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On the Standardization
of Input-Output Multipliers

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Abstract

On the Standardization of Input-Output Multipliers

Variations on the basic input-output based output multiplier have proliferated of late. This paper provides a standard basis upon which these multipliers can be compared and contrasted. Certain inconsistencies are cited, reconciliations offered and standardizations suggested. Examples of each variety are based on the Virginia input-output model.

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Introduction

In a recent address to the Regional Science Association, Karen Polenske warned input-output analysts of the many, potentially inconsistent (or non-conventional) means of calculating input-output based multipliers. West and Jensen have recently offered some suggested standardizations. Agricultural Economists, particularly those in the areas of rural development, regional development and resource economics, make significant use of input-output analysis and stand to benefit as much as any from more standardization.

This paper describes the major varieties of multipliers reported in the literature and offers a few suggested standardizations which agricultural economists might wish to adopt. Where appropriate, inconsistencies in the literature are cited and suggested reconciliations are offered. Examples of the various types of multipliers are taken from the Virginia input-output model, a derivative of the 1972 U.S. model.

Varieties of Multipliers

In the conventional, square, static Leontief input-output model the level of output in each sector over a period of time is the sum of intermediate demands and final demand. Mathematically,

(1) $X = AX + F$,

where $X = [x_i]$ = an n x 1 vector of sectoral outputs;

$A = [a_{ij}]$ = an n x n matrix of interindustry coefficients;

$F = [f_i]$ = an n x 1 vector of levels of final demand; and

n = the number of sectors.

Equation (1) considers only the n endogenous sectors in the economy. Payments are also made to the primary inputs or payment sectors - households, governments, savings and rest of the world (imports). By augmenting vectors X and F and matrix A the following equation of partitioned matrices results:

$$(2)\quad \begin{matrix} X_s \\ X_h \\ X_p \end{matrix} = \begin{matrix} A_{ss}A_{sh}0 \\ A_{hs}A_{hh}0 \\ A_{ps}A_{ph}0 \end{matrix} \quad \begin{matrix} X_s \\ X_h \\ X_p \end{matrix} + \begin{matrix} F_s \\ F_h \\ F_p \end{matrix}$$

where X and P are (m + n + 1) x 1 vectors;
A is an (m + n + 1) x (m + n + 1) matrix;
s refers to the n sectors;
h refers to households; and
p refers to the m primary inputs.

Since X_s is not a function of X_p, the rows of entries in X_p need not be restricted to those things usually included in the definition of primary inputs. For example, X_p may include employment, water use, energy consumption, externalities generated, or any other measure which is produced or used approximately in proportion to the level of sectoral outputs. Furthermore, the household income row, $(A_{hs}A_{hh}0)$ can be repeated here to facilitate the calculation of income multipliers. These affects will be collectively referred to as primary inputs in this paper. All that is required is that A_{ps} include coefficients defined as the ratio of the direct effects to level of output in each sector.

Two assumptions are possible with respect to households. The first is that the level of household expenditures are endogenously determined by income, X_h, and the second is that the level is fixed (exogenously determined). In the first case, the model is said to be open with respect to households and A_{sh}, A_{hh} and A_{ph} are zero. Household expenditures are included in F. In the second case, the model is said to be closed, with respect to households and the h column of A contains consumption coefficients. The first case results in the so-called type I multipliers and the second, type II multipliers. Both types can be calculated from equation (2).

From equation (2) the level of household receipts is

(3) $X_h = (I - A_{hh})^{-1}(A_{hs}X_s + F_h)$.

Substituting this into the partitioned equation for sector levels yields

(4) $X_s = A_{ss}X_s + A_{sh}(I - A_{hh})^{-1}(A_{hs}X_s + F_h) + F_s$,

which has the solution,

(5) $X_s = [I - A_{ss} - A_{sh}(I - A_{hh})^{-1}A_{hs}]^{-1}[A_{sh}(I - A_{hh})^{-1}F_h + F_s]$.

