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INCORPORATION OF SOIL TEST RESULTS IN THE DEVELOPMENT OF OPTIMAL LONG-TERM FERTILIZER POLICIES FOR FARMERS*

by

Arthur L. Stoecker

Associate Professor

Texas Agricultural Experiment Station

Texas Tech University

Cooperative Research Unit

Lubbock, Texas

and

Arthur B. Onken

Professor

Texas Agricultural Experiment Station

Lubbock, Texas

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Abstract

The economics of using soil test results for making fertilizer recommendations in the presence of nutrient carry-over are discussed. A model for maximizing long run returns with nutrient carry-over is presented. Dynamic programming is used to determine fertilizer applications for grain sorghum given an initial soil test value.

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Introduction

The biological and economic concepts related to the response of a crop to applied fertilizer are of interest to both agronomists and agricultural economists. Accordingly, a large body of literature on this topic is available in both the agronomic and economic journals. The suitability of alternative algebraic forms for capturing the biological response and the calculation for determining the optimal amount of fertilizer is thoroughly discussed by Heady and Dillon (1961). Dillon (1977) has compiled an extensive bibliography of fertilizer response research of special interest to agricultural economists.

The importance of considering residual soil nutrients in making fertilizer recommendations has long been recognized. Cate and Nelson (1971) developed a simple graphical procedure for using soil test information to classify soils into groups with high and low probabilities of response to applied nutrients. Anderson and Nelson (1977) developed an analysis of variance equivalent to the graphical Cate-Nelson method. Onken et al. (1979) have used the Anderson-Nelson approach to evaluate the effectiveness of alternative tests for soil phosphorus in predicting yield response. Mombiela et al. (1981) have appraised the effectiveness of various algebraic forms of the response function for the purpose of making fertilizer recommendations.

The research efforts to utilize soil test information, crop response functions, and economic information to make fertilizer recommendations have for the most part operated under a static framework. While it is accepted that soil test information is important in making recommendations, the effect of the fertilizer application on future soil nutrient levels is rarely considered.

The control theory-dynamic programming methodology for determining optimal fertilizer policies when carry-over is significant has been developed in articles by Stauber and Burt (1973), Stauber et al. (1975), and Kennedy (1973). The dynamic programming approach is also discussed by Dillon (1977).

The major purpose in this paper is to discuss the role of soil test information in making fertilizer recommendations in the presence (or absence) of fertilizer carry-over and in the presence (or absence) of an interaction between the applied and residual nutrients. Example data will be used to compare fertilizer recommendations made under an optimal long-term fertilizer policy with those which would be made under the more conventional static analysis.

A Crop Response and Carry-Over Function

Onken and Sunderman (1972) proposed treating soil nutrients as separate variables from the applied nutrients. They specified the following second order function which relates crop response to applied and residual soil nutrients.

$$Y_t = a + bN_t + cS_t + dN_t^2 + fS_t^2 + gN_tS_t \qquad (1)$$

where Y_t is the expected yield in year t;

N_t is the quantity of applied fertilizer in year t;

S_t is the amount of the residual nutrient in the soil in year t.

To allow for nutrient carry-over, a simple linear function which relates the level of soil nutrients (S_t) in year t to previous quantities of applied and residual nutrients can be specified as

$$S_t = h + mN_{t-1} + nS_{t-1} \qquad (2)$$

The initial value of S_t can be determined by a soil test or can be derived implicitly as shown by Stauber and Burt (1973).

It is recognized that more elaborate algebraic forms for equations (1) and (2) might be chosen. These forms are considered in this paper because of their familarity to agricultural economists and because of the ease with which the functions may be manipulated.

Case of No Carry-Over

If statistical analysis indicates carry-over is not significant, then the profit maximizing quantity of fertilizer is determined from the relation in equation 3.

$$N_t = \frac{P_{N_t}/P_{y_t} - b - gS_t}{2d} \qquad (3)$$

The value of soil test information in this case depends on whether or not there is a significant interaction between the applied and residual soil nutrient. In the absence of a significant interaction (g = 0), the optimal level of the applied nutrient is independent of the quantity of nutrients in the soil. The total nutrient level is commonly taken to be the sum of residual and applied nutrients as

$$TN_t = N_t + \alpha S_t \qquad (4)$$

where α is a factor to convert soil nutrients to an equivalent amount of applied nutrient.

in equation (4). This response function can be written as shown in equation (5).

$$Y_t = a' + b'TN_t + c'(TN_t)^2$$

If we substitute $N_t + \alpha S_t$ for TN_t in equation 5, it can be seen that equation 5 is equivalent to equation (1) if $a' = a$, $b' = b = \alpha c$, $c' = d = \alpha^2 f = 2\alpha g$. In other words, we commonly assume the interaction term g is present and negative.

