ABSTRACT

The Possibilities for Possibility Theory

A. Halter and J. Brandstatter

Possibility theory is discussed as an extension of probability
theory arising from the need to express subjective informa-
tion quantitatively in linguistic terms, but without the re-
strictions of probability numbers. The concept of fuzzy sets
is introduced as the basis of possibility theory and the
broad scope of applications is mentioned.

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THE POSSIBILITIES FOR POSSIBILITY THEORY

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The modeling of complex systems must necessarily deal with uncertainty especially if the model is to be used for decision-making purposes. The uncertainties associated with complex systems are reflected in at least three aspects--parameter, system, and human based uncertainty. The first, parameter uncertainty, is due to inexact knowledge of the model parameters assuming a precisely defined system; the second, system uncertainty, is due to the inadequacy of the theoretical model of the proposed system (structure) assuming one has precisely defined parameters; and the last, human based uncertainty is intertwined with the previous ones.

Although parameter uncertainty may often be assessed by use of "objective" statistics (when they are available) to give probability distributions (in which the parameters are random variables), system uncertainty cannot be normally assessed by "objective" statistics and must be assessed "subjectively." This is so because system uncertainty is due to the lack of dependability of a theoretical model when used to describe the behavior of a proposed system. Human subjective models are necessarily involved in the process of developing analytical models; and from these human perceptions of a system one attempts to formulate so-called "objective" models.

But subjective models are quite different from objective ones because intuition, experience and wisdom provide a broader perspective than the sequential analytic processes of thought. The human mind is capable of understanding many subtleties and complexities which are difficult to communicate verbally and analytically to another observer. In the last 15 years there has been a growing recognition that conventional quantitative techniques are

intrinsically unsuited to characterize complex systems [3]. Systems of optimization and operations research provided some advantage but have not had the impact originally expected of them. Many of the techniques are adaptations of methods used for dealing with linear causal mechanistic systems--physical systems such as mechanics. The success of these methods led to the naive hope that they or similar techniques could be applied to human centered systems or to systems which approached them in complexity. This problem was common to many subject areas such as economics, medicine, management science, psychology, sociology where classical quantitative techniques have thus far failed to have significant impact.

Because of this inability to cope with system complexity involving socio/economic/technological ingredients, new approaches are emerging, and most of them are based on "approximate reasoning." The cornerstone of this philosophy is captured in the "principle of incompatibility" which asserts 'that as the complexity of a system increases, our ability to make precise yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and relevance become almost mutually exclusive characteristics.'

The engrained tradition of scientific thinking which equates the understanding of phenomenon with the ability to analyze it quantitatively needs to be re-examined and recognized as having the ability to cope with only about 10 percent of real world problems. The remaining 90 percent requires methodology that can capture qualitative and subjective information, process it and manipulate it in ways that are different from the known quantitative techniques. We need to recognize that humans with their 100 billion cell brain/bio-computer are able to summarize masses of information and extract important items which are relevant to a particular problem because they think approxi-

mately. We think in terms of classes or sets of objects where the transition from membership to nonmembership is not abrupt but gradual. Human reasoning is not based on a two valued logic or even a multivalued logic but rather on a fuzzy logic. This means that human descriptions of complex systems are more comfortably expressed in "fuzzy" terms.

In almost all disciplines it has been recognized that decision making is not an algorithm. Information available for decision making can generally be classified into objective and subjective parts. The objective involves countable information about the external world which experts normally have available to them, while the subjective concerns, wisdom, understanding, knowledge, experience, and intuition are normally not taken into formal consideration. Nevertheless, both kinds of information are essential in decision making.

Although the importance of subjective information in the decision-making process is acknowledged, the lack of a systematic method of incorporating this information into the objective information systems means that much of this wisdom and experience goes unused. Over the years there have been some attempts to capture and measure subjective information, for example, the field of worth assessment contains a variety of techniques of varying degrees of complexity, but they all fall short of a way of incorporating the results into an objective information system.

In 1965, Zadeh introduced the concepts of fuzzy set theory which was intended to be a structure designed to handle the feature of linguistic imprecision [10]. Over the years it became clear that decision making under uncertainty was not just the inclusion of probability for representing uncertainty but that one needed to distinguish uncertainty due to randomness and uncertainty due to vagueness or imprecision.