This rather cumbersome equation can be made considerably simpler without any loss of generality if it is assumed that A_{hh} and F_h are equal to zero. Then

(6) $X_s^{EN} = (I - A_{ss} - A_{sh}A_{hs})^{-1}F_s$,

when households are endogenous. When households are exogenous, the submatrix $A_{sh} = 0$ and

(7) $X_s^{EX} = (I - A_{ss})^{-1}F_s$,

the standard IO solution. The difference between the type I and II multipliers is the induced effect per dollar of final demand, that is

(8) $M_{INDUCED} = (I - A_{ss} - A_{sh}A_{hs})^{-1} - (I - A_{ss})^{-1}$.

At the same time household receipts may be expressed as

(9) $X_h^{EN} = A_{hs}(I - A_{ss} - A_{sh}A_{hs})^{-1}F_s$,

when households are endogenous and

(10) $X_h^{EX} = A_{hs}(I - A_{ss})^{-1}F_s$,

when they are not. Assuming A_{ph} and F_p equal zero, the primary inputs are

(11) $X_p^{EN} = A_{ps}X_s = A_{ps}(I-A_{ss}-A_{sh}A_{hs})^{-1}F_s$, and

(12) $X_p^{EX} = A_{ps}(I-A_{ss})^{-1}F_s$.

Given the equations above, we can define a number of multipliers commonly generated and used by input-output analysts. First note that the term, $(I - A_{ss})^{-1}$, in equations (7), (10), and (12) is the conventional, n x n, matrix of multipliers. Type I sectoral output multipliers can be calculated as follows:

(13) $M = I\ (I - A_{ss})^{-1}$,

where M_o = 1 x n, vector of sectoral output multipliers; and

I = 1 x n row vector of 1's.

Type II output multipliers can be calculated from $(I-A_{ss}-A_{sh}A_{hs})^{-1}$ in the same way.

The jth element of M_o is the ratio of direct plus indirect output in all sectors divided by the direct output in (final demand for) sector j, that is $(\sum_{i=1}^{n} x_i)/f_j$.

Other common multipliers (income, value added, etc.) are calculated by dividing the total change in the appropriate element of X_p (say x_{pi}) by the direct change in the level for each sector. This direct change is the product $a_{ps_{ij}} f_j$. Increments can be introduced for each sector by post multiplying the multiplier matrix by a diagonal matrix of sector final demands, F_s. However, if these increments in final demand equal 1.0 as they do when multipliers are being calculated, then this is the identity matrix and changes in output in equation (7) are simply the n x n matrix, $(I - A_{ss})^{-1}$.

From equation (12), the type I levels of total primary inputs will be

(14) $M_p = A_{ps}(I - A_{ss})^{-1}$,

where M_p = an m x n matrix of total levels of primary inputs i (i = 1, m), for unit changes in each sector j (j = 1, n).

What is left is to divide this total change in each primary input by the direct change in the primary input for each sector. For a given primary input, say income, the direct effects are the income row of submatrix A_{ps}. Thus, the type I income multipliers are calculated as follows:

(15) $M_I = A_{psI} (I - A_{ss})^{-1} A_{psI}^{-1}$ where

M_I = a 1 x n vector of sectoral income multipliers

A_{psI} = 1 x n vector of income coefficients (the income row of A_p).

A_{psI}^{-1} = the inverse of the diagonal matrix formed from the income (I) row of A .

This method can be repeated to calculate each of the various primary multipliers possible. Thus, M_E might be defined as the ratio of total employment generated per direct employee in each sector and M_W might be the total economy wide water use generated by a unit of water embodied in products destined for final demand. Of course, for each of these varieties, type II multipliers can also be calculated. Table 1 gives output and primary input multipliers for selected sectors of the Virginia economy.

In practice, a much more direct method of calculating the type II output multiplier is usually employed. This method involves the merger of A_{ss}, A_{hs}, and A_{sh} into a single coefficients matrix prior to the inversion. This is equivalent to treating the household sector as if it were a producing sector. This method gives equivalent results except that when the addition of columns is performed as in equation (13), an extra row - that of household receipts - is included.

A Problem With Type I and II Output Multipliers

In many studies the above method of calculating type II output multipliers has led to an inconsistency which is quite misleading. These studies (Hoppe, 1978; Hiser and Fisher, 1977; Schaffer, 1976; Stoeker, Wright and Pyles, 1981; and Maki, 1981; Trenchi and Flick) calculate both type I and type II multipliers by adding the columns of the direct plus indirect requirement matrix ($(I - A)^{-1}$) and attribute the difference to the effect of induced spending by households. The problem with this is that the type II multiplier includes household receipts while the type I multiplier does not, even though it exists and can be measured. The result is that the induced effect is seriously overestimated.