Case of Significant Carry-Over

In the presence of significant carry-over, the problem of determining the optimal long-run fertilizer level is that of maximizing long-run profits given knowledge of yields, prices, and costs. A constrained profit maximization problem can be written as

$$\text{Maximize } L\ (N_t, S_t, \lambda_t) =$$

$$\sum_{t=1}^{T} \beta_t[P_{y_t}(a + bN_t + cS_t + dN_t^2 + fS_t^2 + gN_tS_t) - P_{n_t}N_t] + \sum_{t=2}^{T} \lambda_t(h + mN_{t-1} + nS_{t-1} - S_t) + \lambda_o(S_o - S_1) \quad (6)$$

where T is the length of the planning horizon;

β_t is the discount factor for year t;

P_{y_t} is the price of the output in year t;

P_{n_t} is the price of the applied nutrient in year t;

S_o is the initial quantity of residual nutrients in the soil at the beginning of the planning horizon.

The optimal control sequence of fertilizer applications can be found by differentiating the above function (6) with respect to $N_t(t=1, 2 . . T)$, $S_t(t=2, 3, . . . T)$, $\lambda_t(t=2, 3, . . . T, 0)$ setting the derivatives equal to zero and solving N_t, S_t, and λ_t. The resulting set of equations is shown schematically in Figure 1. The control variable is the quantity of nitrogen to be applied in each year while the state variable in each year is the level of soil nutrients.

The Lagrangian multipliers in this problem measure the discounted VMP of residual soil nutrients. In the third equation, if $S_2 > 0$, we have

$$\lambda_2 = \beta_2 P_{y_2}(c + 2fS_2 + gN_2) + n\lambda_3 \qquad (7)$$

The expression in the parenthesis is the MPP of the soil nutrients in year 2. Similarly, λ_3 is equal to the discounted VMP of soil nutrients in year 3 plus $n\lambda_4$. In the model, nutrient applications affect the output in subsequent periods through the level of soil nutrients. In the context of a chain rule, we have

$$\frac{\partial Y_{t+1}}{\partial N_t} = \frac{\partial Y_{t+1}}{\partial S_{t+1}} \frac{\partial S_{t+1}}{\partial N_t} = \frac{\partial Y_{t+1}}{\partial S_{t+1}} m, \quad \text{where } \frac{\partial S_{t+1}}{\partial Nt} = m \qquad (8)$$

Since λ_t represents the discounted sum of all future VMP's of residual soil nutrients from time t+1 until T, the decision rule indicates that a nutrient should be applied until the current cost per unit of the nutrient is equal to the discounted sum of current plus the future value of marginal products. The result is shown graphically in Figure 2. It should be noted that since P_{n_t} is equated to the sum of the discounted MVP's of the applied nutrient, the optimal application rate can be in excess of the quantity required to maximize the total yield in the current period.

Example Application.

An example application was provided. The data were from a three-year grain sorghum fertilizer experiment on Sherm Silty Clay Loam in the Texas High Plains. Residual soil nitrate levels were determined from annual samples taken from each plot by the method developed by Onken and Sunderman (1972). The nitrogen application levels ranged from 0 to 160 pounds of N per acre. The SAS Time Series Cross Section procedure was used.

The regression estimates for the coefficients in equations (1) and (2) are shown in Table 1. The interaction term in the yield equation is positive rather than negative as anticipated. The positive interaction term does imply that applied and residual nutrients interact to increase yields.

The long-term profit function is quadratic in N and S as described above. The control solution to the system of linear first partial derivatives can be obtained by simultaneous equations techniques (assuming convexity and non-negativity of N_t and S_t), by quadratic programming or by dynamic programming with quadratic criteria. More complex algebraic forms can be solved by the use of more general dynamic codes. The results in this paper were obtained by using a quadratic dynamic programming code written in BASIC on a micro-computer. Computation time for a problem with a 10-year planning horizon was about 5 seconds.

Results and Implications.

