $\partial x_1/\partial r_1$ thus hinges on the determination that $\partial E(u'F_{12}) < 0$. This will be the case only when: (a) $\delta > 0$ and either $F_{12} < 0$ when both inputs marginally decrease risk or $F_{12} > 0$ when both marginally increase risk, and (b) $\delta < 0$ when the inputs are substitutes and x_1 is marginally risk increasing. In terms of economics both these cases say that unambiguous results are attained only when the "uncertainty" effect reinforces the "price" effect.

Consider the case in which the input x_1 and x_2 are substitues. In this situation an increase in the price of x_1 (a pesticide in our example) will bring about a decrease in its use due to the "price" effect. Because the inputs are substitutes, use of x_2 will increase as a result of the lower utilization of x_1. If x_2 is a marginally risk increasing input, there will be an "uncertainty" effect calling forth an increase in x_1, the marginally risk reducing input. If this "uncertainty" effect is strong enough, that is, if the farmer is strongly risk averse and $|h_1|$ is large enough, the "uncertainty" effect could overwhelm the "price" effect and one would observe $\partial x_1/\partial r_1 > 0$. On the other hand, if x_2 is marginally risk decreasing, then there will be no "uncertainty" effect on x_1 to offset the "price" effect.

The foregoing discussion suggests that the degree of risk aversion will affect not only the intensity of the use of x_1, and thus the level of production of e, but also the strength of the "uncertainty" effect. Pope and Kramer (p. 495) have shown that one can unambiguously determine how an increase in absolute risk aversion will affect factor demands: (a) when the inputs are complementary and both affect risk in the same direction, and (b) when the inputs have opposite effects on risk and are substitutes. In our example, x_1 is assumed to be risk decreasing. If x_2 is a risk increasing substitute for x_1, then the utilization of x_1, and consequently the production of the externality 3, would be higher for farms with greater risk aversion (risk aversion as measured by $R = -u''/u'$). Higher production of the externality will call forth a higher tax $T = P_B Z_e e'$ only if the marginal cost imposted by e upon farm B's production is an increasing function of e. If both e´ (the marginal product of x_1 in the production of the externality) and Z_e are either constant or declining, then the tax rate per unit output will not increase as risk aversion increases for farm A. Although the tax rate may or may not be higher for higher levels of risk aversion, the optimal production of the externality can be expected to be higher, the greater is the level of risk aversion of farmer A, *ceteris paribus*. This conclusion follows straightforwardly from the observation that for a given tax rate the farmer with greater production of e would have to reduce use of x_1 by a greater amount than would a farmer with lower risk aversion and a correspondingly lower usage of x_1 in order that the same level of production of the externality is attained. However, for the farmer with the higher level of risk aversion, the "uncertainty" effect on x_1 would be higher and thus would require a higher tax to induce him to lower use of x_1 such that his externality production is the same as the farm with the lower level of risk aversion.

Conclusion

The preceding analysis suggests that uncertainty does affect the standard policy prescriptions for attaining Pareto optimality when technological diseconomies are present. The major conclusion is that when the party producing the externality acts to maximize the expected utility of profits rather than acting to maximize profits, it is no longer possible to say that a tax on the externality producing input will necessarily induce a reduction in the level of production of the externality. In certain cases this will indeed transpire, but if the factor which causes the externality is a risk reducing input, then a tax may in fact bring about increased production of the externality. This implies that the standard solution, namely taxing the use of x_1 with the proceeds of the tax going to the affected party, not only will it not achieve Pareto optimality but may not even be stable.

Footnotes

1/ The crucial assumption here is that each farmer is a profit maximizer. The addition of stochastic elements would not change any of the results. Each farmer would base his decisions on expected value rather than the "known" value.

2/ A number of authors (Lin and Whitcomb, Negishi) have argued that only taxes are required to achieve a Pareto optimal allocation. However, see Buchanan and Stubblebine for arguments that both taxes and subsidies are necessary.

3/ That q_A will be lower follows necessarily from the particular form of production function used here. See Whitcomb for generalized joint production functions for which saleable output does not necessarily fall with decreased utilization of the externality producing input.

4/ As in Pope and Kramer, we assume that farmer A is decreasingly risk averse. Thus, $u' > 0$, $u'' < 0$ and $R'(\Pi) = d(u-u''/u')/d\Pi < 0$.

5/ The tax, T, is set by a pollution authority at the precise level that induces both farms to produce at Pareto optimal levels. This *deus ex machina* was borrowed from Lin and Whitcomb. Second order conditions are satisfied when the principal minors of the Hessian, H, alternate in sign. To facilitate the presentation of H, note that at optimum input levels equations (11) can be solved out for t_1 implying:

$$\text{(a)}\quad Pf_2 - r_2 = \frac{h_2}{h_1}\,[Pf_1 - (r_1 + T)].$$

Setting $\delta = h_2/h_1$, equation (a) implies:

$$\text{(b)} \quad \frac{\partial \Pi}{\partial x_2} \equiv \Pi_2 = \delta \Pi_1 \equiv \delta \frac{\partial \Pi}{\partial x_1}$$

To derive H simply differentiate (11), using (b) to simplify:

$$\text{(c)} \quad H = \begin{bmatrix} E[u''\Pi_1 + u'PF_{11}] & E[u''\delta\Pi_1 + u'PF_{12}] \\ E[u''\delta\Pi_1^2 + u'PF_{12}] & E[u''\delta^2\Pi_1^2 + u'PF_{22}] \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Essentially, we have to assume that the second order conditions are indeed satisfied. That H_{11} and H_{22} are both less than zero follows from the twin assumptions of risk aversion $[E(u'' \Pi_1^2) < 0]$ and concavity of the production function $[E(u'PF_{11}) < 0]$. However, the magnitude of H_{12}^2 cannot be determined a priori. We shall simply assume $H_{11}H_{22} > H_{12}$.

6/ This statement is not strictly correct. As Silberberg points out (p. 112) the fact that factor demand curves are downward-sloping follows only for a profit maximizing firm and only in the vicinity of the optimum level of inputs. However, it is deemed not too unreasonable an assumption to maintain.

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