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INFORMATION AND THE CONCEPT OF OPTION VALUE

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INFORMATION AND THE CONCEPT OF OPTION VALUE

I. INTRODUCTION

Ever since the seminal article by Weisbrod [1964], there has been a widespread interest among resource economists in the concept of option value. Weisbrod argued that, in contemplating an action with irreversible consequences such as the destruction of a national park, one must recognize that many people would be willing to pay something to preserve the option of visiting the park in the future. Private market calculations will overlook this option value if there is no mechanism by which the owner of the park can charge for it, but it certainly should be included in any social benefit cost analysis of the destruction of the park. This conclusion has been widely accepted. However, because Weisbrod's argument was developed in a fairly informal manner, it has stimulated a lively debate as to the precise definition and measurement of the option value concept.

Two broad interpretations have emerged. The first, presented originally by Ciccnetti and Freeman [1971] and refined by Schmalensee [1972], Bohm [1974] and Graham [1981], interprets option value as a risk premium arising from uncertainty as to the potential future value of the park if it were preserved. As Smith [1982] and others have pointed out, the passage of time per se is not essential to the formulation of this concept. The second interpretation, advanced by Arrow and Fisher [1974] and independently by Henry [1974], focuses explicitly on the intertemporal aspects of the decision and stresses the irreversibility of the destruction of the park and the flow of information over time. In Henry's words, "[T]he mere prospect of getting fuller information [about the future value of the park], combined with the irreversibility of the non-preservation alternative, brings forth a positive option value in favor of preservation."² Unlike the other one, this interpretation does not require any assumption of risk aversion on the part of park users.

This paper is concerned with the Arrow-Fisher-Henry (AFH) time-dependent concept of option value. Conrad [1980] argues that it is equivalent to another

concept arising in decision theory, the expected value of information. Here I will show that the AFH option value is not necessarily the same as the expected value of information and, indeed, may exceed it. In order to reach this conclusion I first develop a general model of irreversible investment, which is presented in section II. Using this model, I analyze the definition of option value offered by Arrow and Fisher, and then provide an alternative, but equivalent, characterization of option value, in the light of which Conrad's claim is evaluated. In section IV, I go on to consider the effect on option value of an increase in the uncertainty surrounding the future benefits and costs of irreversible development. Arrow and Fisher show for one particular case that an increase in future uncertainty raises the option value and increases the advantage of postponing development initially. I show that this conclusion is not generally true, but I also characterize the conditions under which it becomes valid. Finally, in section V I consider two possible generalizations of the analytical framework adopted to this point. I show that my analysis of option value readily carries over to a situation where one assumes that the passage of time brings partial rather than perfect information about the future consequences of development. However, I show that the concept of option value is not well-defined when one assumes that, instead of a binary choice between development and preservation, there is a continuum of possible levels of development.

II. A MODEL OF IRREVERSIBLE INVESTMENT

In this section I will outline a model of irreversible investment which combines the main features of Henry's [1974] analysis together with that of Arrow and Fisher [1974]. The model will be used in the following sections to characterize the concept of option value and explore some of its properties. Assume that there are two time periods, $t = 1, 2$. The decision concerns d_t , the amount of land developed during period t . The units of measurement will be chosen so that the maximum possible level of development (the minimum possible level of preservation) is unity,

$$(1) \quad d_1 \leq 1, \quad d_1 + d_2 \leq 1.$$

In addition, a key assumption is that any development is irreversible

$$(2) \quad d_t \geq 0, \quad t = 1, 2.$$

Associated with any development program is some level of net benefits, B , representing both the benefits of development and the benefits of preservation. Let B_t be the benefits accruing in period t . For simplicity I assume that the overall benefit function is additive with respect to the two periods' benefits

$$(3) \quad B = B_1 + B_2,$$

although the results discussed below would also hold with a benefit function of the more general form $B = B(B_1, B_2)$. The first period's benefits depend on the amount of land developed during that period, d_1 , while the second period's benefits depend both on the total amount developed over the two periods, $d_1 + d_2$, and on the amount developed specifically during the second period, d_2 . Moreover, I assume that these second period's benefits involve an element of uncertainty, here represented by the random variable θ .

Thus,

$$(4) \quad \begin{aligned} B_1 &= B_1(d_1) \\ B_2 &= B_2(d_1 + d_2, d_2; \theta). \end{aligned}$$

It is immaterial to the argument which follows whether or not there is also uncertainty concerning the first period's benefits.