The correct calculation of induced effects is given in equation (8). The erroneous method referred to adds to this, the level of household receipts in equation (9) but does not subtract household receipts in equation (10), when households are exogenous.

Even if household receipts are defined as output the practice is incorrect since the correct measure is

$$(16)\quad M_{INDUCED} = (I - A_{ss} - A_{sh}A_{hs})^{-1} - (I - A_{ss})^{-1} + A_{hs}(I - A_{ss} - A_{sh}A_{hs})^{-1} - A_{hs}(I - A_{ss})^{-1}$$

Table 2 shows the degree of overestimation involved in several studies in which this inconsistence exists.

Table 2: A comparison of reported and corrected type II output multipliers, selected studies, selected sectors.

Study			Construction	Trade	Finance
Minnesota's Region Six East:	Type I		1.1424	1.1674	1.0933
	Type II	(Reported)	1.7814	2.1662	2.1702
		(Correct)	1.3302	1.4608	1.4097
Clinton County, New York	Type I		1.20	1.13	1.09
	Type II	(Reported)	1.78	1.61	2.02
		(Correct)	1.45	1.31	1.49
Georgia	Type I		1.48	1.22	1.44
	Type II	(Reported)	2.51	2.59	2.94
		(Correct)	1.91	1.79	2.07
Texas High Plains	Type I		1.7456		1.6827
	Type II	(Reported)	2.6862		2.5415
		(Correct)	2.1251		2.0292

The type II multipliers in Table 2 are calculated in the same fashion as the type I multipliers were calculated - that is, they exclude household receipts. This is not the method that most studies use to calculate type II multipliers. Most studies (Carrol and Stoevener; Goldman, et al, Goldman, Wallace and Mammer; Guedry and Smith; Haroldsen and Younmans are just a few) calculate type II output multipliers by summing the columns of the inverted matrix including the household row. This is largely a matter of preference provided the type

I multipliers, if calculated are calculated in the same fashion. It seems appropriate, however, that one method or the other be adopted as the standard practice. It is argued here that the first alternative is preferable.

Whether the profession accepts a standard practice with respect to defining households as an output or not is, perhaps, less important than being cognizant of the potential difference from one study to another. Under no circumstances, however, is it correct to define type I multipliers in one way and type II multipliers in another. It also bears reminding that this is a problem with output multipliers only.

Pseudo-Multipliers

A variation on these primary input multipliers, used in several studies to date (Maki, et. al., Schaffer; Johnson and Kulshreshtha, 1981; 1982; Bourque, Conway, and Howard; West and Jensen), is based on equation (7) rather than equations such as (8). The values in equation (7) are the total changes in economy wide primary inputs (and other measures) associated with a $1.00 change in final demand. The jth element in the income row, for example, is the direct plus indirect income divided by the direct output. These coefficients are really not multipliers, although they are sometimes referred to as such (Schaffer), since their numerators and denominators are in different units.

The advantage of these ratios, or pseudo-multipliers (Johnson and Kulshreshtha, 1982) are as follows: 1) they are all directly derivable from equation (6) in a single step because the implicit final demand level is the same for each ($1.00); 2) they are more readily comparable between sectors of an economy since each is the income, employment, etc., per dollar of direct output; 3) they are less likely to be misused since all of these pseudo-multipliers, including the output multiplier are multiplied by direct output (final demand); and 4) the various pseudo-multipliers for a given sector are comparable and additive. The income pseudo-multiplier plus the tax and savings pseudo-multiplier equal the value-added pseudo-multiplier. Selected pseudo-multipliers for the Virginia economy are presented in Table 1.

These pseudo-multipliers have a number of features which distinguish them from the real multipliers. Sectors with large output multipliers tend to have large pseudo-multipliers since the indirect effects are more widespread. Pseudo-multipliers are much smaller than their multiplier counterparts because the denominators are larger. Only type II pseudo-multipliers may exceed 1.0.