The temporal sequence of fertilizer applications which would maximize profits over the first 5 years of a 10-year planning horizon are shown in Table D. In this analysis, it was assumed that the price of grain sorghum would remain at $4.25 per cwt. and nitrogen would cost $.1215 per pound.

The rate of interest was assumed to be 10 percent. The results in Table 2 show the recommendation for a farmer whose soil test indicated 10 lbs. of N would be to apply 161 pounds of fertilizer the first year. The expected carry-over the second year would be 29.6 pounds, while the nitrogen application in the second year is expected to be 196 pounds. However, it is anticipated the soil would be retested and a new multi-year planning sequence would be started each year.

A sensitivity analysis indicated in this case the first year fertilizer recommendation would be sensitive to extending the planning horizon from one to 5 years and to changes in the coefficients of the carry-over function. The first year recommendations were found to be relatively insensitive to changes in relative prices, the rate of interest, and extension of the planning horizon from 5 to 10 years.

In Figure 3, a comparison is made between the time sequence of fertilizer applications, residual carry-over, and crop yields for long-term profit maximization and for the same variables for a sequence of single year profit maximizations. The quantity of applied nutrient lies above the static ridgeline in the optimal control sequence. These results must be interpreted with caution because of the variability of the carry-over function.

However, the results do imply that for soil situations where carry-over does occur single year fertilizer recommendations will, in general, understate the quantity of fertilizer necessary to maximize profits and to maintain an optimal inventory of residual soil nutrients. Whereas considerable effort has gone into the choice of experimental designs to measure crop response to applied nutrients, little attention has gone into selection of experimental designs to measure the relationship between changes in treatment levels and the subsequent level of residual nutrients. The results also demonstrate that fairly complex analyses can be routinely conducted on low cost micro-computers in remote field stations.

References

Brown, W.G., T.L. Jackson, and R.G. Petersen. "A Method for Incorporating Soil Test Measurments into Fertilizer Response Functions." Agronomy Journal. 54(1962): 152-154.

Cate, B.C. Jr, and L.A. Nelson. "A Simple Statistical Proceedure for Partitioning Soil Test Correlation Data into Two Classes." Soil Sci. Soc. Amer. Proc. 35(1971): 658-660.

Dillon, J.L. The Analysis of Response in Crop and Livestock Production. New York: Pergamon Press 1977.

Dreyfus, S.E., and A.M. Law. The Art and Theory of Dynamic Programming. New York: Academic Press 1977.

Heady, E.O., and J.L. Dillon. Agricultural Production Functions. Ames: Iowa State Univ. Press 1961.

Nelson, L.A. and R.L. Anderson. "Partitioning of Soil Test-Crop Response Probability." in Soil Testing and Interpreting the Analytical Results. Amer. Soc. of Agron. ASA Special Pub. 29(1977): 19-37.

Mombiela, F., J.J. Nicholaides, III, and L.A. Nelson. "A Method to Determine the Appropriate Mathematical Form for Incorporating Soil Test Levels in Fertilizer Response Models for Recommendation Purposes." Agronomy Journal. 73(1981) 937-941.

Nelson, L.A., and R.L. Anderson. "Partitioning of Soil Test-Crop Response Probability." in Soil Testing: Correlating and Interpreting the Analytical Results. ASA Special Publication No. 29 (1977) 19-38.

Onken, A.B., R. Mathesen, and E.J. Williams. "Soil Testing for Phosphorus on the Texas High Plains." Texas Agric. Exp. Sta., Texas A&M Univ.: PR-3549 (1979).

Onken, A.B., and H.D. Sanderman. "Applied and Residual Nitrate-Nitrogen Effects on Irrigated Grain Sorghum Yield." Soil Sci. Soc. Amer. Proc.: 36(1972) 94-97.

Stauber, M.S., and O.R. Burt. "Implicit Estimate of Residual Nitrogen Under Fertilized Range Conditions in the Northern Great Plains." Agronomy Journal: 65(1973) 897-901.

Stauber, M.S., O.R. Burt, and F. Linse. "An Economic Evaluation of Nitrogen Fertilization of Grasses When Carry-over is Significant." Amer. Journ. Agr. Econ.: 57(1975) 463-471.