The central feature of fuzzy set theory is the membership function which represents numerically the degree to which an element belongs to a set. Instead of using only unity (certainty) and zero (impossiblity) when dealing with objective information, the degree of membership can now take on values between one and zero in order to more fully describe subjective concepts. The degree of membership is a measure of the relative potential for occurrence of various events and outcomes, hence, the new name of possibility theory [2].

One of the first to suggest the use of the concept of possibility rather than probability in subjective estimation was the economist, Schackle [8]. He discussed the use of a degree of 'potential surprise' according to a measure of possibility and even presented axioms for its definition. Corresponding to perfect possibility there is a zero degree of surprise, and corresponding to impossibility, an absolute maximum degree of surprise. The greatest surprise is caused by the occurrence of a seemingly impossible event, and a very slight degree of surprise is associated with an event which we know could very well happen. Potential surprise and actual surprise may be quite different as they are assessed at different times, they do not coexist. Schackle wanted to get away from the restrictions of probability theory caused by the necessity for values to sum to unity. He asserted that zero potential surprise could be assigned to an unlimited number of rival hypotheses all at once; in other words, any number of distinct happenings arising out of a set of circumstances could all be regarded as perfectly possible. Perfect 'possibility,' however, is not perfect 'certainty.' In this context the degree of belief is given an interpretation quite different from that of probability theory. A simple example is given by a person using a telephone who could have a zero degree of surprise, both for getting the right number and the wrong number. Thus an event A and its negation, not A, can both be assigned zero surprise. In

another context, not A might often cover a multitude of possibilities, for example, if A is "it will rain tomorrow," then not A will be true if it is sunny, foggy or if it hails. If we give a probability of 1/2 to rain, then the other 1/2 is left to share amongst the other events which it may be felt deserve a greater consideration in the assessment. In effect, a hypothesis may rate a low probability because it is crowded out by other hypotheses and not because anything in its own nature disqualifies it from attention.

Since a great deal of expert opinion or judgment is mental and can only be expressed verbally, possibility theory exploits this wisdom--it permits the organized manipulation of vague concepts and "linguistic variables".

These are variables which take on linguistic rather than numerical values. For example, "willingness to pay," or "consumer appeal" may be considered linguistic variables with the possible values low, medium, and high [4]. Any selection of values for linguistic variables is vague or fuzzy since such concepts as "willingness" or "appeal" do not have clearly defined measures. To cope with this vagueness one assigns (subjectively) degrees or grades of membership, or possibility ratings, to each possible value. Even for numerical variables such as 'hog demand,' a possibility distribution may be used to express uncertainty about their values.

Mathematically, a possibility measure is distinguishable from a probability distribution in that the former is based on ordinal information, the latter on cardinal information. That is, to obtain a possibility function all we need is an ordering on the elements of a set with respect to the ability of a variable (e.g., a linguistic variable) to assume a value. Probability theory requires intensity as well as ordering. That is, we must know how many times more possible it is for the variable to assume one value than another. This means we can use possibility theory with much less information than is

required to use probability theory. In many cases, all that the available information may permit is possibilistic statements. Hence, we make a tradeoff between precision and an ability to make some statement. However, possibility theory is not a weaker form of probability theory. There are many situations in which one can speak in terms of possibilities but no concept of probability exists.

Scope of Applications

The recognition that uncertainty due to imprecision, vagueness or fuzziness is another dimension of our perceived reality and that uncertainty due to randomness does not include all the possibilities has led to the opening of new frontiers in essentially all disciplines [1]. The Carnap-Popper controversy in philosophy over inherent imprecision takes on new meaning across logic, language, linguistics, and mathematics where the developments in fuzzy set theory and its foundations have been fast and exploding.

Specific applications of the concept of fuzziness are already too numerous to mention in this short paper and the notion of "linguistic variables" allows the imagination to range from automata through decision making and medical sciences, to system theory itself [6]. There are two general categories of application. In the first category there are applications which call for the development of new techniques and new methods which can be based on the fuzzy set theory. In the second category, classical techniques, as well as, fuzzy techniques can both be applied. In the latter case the fuzzy techniques are usually simpler, intuitively more appealing and actually complement the classical techniques. Application of this category include linear programming, resource allocation, PERT, and other scheduling techniques from the field of operation research.