<u>Output</u> <u>vs</u>. <u>Final</u> <u>Demand</u>

Another area of potential confusion relates to a distinction sometimes made between final demand and output multipliers. Stoeker, Wright, and Pyles (1981) define a sectoral final demand multiplier as the ratio of total outputs to direct output, in a sector. This definition is adopted since direct output is synonomous with final demand if the latter is entirely domestic. They define a sectoral output multiplier as the ratio of total economy output to total output in a sector. Since total output in a sector is larger than direct output by a factor equal to the appropriate diagonal element of the multiplier matrix, the two multipliers are also related by this factor.

This distinction means that equation (5) represents the final demand multipliers while the output multipliers are

(17) $M_o^* = I(I - A_{ss})^{-1}D_o$

where D_o = an n x n diagonal matrix of inverses of the diagonal of $(I - A_{ss})^{-1}$

This distinction is useful since it draws attention to a potential misuse of multipliers. When a direct output change is known, equation (13) is appropriate, but often it is the level of output in a sector which is of concern, in which case equation (18) should be used.

Other authors, noting this point, have taken other approaches (Johnson and Kulshreshtha, 1981, 1982; Petkovich and Ching, 1978). The Johnson and Kulshreshtha approach is to exogenize the sector in question and set it at its known level, while the Petkovich and Ching method uses linear programming to constrain the level of output. These latter approaches are more general than the Stoeker, Wright and Pyles method since they allow the analyst to control the expenditure and trade patterns of the sector as well as the level of output.

While this distinction is useful, the choice of names is somewhat misleading since the distinction being made can and should be extended to each of the varieties of multipliers discussed above. For example, any primary input multipliers, such as the income multipliers in equation (15) can be converted from its present "final demand" or direct format to an "output" or total sectoral format. In equation (15) the ratio of direct income to total sectoral income is $A_{psI} D_I$

where D_I = an n x n diagonal matrix of inverses of the diagonal elements in $A_{psI}(I - A_{ss})^{-1}$.

The alternate income multipliers then are

(18) $$M_I^* = A_{psI}(I - A_{ss})^{-1} A_{psI}^{-1} A_{psI} D_I = A_{psI}(I - A_{ss})^{-1} D_I.$$

The pseudo-multipliers can be converted from "final demand" to "output" terms in precisely the same way that the original calculation was performed, that is

(20) $M_p^* = A_{ps}(I - A_{ss})^{-1}D_o$.

There is obviously a need for standardization in the naming of these myriad of multipliers. The terms output, income, value-added and employment are all too well entrenched in the literature to be changed as proposed by Stoecker, Wright and Pyles, yet there is a need to distinguish the two concepts. Perhaps the multipliers described by equations (17), (18), and (19) could be referred to as expansion factors or expansion ratios rather than multipliers. Equation (17) then would describe output expansion ratios.

Table 1 distinguishes the conventional output multipliers from those proposed by Stoeker, Wright, and Pyles (here called output expansion ratios). The concept is also expanded to include income and value added expansion ratios.

Conclusions

Despite their widespread use and acceptance as an important economic tool, input-output multipliers have become less rather than more standardized.

The original output multiplier has been joined by various primary input and physical multipliers such as income, value-added, and water multipliers. Each of these has numerous variations.

The need for a standard nomenclature to distinguish between them is obvious. In addition, input-output analysts must be clear about what varieties of multipliers they are reporting, the assumptions underlying each and the appropriate multiplier(s) for each particular need.

List of References

Bourque, P.J., Richard Conway, Jr., and C.T. Howard. The Washington Projection and Simulation Model. Input-Output Series, University of Washington Graduate School of Business Administration, September 1977.

Carrol, T.M. and H.H. Stoevener, "A 1977 Input-Output Model for Clatsop County, Oregon." Agricultural Experiment Station, Oregon State University, Corvallis. Special Report #525, December 1978.

DiPasquale, Denise and Karen R. Polenske. "Output, Income, and Employment Input-Output Multipliers." In Economic Impact Analysis: Methodology and Applications. pp. 85-113, Edited by Saul Pleeter. Boston: Martinas Nijhoff Publishing, 1980.