The social decision involves the maximization of expected benefits with respect to d_1 and d_2 subject to (1) and the irreversibility constraints (2). The decision on d_1 is made at the start of the first period, but there are two possible scenarios for the decision on d_2 . One scenario is that d_2 is also determined at the start of the first period or, equivalently, it is determined at the start of the second period but no more information about θ is available then than before. The other scenario is that the specific value of θ is known at the start of the second period, and the choice of d_2 can be postponed until that time in order to incorporate this information.

In the first scenario, d_1 and d_2 are chosen so as to maximize

$$V(d_1, d_2) = B_1(d_1) + E\{B_2(d_1 + d_2, d_2; \theta)\}$$

subject to (1) and (2). Denote the solutions by d_1^* and d_2^* , and let $V^* \equiv V(d_1^*, d_2^*)$ be the expected overall benefit under this optimal decision. In the second scenario where information becomes available over time, whatever the level of d_1 , d_2 will be chosen in the second period so as to maximize $B_2(d_1 + d_2, d_2; \theta)$ subject to $d_2 \geq 0$ and $d_1 + d_2 \leq 1$. Denote the solution as a function of d_1 and θ by $d_2 = f(d_1; \theta)$. In the first period, when the value of θ is not yet known, the decision is to maximize

$$(5) \quad \hat{V}_1(d_1) = B_1(d_1) + E\{B_2[d_1 + f(d_1; \theta), f(d_1; \theta); \theta]\}$$

subject to $0 \leq d_1 \leq 1$. Denote the solution by $\hat{d}_1$. Given θ , the second period decision will be $\hat{d}_2 = f(\hat{d}_1; \theta)$. The expected overall benefit associated with this solution is $\hat{V} \equiv \hat{V}(\hat{d}_1)$.

In order to compare the two scenarios, it is useful to reformulate the decision problem in the first scenario by applying the two-stage maximization principle. Define $V^*(d_1)$ by

$$(6) \quad V^*(d_1) = B_1(d_1) + \max_{\substack{d_2 \\ 0 \leq d_1 + d_2 \leq 1 \\ 0 \leq d_2}} \left[E\{B_2(d_1 + d_2, d_2; \theta)\} \right].$$

Then, d_1^* maximizes $V^*(d_1)$ subject to $0 \leq d_1 \leq 1$, and $V^* = V^*(d_1^*)$. For comparison, the maximand in the second scenario, (5), can be written as

$$(5') \quad \hat{V}(d_1) = B_1(d_1) + E \left\{ \max_{\substack{d_2 \\ 0 \leq d_1 + d_2 \leq 1 \\ 0 \leq d_2}} \left[B_2(d_1 + d_2, d_2; \theta) \right] \right\}.$$

As noted above, $\hat{d}_1$ maximizes $\hat{V}(d_1)$ subject to $0 \leq d_1 \leq 1$. Thus, the difference between the scenarios depends on the difference between the two functions $V^*(d_1)$ and $\hat{V}(d_1)$. Each of these functions measures the expected benefits over both periods as a function of the initial amount of development, d_1 , given that the amount of development in the second period is optimally chosen subject to the irreversibility constraint and the limitation of the information structure in the scenario. The quantities V^* and $\hat{V}$ are the expected benefits with an optimal choice of development in both periods. Their difference, $\hat{V} - V^*$, is known in the literature on decision theory as the expected value of perfect information (EVPI).

Without imposing some further structure on the model, one can say only a few things about the two scenarios. For any amount of development in the first period, it is clear that one cannot be worse off under the second scenario than under the first since, from (5') and (6),²

$$(7) \quad \hat{V}(d_1) - V^*(d_1) = E \{ \max B_2(d_1 + d_2, d_2; \theta) \} - \max E \{ B_2(d_1 + d_2, d_2; \theta) \} \geq 0.$$

However, if there is full development in the initial period, then it makes no difference whether or not information becomes available in the second period because it is impossible to adjust the stock of developed land further:

$$(8) \quad \hat{V}(1) = V^*(1) = B_1(1) + E \{ B_2(1, 0; \theta) \}.$$

It follows from (7) and (8) that $\hat{V} \geq V^*$. Hence, $EVPI = \hat{V} - V^* \geq 0$.

An interesting question is how the irreversibility of development and the possibility of obtaining information over time affect the initial development decision, d_1 . Since $V^*(d_1)$ and $\hat{V}(d_1)$ are two different functions, in general

$d_1^* \neq \hat{d}_1$. Thus, the decision on how much land to develop currently cannot be decoupled from the decision on how much land to develop in the future or from the timing of the information on the consequences of the latter decision.

