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1982

Portfolio Analysis Under Risk and Imperfect Markets

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Presented at AAEA meetings, Logan, Utah, Aug. 1-4, 1982.

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ABSTRACT

Rural banks face an imperfect and uncertain demand for non-farm real estate agricultural loans. Maximization of a bank's expected utility for a negative binomial is solved by quartic programming. Empirical results show diversification between competitive and imperfectly competitive assets. Uncertainty about expected return parameters is an important risk component.

Key words: portfolio analysis, imperfectly elastic asset, rural banks.

PORTFOLIO ANALYSIS UNDER RISK AND IMPERFECT MARKETS

This paper extends the mean-variance portfolio model in order to account explicitly for the effects of assets traded in imperfectly competitive markets on an expected utility maximizing portfolio. We illustrate this phenomenom in terms of a banking firm, review some relevant literature, and then explore the analytical properties of the imperfect-risk model with a simple three-asset case. Risk is measured as the sum of variation from an asset's mean plus the variation due to uncertainty about the true mean. The conceptual model is operationalized with a numerically specified, non-linear program that demonstrates the derivation of a risk-efficient set and its response to changes in parameters reflecting risk and market characteristics. The programming results show that the effects of these model specifications are not trivial and warrant further consideration in more comprehensive banking models.

Agricultural Banking, Risk, and Market Imperfection

Micro modeling of financial intermediaries has become a useful means for evaluating the availability and cost of financial capital to the farm sector in light of an intermediary's unique regulatory and financial environment. Modeling of agricultural banks, in particular, has provided a rich setting for evaluating their possible responses to changes in costs of funds, competitive pressures, interest rate controls, structural regulations, etc., and the implications for their involvement in farm lending (Robison and Barry; Boehlje et al). The modeling approach requires as complete a specification as possible of the bank's decision criteria, choices for structuring assets and liabilities, limits on resource availability, other regulatory effects, and the influences on pricing and allocative decisions of risk and competitive position in the local banking market.

Modeling the effects of risk and competitive phenomena are especially challenging. Banking risks arise from the combined effects of variations in rates of return (current returns and capital gains) on loans and securities, variations in costs of the banks' sources of funds, and liquidity risks associated with unanticipated changes in deposits and other sources of funds. Banking competition is distinguished by the multi-product characteristic of bank portfolios in which the bank's loan market is predominantly an imperfectly competitive market, while many of the securities the bank trades in are characterized by perfect elasticity (Mason). Moreover, the degree of competition in rural banking markets, organized in unit banking systems, is considered less than in their urban counterparts. Under these conditions banks are expected to segment their loan customers according to differences in risk and competition, and derive pricing policies and loan allocations in accord with these customer characteristics. Thus, bank portfolios will indicate varying degrees of diversity between securities and loans, and among types of loans, reflecting the combined effects of the differences in risk and competition.

Related Studies

Combining the effects of risk and market imperfections in bank models is a demanding task (Baltensperger). Mean-variance portfolio theory provides one modeling approach, but it was originated by Markowitz under the assumption that all assets are traded in perfectly competitive markets.

Studies by Klein (1970) and James offer important insight about the theoretical effects of including assets traded in imperfect markets in EV efficient sets. Klein's approach derived an equilibrium ratio of loans to total assets for an expected utility maximizing banker whose utility function is modeled as a quadratic. The optimal loan-to-asset ratio explicitly accounted for the effects of lending risks, differences in loan demand, and differences in the elasticity of demand for bank loans, under the assumption of a linear demand function. An important result was the loss in applicability of Tobin's separation theorem; the optimal combination of risky assets, relative to holding a risk-free asset, is no longer independent of the decision maker's utility function. If one of the risky assets (loans) has less than perfect elasticity, then the expected returns on loans cannot be determined without first knowing the amount of risky assets relative to the risk-free one, and this requires knowledge about the bank utility function (Klein, p. 494).

James extends Klein's analysis by explicitly showing the relationship between risk and return in a portfolio model with an imperfect risky asset. His results show that the introduction of market imperfections (specified as a monopoly position), subject to a downward sloping demand curve, does not affect the upward slope of an EV efficient set; however, the EV set changes from a linear to a concave function. Moreover, the difference between the expected return on the imperfectly competitive asset and a risk-free rate can be clearly expressed as the standard risk premium from the capital asset pricing model plus a monopoly premium determined by the demand elasticity. An interaction between the risk and monopoly premiums means that expanded holdings of the imperfect asset bring greater risk.