In the first category are those applications which are made interesting and useful by the broadened perspective given to them by the notion of fuzziness and the feasibility of computing with linguistic variables. One area is decision making where it seems that most human decisions (90 percent) are made in a fuzzy environment [1]. Most farm and business decision problems involve many imprecisely defined states and vaguely specified multiple payoffs which make these types of problems too complex to handle in a deterministic or even in a probabilistic manner. Fuzzy set theory has given a new outlook to utility theory or worth assessment that should make decision-making applications in business and government more productive and stimulating to researchers [6].

Other areas in which the new outlook for worth assessment has been given renewed emphasis is in policy analysis, risk assessment, cost-benefit analysis, and environmental assessment. These areas all involve the assessment of worth or utility in subjective terms and the feasibiilty of computing with linguistic variables make them fertile fields for imaginative applications [7].

Applications of fuzzy sets in the field of engineering already abound and much productive work has been done in failure/safety analyses and the evaluation of reliability in man-machine systems [9]. Our ability to capture the imprecision inherent in such concepts of safety, risk, and reliability has been given greater scope by the developments in fuzzy set theory. Such a broadened perspective should give renewed insights into how to incorporate expert opinion and professional judgment into engineering calculations and thereby utilize information heretofore ignored in the design and building of structures, machines, and the development of new technology.

Finally, the diciplines of social sciences, medical, and biological sciences have perhaps the greatest potential for application of the new

perspective and its methodology and techniques [5]. These areas of study for which human communications and interactions are basic should be given renewed power in handling imprecision, vagueness, and fuzziness. Whether the recognition of inherent imprecision in human centered systems gives a broader foundation to these fields remains to be seen.

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INTUITIVE DECISION MAKING

CHOICE OF HOME COMPUTER: APPLE 2

                                      IBM PC

PERFORMANCE VARIABLES:

      COST

      RELIABILITY

      QUALITY OF GRAPHICS

                (MEMORY

      OTHER   (EASE OF USE

                (SOFTWARE

UTILITY OF PERFORMANCE VARIABLES:

   E.G. (MEDIUM, MEDIUM HIGH, VERY HIGH)

   AGGREGATE UTILITY: MAYBE HIGH

DECISION BASE: COMPARISON OF AGGREGATE

                UTILITY OF EACH ALTERNATIVE

UNDERLYING PHILOSOPHY OF INTUITIVE
DECISION MAKING

PARTIAL UTILITIES (I.E., THE UTILITY OF EACH
INDIVIDUAL PERFORMANCE VARIABLE) CANNOT BE
AGGREGATED BY GENERAL MATHEMATICAL FUNCTIONS
BECAUSE AGGREGATION METHODS ARE UNIQUE TO EACH
INDIVIDUAL AND TOO COMPLEX TO BE SATISFACTORILY
DESCRIBED BY AN ARBITRARY COMBINATION RULE.

3

UNIVERSE OF DISCOURSE

$$U = \{ \text{LOW, MEDIUM LOW, MEDIUM, MEDIUM HIGH, HIGH} \}$$

EACH PERFORMANCE (OR CRITERIA) CAN ASSUME VALUES WITHIN THE UNIVERSE OF DISCOURSE

FOR THIS EXAMPLE THE UNIVERSE OF DISCOURSES ARE:

$$C = \{ MH, H \}$$

$$R = \{ MH, H, VH \}$$

$$G = \{ H, VH \}$$

TO GENERATE THE PERFORMANCE SPACE OR DECISION SPACE, WE CONSTRUCT ALL POSSIBLE COMBINATIONS OF THE UNVIERSE OF DISCOURSE OF THE PERFORMANCE VARIABLE

PERFORMANCE SPACE, BY DEFINITION, IS THE CARTESIAN PRODUCT: CXRXG

COST	RELIABILITY	GRAPHICS	AGGREGATE UTILITIES
MH	MH	H	FMH
MH	MH	VH	M
MH	H	H	FMH
MH	H	VH	H
MH	VH	H	M
MH	VH	VH	M
H	MH	H	MH
H	MH	VH	VMH
H	H	H	FH
H	H	VH	VH
H	VH	H	VVH
H	VH	VH	VVVH