In particular, it is not necessarily true that $d_1^* \geq \hat{d}_1$, i.e. that if the future development decision could be postponed until more information becomes available one would be more conservative in deciding on the current level of investment.

Suppose, however, one assumes that there are only two possible levels of development so that $d_t = 0$ or 1 . This assumption, in fact, is invoked in most of the the discussion of the option value concept. One can regard it as a separate restriction on the model or, as in Arrow and Fisher, one can assume that $B_1(\cdot)$ and $B_2(\cdot)$ are linear functions of d_1 , d_2 , and $d_1 + d_2$, in which case the maximization decisions yield corner solutions of either zero or full development. With this assumption, the outcome of the two optimization problems depends on four values, $V^*(1)$ and $\hat{V}(1)$, as defined in (8), and $V^*(0)$ and $\hat{V}(0)$, which are given by

$$(9) \quad \begin{aligned} V^*(0) &= B_1(0) + \max\{E\{B_2(0, 0; \theta)\}, E\{B_2(1, 1; \theta)\}\} \\ \hat{V}(0) &= B_1(0) + E\{\max\{B_2(0, 0; \theta), B_2(1, 1; \theta)\}\}. \end{aligned}$$

Under the first scenario, where no information is available before deciding on the future level of development, the solution is

$$(10) \quad d_1^* = \begin{cases} 0 & \text{if } V^*(0) - V^*(1) \geq 0 \\ 1 & \text{otherwise} \end{cases} \quad \left| \quad V^* = \max[V^*(0), V^*(1)]. \right|$$

Under the second scenario, where information is available, the solution is

$$(11) \quad \hat{d}_1 = \begin{cases} 0 & \text{if } \hat{V}(0) - \hat{V}(1) \geq 0 \\ 1 & \text{otherwise} \end{cases} \quad \left| \quad \hat{V} = \max[\hat{V}(0), \hat{V}(1)]. \right|$$

Since $\hat{V}(0) \geq V^*(0)$ while $\hat{V}(1) = V^*(1)$, it follows immediately that $\hat{d}_1 \leq d_1^*$.

III. OPTION VALUE

Arrow and Fisher introduce their concept of option value in the following manner. Suppose that, in contemplating whether to permit development, a

decision-maker behaves according to (10), ignoring the possibility of improved information and setting d_1 and d_2 on the basis of his current expectation of the future benefits and costs of development. If it is in fact possible to wait and determine d_2 after the value of θ is known, this is not an optimal decision. The inefficiency can be corrected, in principle, by introducing a "shadow tax" on development, τ , so that instead of comparing $V^*(0)$ and $V^*(1)$ the decision-maker will compare $[V^*(0) + \tau]$ and $V^*(1)$. This shadow tax is the option value, which I will denote by OV . In order to elicit the optimal decision it must satisfy the condition

$$[V^*(0) + OV] - V^*(1) = \hat{V}(0) - \hat{V}(1).$$

Hence,

$$(12) \quad OV = [\hat{V}(0) - \hat{V}(1)] - [V^*(0) - V^*(1)].$$

Option value is thus the difference in the advantage of initial preservation ($d_1 = 0$) over initial development ($d_1 = 1$) as between the scenario where information about the future consequences of development becomes available and the scenario where there is no new information.

An alternative, but equivalent, characterization of this concept of option value can be obtained by substituting (8) into (12):

$$(13) \quad OV = \hat{V}(0) - V^*(0) \\ = E\{\max[B_2(0, 0; \theta), B_2(1, 1; \theta)]\} - \max[E\{B_2(0, 0; \theta)\}, E\{B_2(1, 1; \theta)\}],$$

i.e., option value is the gain from being able to learn about the future consequences of development if one does not develop initially. The right-hand side of (13) will be recognized from decision theory as a formula for the expected value of perfect information. It is, in fact, a conditional value of perfect information--it is the gain from information with respect to the choice of d_2 , conditional on $d_1 = 0$. However, it is not the unconditional gain from information in the overall decision problem, as Conrad suggests. The latter is given by the quantity $EVPI$ defined earlier as $EVPI = \hat{V} - V^*$. Observe from (7), (8), (10) and (11) that this can take one of three

possible values:

$$(14) \quad EVPI = \begin{cases} \hat{V}(0) - V^*(0) & \text{if } \hat{V}(0) \geq V^*(0) \geq V^*(1) \\ 0 & \text{if } V^*(1) \geq \hat{V}(0) \geq V^*(0) \\ \hat{V}(0) - \hat{V}(1) & \text{if } \hat{V}(0) \geq V^*(1) \geq V^*(0). \end{cases}$$