Theoretical Framework

We illustrate the effects of an imperfect asset on an optimal portfolio for a risk averse banker under the assumption that the returns are normally distributed and the bankers' utility function is approximated by the negative exponential $U(\Pi) = 1-e^{-2\rho\Pi}$ where Π represents the rate of return and ρ is the degree of risk aversion. Maximizing the expected value of a negative exponential integrated over a normal density function is equivalent to maximizing

$$(1) \quad E[U(\Pi)] = E(\Pi) - \rho\,\sigma_{\Pi}^{2}$$

where $E(\Pi)$ and σ_{Π}^{2} represent a portfolio's expected returns and variance respectively (Freund).

Consider that the bank may allocate a fixed amount (A) of funds among three assets. Asset X_1 is a risk-free asset with return $\bar{r}$. Asset X_2 is a risky asset traded in a competitive market with a return of $r_2^* = r_2 + e_2$ where r_2 is the mean of r_2^* and e_2 is a random variable with mean zero and variance σ_2^2. Asset X_3 is a risky asset traded in an imperfect market with pricing based on a linear loan demand relationship with a return of r_3^* such that $r_3^* = a + bX_3 + e_3$. Let a and b be unknown population constants and e_3 be a random variable with mean zero and variance σ_3^2. The traditional mean-variance aproach when r_2, a, and b are unknown is to estimate these values and use estimates of σ_2^2 and σ_3^2 as the measure of variance. This procedure underestimates portfolio risk because the error in estimating the unknown parameters is ignored, Fried (1970); Klein and Bawa. This risk component is called estimation risk. The risk generated by the variability of e_2 and e_3 is called market risk.

In estimating r_2, a and b either a classical or Bayesian approach may be adopted. The classical approach is less satisfactory for maximizing expected utility because the means and variances are needed to obtain an optimal solution. The classical approach only estimates these parameters. A Bayesian approach maximizes expected utility given the posterior distribution of the returns, as illustrated by Klein and Bawa. However, if linear regression is used in a classical approach and the sample is large for both r_2^* and r_3^*, the expected utility solution will approach the Bayesian solution if the error terms are normal and diffuse priors are assumed on the parameters.

For the solutions to converge the sample variances of the estimated parameters in the linear regression method must be combined with the estimates of market risk. For example, assume r_2 is estimated by its sample mean for n observations and σ_2^2 is estimated by s_2^2, the customary unbiased estimator of the population variance. The total risk of r_2^* is given as $s_2^2(1 + 1/n)$. The second component is the error of the sample mean as an estimator of the population mean.

For r_3^* the estimation variance is more complex to compute. Using a Bayesian approach where e_3 is normally distributed and only diffuse priors are available for the parameters, r_3^* for a given level of X_3 has a t distribution with mean equal to $\tilde{a} + \tilde{b}X_3$ where $\sim$ denotes the least squares estimate of the parameter. The variance of the predicted r_3^* is approximately $s_3^2(1 + (1\ X_3)(Z'Z)^{-1}(1\ X_3)')$ where Z is the matrix of regressors and s_3^2 is the conventional unbiased estimate of σ_3^3. With a large number of observations the posterior distribution is approximated by the normal distribution.

Using the Bayesian approach and assuming the posterior distributions are normal, the expected utility problem maximizes J:

$$(2)\quad J = r_1X_1 + r_2X_2 + aX_3 + bX_3^2 - \rho[s_2^2X_2^2 + 2s_{23}X_2X_3 + s_3^2X_3^2 + s^2_{\frac{2}{n}}X_2^2 + 2s_{4_2a}X_2X_3 + s_a^2X_3^2 + 2s_{r_2b}X_2X_3^2 + 2s_{ab}X_3^3 + s_b^2X_3^4]$$

subject to

$$X_1 + X_2 + X_3 < A \qquad X_1, X_2, X_3, > 0.$$

where s_i^2, i = 2,3 are the estimated variances of the e_i and $s_{2,3}$ is the estimated covariance of e_2 and e_3. The s with a double subscript denotes the covariance between the two subscripted estimates and s^2 with a single subscript denotes the estimated variance of the subscripted parameter. The parameters r_2, a and b are posterior means.

The variance of the expected return in (3) is the sum of the nine terms in the brackets. The first three terms are the terms conventionally found in EV problems. The last six terms arise due to estimation error. The middle three terms arise because of estimation error for the mean of r_2^* and the intercept for r_3^*. These terms account for estimation error due to exogenous variables that influence the mean return (Fried). The last three terms are attributed to the imperfect asset. Cubic terms reflect any correlation between the estimate of the slope coefficient and the mean of r_2^* or the intercept of the demand equation. The variance of b is multiplied by a quartic term. Thus the imperfectly competitive asset problem with a linear demand and uncertain parameters results in a portfolio model that is solved by quartic programming.