Table 1. Regression Results for Yield and Carry-over Equations

Variable	Coefficient	Coef/Stand. error	Standard Error
'Yield equation			677
intercept	1550.33		
N_t	19.18	2.1	
N_t^2	-.091	2.2	
S_t	86.15	2.2	
S_t^2	-.858	2.4	
$N_t S_t$	.356	1.9	
Carry-over equation			10.9
intercept	14.58		
N_t-1	.074	2.2	
S_t-1	.317	1.3	

Table 2. Projected Levels of Applied Nitrogen and Residual Nitrate Nitrogen for Long Term Profit Maximization From Irrigated Grain Sorghum Production

Year	Residual Nitrogen	Applied Nitrogen	Yield Grain Sorghum	Total Return Less Fertilizer Cost
		pounds per acre		dollars/acre
1	10.0	161	3629	133
2	29.6	195	5686	216
3	38.4	210	6475	248
4	42.3	217	6802	261
5	44.0	220	6943	267

net present value = $829.80

	Variables											
Derivative	N_1	$\bar{S}_1$	N_2	S_2	N_3	S_3 . . .	N_T	S_T	λ_2	$\lambda_3 \ldots \lambda_T$	λ_o	Constants
$\partial L/\partial N_1$	$2\beta_1 P_{y_1} d$	$\beta_1 P_{y_1} g$							m			$\leq \beta_1 (P_{y_1} b - P_{n_1})$
$\partial L/\partial S_1$	$\beta_1 P_{y_1} g$	$2\beta_1 P_{y_1} f$							n			$\leq \beta_1 P_{y_1} C$
$\partial L/\partial N_2$			$2\beta_2 P_{y_2} d$	$\beta_2 P_{y_2} g$						m		$\leq \beta_2 (P_{y_2} b - P_{n_2})$
$\partial L/\partial S_2$			$\beta_2 P_{y_2} g$	$2\beta_2 P_{y_2} f$					-1	n		$\leq \beta_2 P_{y_2} C$
$\partial L/\partial N_3$					$2\beta_3 P_{y_3} d$	$\beta_3 P_{y_3} g$						$\leq \beta_3 (P_{y_3} b - P_{n_3})$
$\partial L/\partial S_3$					$\beta_3 P_{y_3} g$	$2\beta_3 P_{y_3} f$				-1		$\leq \beta_3 P_{y_2} C$
⋮												
$\partial L/\partial N_T$							$2\beta_T P_{y_T} d$	$\beta_T P_{y_T} g$				$\leq \beta_T (P_{y_T} b - P_{n_T})$
$\partial L/\partial S_T$							$\beta_T P_{y_T} g$	$2\beta_T P_{y_T} f$				$\leq \beta_T P_{y_T} C$
$\partial L/\partial \lambda_2$	-m	-n		1								= h
$\partial L/\partial \lambda_3$			-m	-n		1						= h
⋮												
$\partial L/\partial \lambda_T$							-m	-n				= h
$\partial L/\partial \lambda_o$		1										$= S_o$

Figure 1. Partial Derivative Equations Associated with Determination of Long-Term Fertilizer Policy.

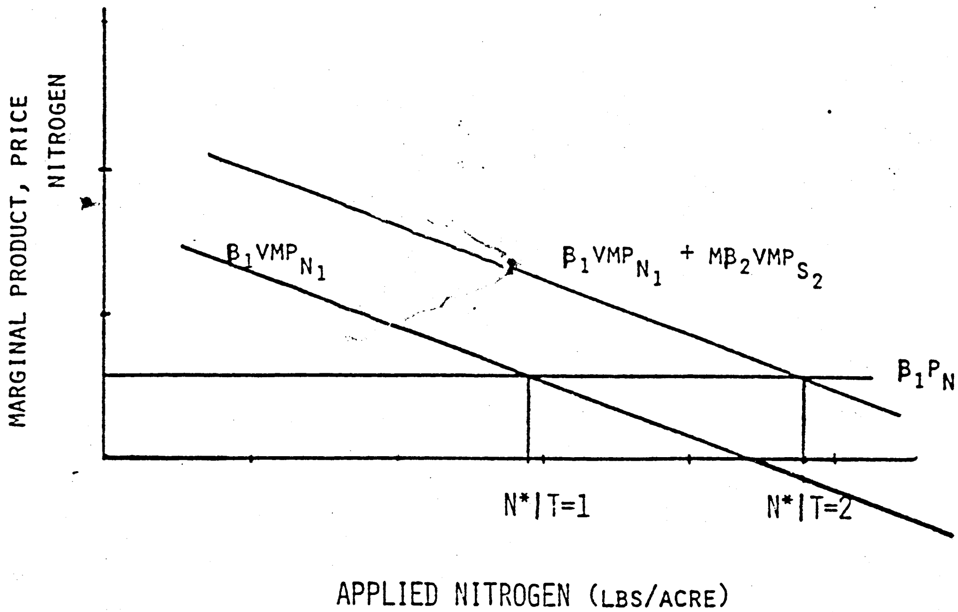

FIGURE 2. OPTIMAL QUANTITY OF NITROGEN TO APPLY IN YEAR 1 FOR A FARMER WITH A ONE AND WITH A TWO YEAR PLANNING HORIZON.