In the first case $d_1^* = \hat{d}_1 = 0$, and in the second case $d_1^* = \hat{d}_1 = 1$; in both of these cases EVPI coincides with a conditional value of information with respect to the choice of d_2 . In the third case $d_1^* = 1$ while $\hat{d}_1 = 0$, and EVPI cannot be identified with a conditional value of information.³ Only in the first case does the value of information coincide with the AFH concept of option value. Moreover, in the other two cases the AFH option value exceeds the value of information.⁴ In short, the relationship between option value and the expected value of perfect information may be summarized thus

$$(15) \quad OV \geq EVPI \geq 0.$$

IV. INCREASING UNCERTAINTY

In this section I will discuss the effect on option value of an increase in the uncertainty concerning the future benefits and costs of development in the sense of a change in the probability distribution of the random variable θ . I will also consider how this change in uncertainty affects the current development decisions, d_1^* and $\hat{d}_1$. These questions were originally raised by Arrow and Fisher for the special case where $B_2(d_1 + d_2, d_2; \theta)$ is linear in θ . They compare two situations: no uncertainty at all (θ is a known constant) and some uncertainty (θ is a random variable). I will employ here a more general notion of an increase in uncertainty, a Rothschild-Stiglitz [1970] mean-preserving spread in the distribution of θ . Arrow and Fisher's result is as follows:

THEOREM 1. If $B_2(d_1 + d_2, d_2; \theta)$ is linear in θ , a mean-preserving spread in the distribution of θ has no effect on d_1^* , lowers $\hat{d}_1$, and raises OV.

Suppose that $B_2(\cdot)$ is not linear in θ . Is it still true that an increase in uncertainty raises the option value and increases the advantage of postponing development? In general, the answer is negative: the increase in uncertainty may reduce the option value and hasten development. After all, as noted above, OV is a conditional value of information, and Gould's [1974] analysis shows that, in some circumstances, an increase in risk can reduce the value of information. The problem is that, whereas an increase in uncertainty raises $E\{\max[B_2(0, 0; \theta), B_2(1, 1; \theta)]\}$, it may raise $\max\{E\{B_2(0, 0; \theta)\}, E\{B_2(1, 1; \theta)\}\}$ even more. Therefore, there is no guarantee that OV will increase.

In order to obtain useful results, it is necessary to impose some further conditions on the model. Instead of linearity, I will assume that

$$(16) \quad W(\theta) \equiv B_2(0, 0; \theta) - B_2(1, 1; \theta) \text{ is monotone in } \theta.$$

By also restricting the curvature of $B_2(\cdot)$ as a function of θ , one obtains the following two theorems which generalize Arrow and Fisher's result:⁵

THEOREM 2. Given assumption (16) and either

(i) $E\{W(\theta)\} > 0$ and $W(\theta)$ is concave in θ or (ii) $E\{W(\theta)\} < 0$ and $W(\theta)$ is convex in θ , then a small mean-preserving increase in uncertainty raises OV.

THEOREM 3. Assume (16) and that $B_2(0, 0; \theta) - B_2(1, 0; \theta)$ and $B_2(1, 1; \theta) - B_2(1, 0; \theta)$ are each convex in θ . Then a mean-preserving increase in uncertainty lowers both $\hat{d}_1$ and d_1^* .

As a corollary, it follows that, if $B_2(\cdot)$ is a differentiable function of θ and either $E\{W\} > 0$ and

$$(17) \quad B_{2\theta\theta}(1, 0; \theta) \leq B_{2\theta\theta}(0, 0; \theta) \leq B_{2\theta\theta}(1, 1; \theta)$$

or $E\{W\} < 0$ and

$$(18) \quad B_{2\theta\theta}(1, 0; \theta) \leq B_{2\theta\theta}(1, 1; \theta) \leq B_{2\theta\theta}(0, 0; \theta),$$

then a small mean-preserving increase in uncertainty simultaneously raises OV and lowers $\hat{d}_1$ and d_1^* . Thus, the effects of an increase in uncertainty depend crucially

on the curvature of the second-period benefit function and on the way in which this curvature varies with changes in the levels of development.

V. SOME EXTENSIONS

There are two restrictions in my analysis up to this point which are taken over directly from the earlier literature on option value. I have assumed that the future brings perfect information about the consequences of development, and I have assumed that the decision-maker faces a binary choice of either no development or full development.

Suppose that at the end of the first period, instead of obtaining perfect information, the decision-maker obtains partial information about the consequences of development which he combines with his prior distribution for θ in a Bayesian manner to obtain a posterior distribution for θ .