EVERY COMBINATION REPRESENTS AN ALTERNATIVE IN THE DECISION SPACE

TRADE OFFS BETWEEN ALTERNATIVES

DECISION MAKER FINDS AFTER SEARCH THAT THE ALTERNATIVES SATISFY
HIS IDEAL TO THE FOLLOWING SUBJECTIVE ESTIMATES OF THE PERFORMANCE
VARIABLES:

APPLE 2:   C = (1.0│H),   R = (.4│MH,.6│H),   G = (.3│H,.7│VH)

IBMPC: C = (1.0│MH),   R = (.5│H),   G - (.2│H,.8│VH)

TO COMPARE THE WORTH OF THE ALTERNATIVE COMPUTERS WE EXPRESS THE
ALTERNATIVES AS FUZZY SETS.

TO GENERATE THE FUZZY SETS OF THE TWO ALTERNATIVES, TAKE THE
CARTESIAN PRODUCT ACROSS C, R, & G: CXRXG USING THE MINIMUM
RULE FOR THE MEMBERSHIPS:

$$\text{APPLE 2} = \left\{ 0.3 \middle| (H, MH, H), 0.4 \middle| (H, MH, VH), 0.3 \middle| (H, H, H), 0.6 \middle| (H, H, VH) \right\}$$

$$\text{IBMPC} = \left\{ 0.2 \middle| (MH, H, H), 0.5 \middle| (MH, H. VH) \right\}$$

GOING BACK TO THE TABLE OF COMBINATIONS WE SEE THAT THE VALUE OF
APPLE 2 IS WITHIN THE SPACE OF ALTERNATIVES GENERATED,
I.E. $\left\{ H, MH, H \right\}$ = MH

BY REPLACING THE COMBINATIONS WITH THE TABLE VALUES WE HAVE IN TERMS OF AGGREGATE UTILITY (OR WORTH IN TERMS OF OUR OWN VOCABULARY OR UNIVERSE OF DISCOURSE

$$APPLE\ 2\ =\ \left\{\ .3\Big|\ \text{MH},\ .4\Big|\ \text{VMH},\ .3\Big|\ \text{FH},\ .6\Big|\ \text{VH}\ \right\}$$

$$IBMPC\ =\ \left\{\ .2\Big|\ \text{FMH},\ .5\Big|\ \text{H}\ \right\}$$

GOING BACK FROM THE FUZZY SET REPRESENTATION TO A LINGUISTIC INTERPRETATION WE COULD READ THIS AS:

APPLE 2 = MEDIUM HIGH OR HIGH
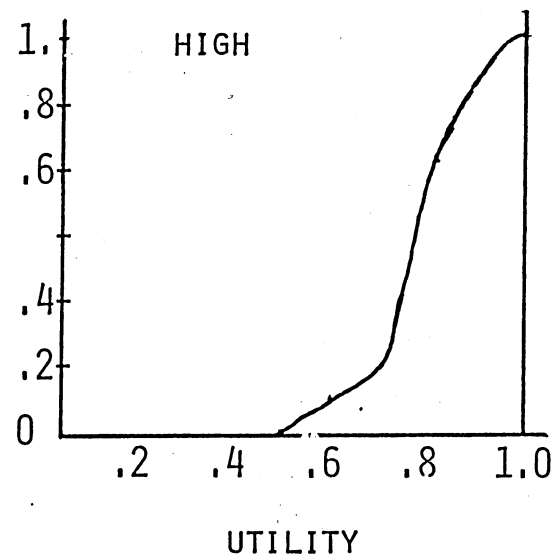
IBMPC = MEDIUM HIGH

CHOICE: APPLE 2

A MORE FORMALISTIC WAY OF COMPARING THE WORTH IS TO USE THE
FUZZY SET REPRESENTATION AND CARRY OUT THE APPROPRIATE FUZZY
SET CALCULATION:

TO COMPUTE WE NEED THE FUZZY SET REPRESENTATIONS OF TWO OF
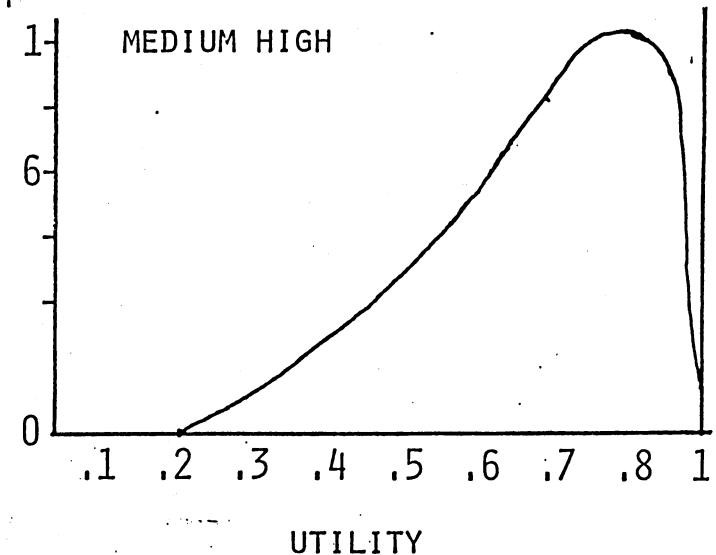THE LINGUISTIC VALUE.

HIGH $= (\ 0.1|0.6\ ,0.2|0.7\ ,0.6|0.8\ ,0.9|0.9\ ,1|1\ )$

MEDIUM HIGH $= (\ 0.2|0,\ 0.6|0.6,\ 0.8|0.7,\ 1|0.8,\ 0.9|0.9,\ 0.2|1\ )$
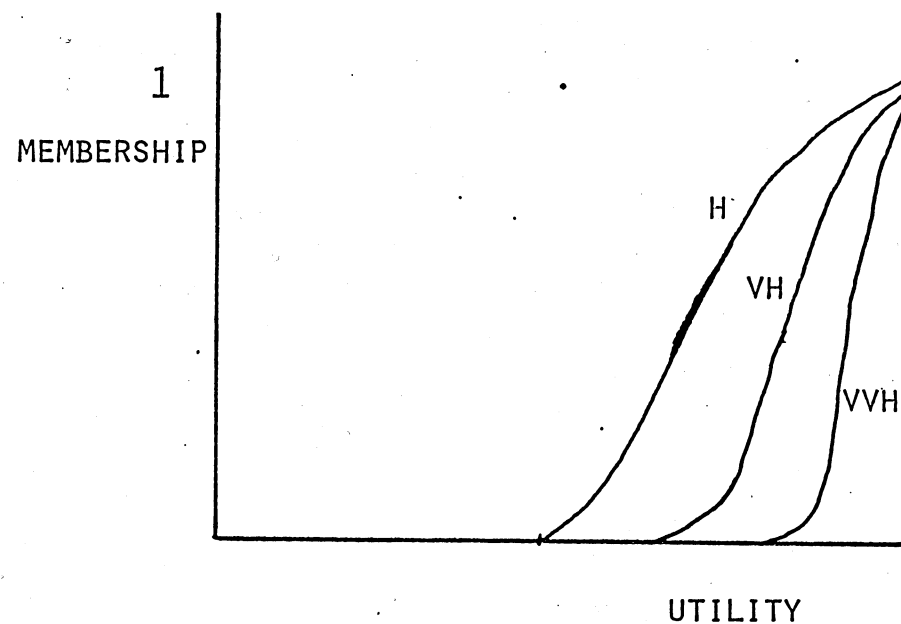
MEMBERSHIP

UTILITY

MEMBERSHIP

UTILITY

ALL OTHER LINGUISTIC VALUES ARE OBTAINED FROM THESE BY

HEDGING RULES:

$$VH = H^2 \quad \text{(SQUARE THE MEMBERSHIPS)}$$

$$FH = \sqrt{VH} \quad \text{(MORE OR LESS HIGH, SQUARE}$$
$$\text{ROOT OF MEMBERSHIPS)}$$

UTILITY

THE FINAL RESULT USING THE RULES OF THE FUZZY SET CALCULUS GIVES:

APPLE 2 = (0.2|0.5, 0.36|0.6, 0.4|0.7, 0.4|0.8, 0.6|0.9, 0.6|1.0)

IBMPC = (0.2|0.5, 0.2|0.6, 0.2|0.7, 0.5|0.8, 0.5|0.9, 0.5|1.0)

MEMBERSHIP