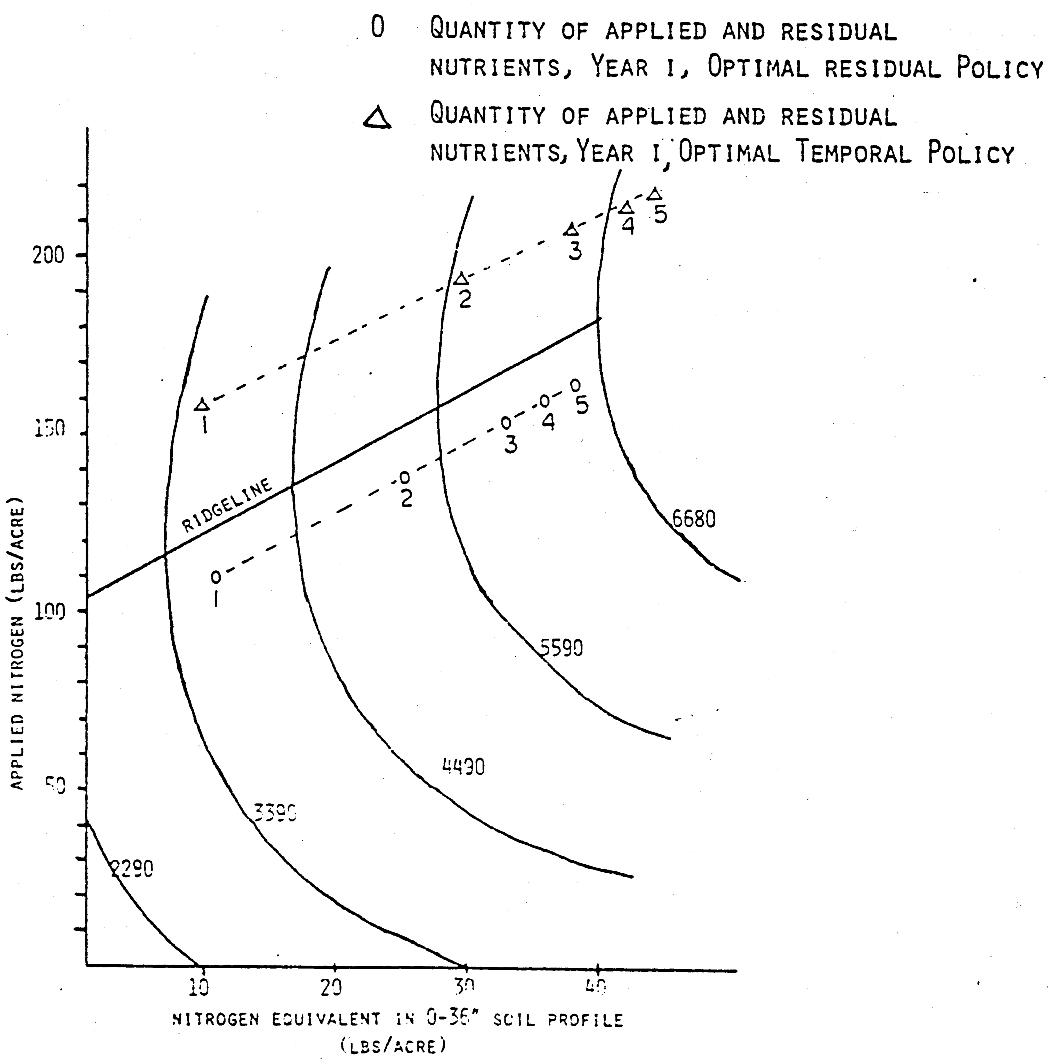

FIGURE 3. COMPARISON OF OPTIMAL TIME PATHS FOR FERTILIZER USE WHEN OPTIMAL TEMPORAL AND OPTIMAL RECURSIVE POLICIES ARE FOLLOWED.