By retracing the analysis of section II with this modification, one obtains a measure of option value which can be shown to be smaller than the AFH concept based on perfect information, but with similar properties: it is non-negative, can be characterized as a shadow tax on development or as a conditional gain from information, and is distinct from, but bounded by, the expected value of information. Thus the assumption of perfect information is not crucial to the formulation of the concept of option value. However, I will now show that the assumption of only two development alternatives is crucial.

For simplicity, I assume perfect information and employ the model of irreversible development outlined in section II in which d_t can range over the interval $[0, 1]$ subject to the constraint (1); the argument can readily be generalized to the case of partial information. Given the value function $V^*(\cdot)$, let $\bar{d}_1^*$ be the solution to: maximize $V^*(d_1)$ subject to $0 < d_1 \leq 1$. Similarly, let $\hat{d}_1$ be the solution to: maximize $\hat{V}(d_1)$ subject to $0 < d_1 \leq 1$. Thus, these are the optimal levels of development conditional on having some development during the first period. The unconditionally optimal development levels, d_1^* and $\hat{d}_1$, can be expressed in a manner similar to (10) and (11) as

$$(19) \quad d_1^* = \begin{cases} 0 & \text{if } V^*(0) > V^*(\bar{d}_1^*) \\ \bar{d}_1^* & \text{otherwise} \end{cases} \quad V^* = \max [V^*(0), V^*(\bar{d}_1^*)]$$

$$(20) \quad \hat{d}_1 = \begin{cases} 0 & \text{if } \hat{V}(0) > \hat{V}(\bar{\hat{d}}_1) \\ \bar{\hat{d}}_1 & \text{otherwise} \end{cases} \quad \hat{V} = \max[\hat{V}(0), \hat{V}(\bar{\hat{d}}_1)].$$

As pointed out in section II, the expected value of perfect information for this decision model is $EVPI = V^* - \hat{V} \geq 0$.

By analogy with (12) one possible concept of option value in the present context is the shadow tax which would have to be imposed on development in order to correct the decision of an agent who myopically behaves according to (19) and ignores the possibility of obtaining information about the future consequences of development. This concept of option value, which I will denote by OV_a , satisfies

$$(21) \quad OV_a = [\hat{V}(0) - \hat{V}(\bar{\hat{d}}_1)] - [V^*(0) - V^*(\bar{d}_1^*)].$$

By analogy with (13) an alternative concept of option value, which I will denote by OV_b , is the conditional gain from information with respect to the choice of d_2 if there is no development initially

$$(22) \quad OV_b = \hat{V}(0) - V^*(0).$$

These two concepts are no longer equivalent. Indeed,

$$(23) \quad OV_a = [\hat{V}(0) - V^*(0)] - [\hat{V}(\bar{\hat{d}}_1) - V^*(\bar{d}_1^*)] \leq OV_b,$$

since it follows from (7) and the definitions of $\bar{\hat{d}}_1$ and $\bar{d}_1^*$ that $\hat{V}(\bar{\hat{d}}_1) \geq V^*(\bar{d}_1^*)$. Therefore, the analyst would have to make a choice between the two concepts.

The concept OV_a may seem intuitively more appealing since it is patterned after Arrow and Fisher's original notion of option value as a shadow tax. However, unlike the AFH concept, there is no guarantee that OV_a is

nonnegative. For example, in the following case which is perfectly compatible with (7),

$$(24) \quad \hat{V}(\hat{d}_1) \geq \hat{V}(0) \geq V^*(0) \geq V^*(\bar{d}_1^*),$$

application of (23) shows that $OV_a \leq 0$. By contrast, the quantity OV_b must be nonnegative, but it is not necessarily bounded from below by EVPI.

For example, in (24) $EVPI = \hat{V}(\hat{d}_1) - V^*(0)$ and $OV_b \leq EVPI$. More

generally, it can be shown that there is no determinate relationship between either OV_a or OV_b and EVPI. In short, there appears to be little basis for choosing either concept of option value. Accordingly I would argue that when development is not a binary choice option value is not a well-defined concept.

FOOTNOTES

1. If there is uncertainty regarding the first period's benefits, the term $B_1(d_1)$ will be replaced by $E\{B_1(d_1)\}$ without affecting any of the analysis.

2. The inequality follows from the convexity of the maximum operator and the application of Jensen's inequality.

3. The case of $d_1^* = 0$, $\hat{d}_1 = 1$ is ruled out by the result of the preceding section that $\hat{d}_1 \leq d_1^*$.

4. This follows from the inequalities on the right-hand side of (16).

5. The proofs of these theorems are contained in an appendix, which is available on request.

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