Doeksen, Gerald A. and Dean F. Schreiner. "A Simulation Model for Oklahoma With Economic Projections from 1963 to 1980." Bulletin B-693, Oklahoma State University, May 1971.

Goldman, G., D. MacLeod, J. Mamer, J. McReynolds. "San Francisco Bay Area Input-Output Model, 1967 and 1974." Special Publication #3248, Division of Agricultural Sciences, University of California, August 1978.

Goldman, George, L.T. Wallace and John Mamer. Lake County Economic and Resource Use Study for Base Year 1965. Agricultural Extension 1-05. Cooperative Extension, University of California, November 1973.

Guedry, Leo J. and David W. Smith. "Impact of Industry in Rural Economies: An Input-Output Approach." Southern Journal of Agricultural Economics (December 1980):19-24.

Haroldsen, A. and R. Youmans. "Grant County: Structure of the County Economy." Extension Service, Oregon State University, Corvallis, Special Report #358, May 1972.

Hoppe, Robert. Building A Non-Metropolitan Input-Output Model: Minnesota's Region Six East Agricultural Experiment Station, University of Minnesota Technical Bulletin #313, 1978.

Ives, Edward and Russell Youmans. "Business Interrelationships of the Tillamook County Economy: A Study for Analysis of Economic Change." Circular of Information 672. Agricultural Experiment Station, Oregon State University, Corvallis, October 1978.

Jensen, R.C. and G.R. West. "On the Nature of Regional Multipliers." Paper presented at the North American Meetings of the Regional Science Association, Pittsburgh, Pennsylvania, November 1982.

Johnson, Thomas G. and Surendra N. Kulshreshtha. Nature of Intersectoral Relations of Saskatchewan Agriculture: An Input-Output Analysis. Technical Bulletin 81-06. Department of Agricultural Economics, University of Saskatchewan, November 1981.

Johnson, Thomas G. and Surendra N. Kulshreshtha. "Exogenizing Agriculture in an Input-Output Model to Estimate Relative Impacts of Different Farm Types." Western Journal of Agricultural Economics 7 (December 1982).

Maki, Wilbur R. "Input-Output Methods for Labor Market Analysis and Projection." Staff Paper P81-20, Department of Agricultural and Applied Economics, University of Minnesota, St. Paul, August 1981.

Miernyk, William H. The Elements of Input-Output Analysis. New York: Random House, 1965.

Miernyk, William H., Ernest R. Bonner, John H. Chapman, Jr., and Kenneth Shellhammer. Impact of the Space Program on a Local Economy. Parsons, WV: McClain Printing Company, 1967.

Moore, Frederick T. and James W. Petersen. "Regional Analysis: An Interindustry Model of Utah," The Review of Economics and Statistics, Vol. XXXVII, No. 4 (1955), 368-83.

Petkovich, M.D. and C.T.K. Ching. "Modifying A One Region Leontief Input-Output Model to Show Sector Capacity Constraints." The Western Journal of Agricultural Economics 3 (December 1978):173-179.

Polenske, Karen R. "Analyses of Output, Income and Employment Impacts in Regional and Multiregional Models." Paper presented at the Twenty-ninth North American Meetings of the Regional Science Association, Pittsburgh, Pennsylvania, November 1982.

Regional Science Research Institute. Basic Regional Input-Output for Transportation Impact Analysis: Handbook One. Revised Preliminary Draft, March 1981.

Schaffer, William A., ed. On the Use of Input-Output Models for Regional Planning. Leiden, Netherlands: Martinus Nyhoff Social Sciences Division, 1976.

Stoecker, Arthur L., Mickey L. Wright, and David A. Pyles. An Input-Output Model of the Texas High Plains Economy in 1977. Department of Agricultural Economics, Texas Tech University, College of Agricultural Sciences, Publication No. T-1-199, August 1981.

Trenchi, Peter and Warren A. Flick. "An Input-Output Model of Alabama's Economy: Understanding Forestry's Role." Bulletin 534, Department of Forestry, Auburn University, Auburn, Alabama, July 1982.

West, G.R. and R. C. Jensen, "Some Reflections on Input-Output Multipliers." Annals of Regional Science 14, No. 2(1980):77-89.