Programming Analysis

The effects of risk and market imperfections are illustrated in a non-linear programming analysis of the three asset case in which solutions are obtained for five levels of risk aversion under alternative numerical specifications of the parameters in equation (3). The empirical setting involves a small agricultural bank with $6 million of funds (A) available for investment in risk-free treasury bills (X_1) having a 5% return, corporate securities (X_2) having an expected return of 5.714% and a variance of 0.3386%, and farm loans (X_3), subject to loan demand specified as

(3) $\bar{r}_3 = 8.024 - .07546X_3$
$$\quad (.230) \quad (.0274)$$

with standard errors in parentheses. The parameters of the loan demand function were estimated from a sample of agricultural banks based on data about their amounts and interest rates on farm loans over the 1972 to 1979 period (Barnard). The constant term shown in (3) is the sum of an intercept term plus six independent variables evaluated at their sample means multiplied by their respective estimated coefficients. The results show a highly elastic demand for farm loans.

Based on these data, the optimal portfolio of bank assets results from the maximization of

$$5.0X_1 + 5.714X_2 + 8.024X_3 - 0.07546X_3^2$$
$$- \rho[.3322X_2^2 + .5151X_3^2 + .006388X_2^2 + .05313X_3^2 - 2(.001909)X_3^3$$
$$+ .0007539X_3^4]$$

subject to

$$X_1 + X_2 + X_3 \leq 6.0 \text{ and } X_1, X_2, X_3 \geq 0$$

Activity levels are shown in Table 1 for five levels of risk aversion. The results show a clear pattern of emphasis on the risk-free asset for higher levels of risk aversion toward increasing specialization in the higher yielding farm loans for smaller ρ. The risk neutral solution ($\rho = 0.0$) shows complete specialization in the farm loan activity, despite its less than perfectly elastic demand curve. The highly elastic loan demand together with other numerical values on returns and fund availability warrant complete specialization.

For each level of ρ in Table 1 the corresponding quadratic programming solution that considers only market risk is given. As expected, the mean return is lower when estimation risk is included. Greater investment in the imperfectly elastic asset occurs in the QP solution than the quartic problem. When $\rho = 0.5$

the true riskiness of the QP solution, 11.5, is underestimated by about 8.7%. This underestimation of the true variation holds for all the values of ρ greater than zero. Moreover, all the QP solutions show a greater investment in the risky assets relative to in the risk-free asset. Estimation error for the competitive risky asset X_2 is a negligible part of its contribution to the portfolio's total variance. This is not true for X_3 where the estimation variance of the constant term in (4) is roughly ten percent the variance of e_3.

Results in Table 2 reflect the effects of a less competitive, more volatile market for farm lending. The slope-coefficient for the loan demand function is multiplied by 10, giving a more inelastic demand, and the variance of the slope coefficient is also increased to make the estimated coefficient twice its standard error. This change in elasticity is maintained throughout the remaining models. The activity levels in Table 2 indicate increased holdings of the risk-free and risky-competitive assets, compared to the base problem, and diversity between the two risky assets in the risk neutral solution. Thus the combined effects of more inelastic loan demand and greater risk reduce the attractiveness of the imperfect asset. For $\rho > 0$, the amounts of X_2 in Tables 1 and 2 are very similar. A rapidly decreasing return on the imperfect asset tends to shift funds into the riskless asset and not the competitive asset. When estimation error is ignored, portfolio variance is underestimated by about 20 percent. Moreover, investment in the imperfect asset exceeds the optimum by at least 20 percent.

Solutions in Table 3 reflect a revision in the estimate of b so that its t ratio equals one, indicating statistical insignificance. Compared to Table 2, solutions to the quartic problem for $\rho > 0$ indicate a shift of investment out of X_3 into the riskless asset. The insignificance of b implies in a typical regression approach X_3 would be deleted from r_3^* so it would be assumed that b = 0 with certainty. Here the mean return on X_3 would be slightly less than 8.024 and investment in X_3 would be greater than indicated in Table 3. Thus the optimal portfolios are very sensitive to the modeling of knowledge about market imperfections.

The results in Table 4 are for the same model as Table 2 except that covariances between the estimates of r_2 and a, and r_2 and b, correspond to a correlation coefficient of .75. This positive covariation decreases the benefit of diversification, thus more of the riskless asset is held and less of the risky assets with the competitive asset showing the greater decline.

Conclusions

The numerical results show that ignoring estimation risk may result in portfolios substantially different from the optimal portfolios. Moreover, inclusion of an imperfectly elastic asset means the optimum portfolio for a risk neutral solution may result in a diversified portfolio instead of only one asset as when all assets are perfectly competitive. Comparison of the solutions to various problems shows that the degree of elasticity has a marked effect on the optimal portfolio composition. Thus including the effects of market imperfection provides a richer, although more complex, analytical framework for evaluating portfolio response.

Table 1. Solutions to Base Problem*

Risk Coefficient (ρ)	X_1	X_2	X_3	Mean Return	Additive	Variance Estimation	Total
0.0	0.0	0.0	6.0	45.4	18.5	2.07	20.6
	0.0	0.0	6.0	45.4	18.5	2.07	20.6
0.5	0.0	1.90	4.10	42.48	9.85	.865	10.7
	0.0	1.69	4.31	42.83	10.5	.960	11.5
1.0	2.58	1.05	2.37	37.5	3.26	.278	3.54
	2.37	1.07	2.56	38.0	3.76	.324	4.08
1.5	3.65	.703	1.64	35.26	1.56	.135	1.69
	3.50	.716	1.78	35.7	1.81	.158	1.97
2.0	4.22	.527	1.26	34.1	.907	.080	.987
	4.10	.537	1.37	34.4	1.06	.094	1.15

*The first row for a given value of the risk coefficient gives the solution to the portfolio problem acknowledging estimation risk. The second row is the solution when estimation risk is assumed to be zero.

Source: Computed.

Table 2. Solutions to Portfolio Problem with Decreased Elasticity for the Imperfectly Elastic Asset*

Risk Coefficient (ρ)	X_1	X_2	X_3	Mean Return	Additive	Variance Estimation	Total
0.0	0.0	4.47	1.53	36.1	7.84	1.02	8.86
	0.0	4.47	1.53	36.1	7.84	1.02	8.86
0.5	2.68	2.11	1.21	34.1	2.24	.409	2.64
	2.36	2.15	1.49	34.3	2.68	.844	3.53
1.0	3.99	1.05	.958	33.0	.842	.172	1.01
	3.73	1.07	1.19	33.3	1.11	.363	1.48
1.5	4.49	.703	.805	32.4	.498	.096	.594
	4.29	.716	.990	32.8	.675	.188	.864
2.0	4.77	.527	.699	32.1	.344	.061	.405
	4.62	.537	.847	32.4	.466	.111	.577

*The entries are organized as in Table 1. The slope of the demand function is -.7546 for the above solutions instead of -.07546 in the base problem. The ratio of the slope coefficient to its standard error is two.

Source: Computed.

Table 3. Solutions to Portfolio Problem with Decreased Elasticity and Greater Uncertainty for the Imperfectly Elastic Asset*

Risk Coefficient (ρ)	X_1	X_2	X_3	Mean Return	Additive	Variance Estimation	Total
0.0	0.0	4.47	1.53	36.1	7.84	3.36	11.2
	0.0	4.47	1.53	36.1	7.84	3.36	11.2
0.5	2.93	2.11	.965	33.7	1.96	.569	2.53
	2.36	2.15	1.49	34.4	2.68	2.97	5.65
1.0	4.18	1.05	.763	32.6	.669	.229	.898
	3.73	1.07	1.19	33.3	1.11	1.22	2.34
1.5	4.65	.703	.651	32.1	.382	.127	.509
	4.29	.716	.990	32.8	.675	.599	1.27
2.0	4.90	.527	.574	31.9	.262	.080	.342
	4.62	.537	.847	32.4	.466	.331	.797

*The entries are organized as in Table 1. The slope of the demand function is -.7546 and its t ratio is one implying the slope is statistically insignificant.

Source: Computed.

Table 4. Solutions to Portfolio Problem with Decreased Elasticity and Positive Covariation Between r_2, a and b*

Risk Coefficient (ρ)	X_1	X_2	X_3	Mean Return	Additive	Variance Estimation	Total
0.0	0.0	4.47	1.53	36.1	7.84	1.68	9.53
	0.0	4.47	1.53	36.1	7.84	1.68	9.53
0.5	2.86	1.97	1.17	33.9	2.00	.549	2.55
	2.36	2.15	1.49	34.4	2.68	1.15	3.83
1.0	4.11	.958	.932	32.8	.752	.219	.971
	3.73	1.07	1.19	33.3	1.11	.467	1.58
1.5	4.58	.630	.786	32.4	.450	.119	.569
	4.29	.716	.990	32.8	.675	.240	.915
2.0	4.85	.468	.684	32.0	.314	.075	.389
	4.62	.537	.847	32.4	.466	.141	.607

*The entries are organized as in Table 1. The slope of the demand function is -.7546 and its t ratio is two. The covariances between r_2 and a and then r_2 and b are such that the corresponding correlation coefficients equal .75.

Source: Computed.

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