Production Economics

University of California, Davis
Department of Agricultural Economics

INCORPORATING RISK IN PRODUCTION ANALYSIS

BY

JOHN M. ANTLE

February 1983
Revised May 1983

(Invited paper for presentation at the annual meeting of the AAEA, Purdue, August 1-4, 1983)

INCORPORATING RISK IN PRODUCTION ANALYSIS

Risk is the farmer's perennial problem. Unfortunately, agricultural economists have made little progress in analyzing or measuring production risk in ways that provide useful information for farm management. Therefore, it is fair to say that the social returns to existing research on production risk are probably low. It is from this perspective that I will discuss incorporating risk in production analysis. My purpose is not to survey the literature or to catalogue all the ways risk concepts have been incorporated in production models. Rather, I intend to critically evaluate some of the basic concepts and approaches in the literature, and to indicate the directions I believe research must take to arrive at a paradigm for incorporating risk in production analysis that could yield higher social returns.

In my view the conventional conceptual framework used for risk analysis has not led agricultural economists to ask the most important questions about the effects risk has on agricultural decision making. The extension of the static neoclassical production model to incorporate price and production uncertainty has led agricultural economists to rationalize observed behavior in terms of the Arrow-Pratt risk aversion concept. This approach has in turn led to qualitative comparative static theorems which are appealing to theorists but have little relation to the decision problems farmers face. To increase the relevance of their models and methods, I believe agricultural economists need to understand specifically how risk affects agricultural production. This will in turn suggest how risk affects agricultural decision making, and why farmers are or should be concerned with it.

My analysis of these issues leads me to discount the hypothesis that risk matters primarily because of the "psychic disutility" it causes. Rather, I hypothesize that risk matters primarily because production is a dynamic phenomenon and, therefore, production and price uncertainty affect expected productivity and expected income. I hypothesize that these "first-order" effects are why farmers are and should be concerned with risk. The analysis of dynamic, uncertain models shows that farmers' optimal decisions are affected by risk, whether they are "risk neutral" or "risk averse. This suggests that dynamic, risk-neutral models may be more useful than conventional static, risk-averse models for understanding the role production risk plays in farm management.

DEFINING RISK

A considerable literature on defining "risk" spans the past several decades. However, how one defines risk is not an important issue within the modern analytical framework that is used to model decision making under uncertainty. What is important is the functional structure of the relationships in the model. The modern approach is based on the assumption that decision makers behave as if they maximize the mathematical expectation of utility, and utility is assumed to be a function of profit and possibly other variables. Expected utility is based on the decision maker's subjective probability distributions of the random variables in profit. I restrict my discussion to this class of parametric maximization models. It will become evident shortly why I define these models as "parametric."

In this class of models, "risk" or "uncertainty" are equivalent and mean very simply that some variables in the objective function are random variables. Thus, incorporating risk in production analysis means incorporating random variables in production models and in the decision problems faced by farm managers. This is the only meaning that risk has in the class of parametric maximization models.

Once random variables are included in maximization models, the decision maker's objective function cannot be defined in terms of the variables themselves because the objective function is then a random variable and cannot be maximized. Therefore, in general, parametric maximization models are defined in terms of the <u>parameters</u> of the random variables in the model. To illustrate, consider a simple model of the firm. Let p be output price, Q output, x a vector of inputs which are to be chosen, and w a vector of input prices. Both p and Q are random varibles when input choices are made, and the distribution of Q is conditionally defined on the input vector x. The joint probability distribution of p and Q is $g(p,Q|x,\alpha)$, where α is the vector of parameters which with x define the probability distribution.[1] The decision maker's utility function is $U = U[\pi;\omega]$, where π is profit and ω is the parameter vector defining the utility function. The mathematical expectation of utility, taken with respect to the distribution of p and Q, can be expressed as a function of x, α, and ω.

(1) $$EU[\pi] = EU[pQ - wx] \equiv u[x, \alpha, \omega]$$

The solution to the expected utility maximization problem, therefore, is $x^* = x^*[\alpha, \omega]$.

My contention that risk definitions are not important is defended as follows. The optimal solutions of parametric maximization models are defined in terms of the parameters of the probability distributions and the utility function. This is true regardless of what characteristics of the probability distribution, i.e., what parameters or functions of parameters, are defined as measuring "risk." What matters in incorporating risk (random variables) in production analysis is how the objective function and the solution of the decision problem depend on the parameters of the random variables in the model.

To further illustrate this point, consider the definition of "risk" given by Rothschild and Stiglitz. For two random variables X and Y such that E(X) = E(Y), random variable X is riskier than random variable Y if and only if expected utility of X is less than expected utility of Y. They use this definition to show that variance is not generally a valid measure of risk, because expected utility of X may be greater than expected utility of Y even though the variance of X is greater than the variance of Y. This result is easily demonstrated without resorting to the complex mathematics of their proof. Referring to the above example, it is clear that ordering random variables in terms of mean and variance would be appropriate only when $u[x, \alpha, \omega]$ in equation (1) depends only on the mean and variance of profit.

WHEN DOES RISK MATTER?

The above discussion suggests that two conditions are relevant in determining whether or not risk should be incorporated in production analysis: (a) the farmer's objective function depends on the parameters of the probability distributions of random variables; (b) the farmer knows enough about the probability distributions to use them in decision making. Whether or not risk need be incorporated in production analysis depends on the objectives of the research. Research aiming to explain or predict farmer behavior need incorporate risk only if both (a) and (b) hold. Indeed, a model based on the assumption that farmers are risk neutral may explain or predict better than a risk-averse model if farmers do not know how risk affects production. This may be true even if farmers are very risk averse!

By the same token, research aiming to help farmers make better management decisions need incorporate risk if (a) is true, whether or not (b) is true. Therefore, the goal of research aiming to improve farm management should be to develop means of measuring and evaluating the properties of price and output distributions faced by farmers. Then extension specialists can help farm managers incorporate information about price and output distributions into their management decisions.

The reader should note that I did not say that risk matters if and only if the decision maker is risk averse in the Arrow-Pratt sense. In contrast, the professional consensus appears to be that risk does matter if and only if farmers are risk averse. The spirit of the conventional wisdom is well represented by Binswanger's statement (p. 392):

> Suppose it can be established for many nonprotective modern inputs that they do indeed increase riskiness of net returns. Whether this is of any consequence depends on the extent of risk aversion among farmers. Hence, for this question, as for many others, empirical estimations of risk aversion. . . becomes crucial.

In the following sections I show that this view is valid only in the conventional static theoretical framework. Moreover, I show that dynamic production relations are sufficient to validate (a). Therefore, we can conclude that risk should be incorporated in production analysis when either the decision maker is risk averse or production is dynamic.

LIMITATIONS OF STATIC MODELS

The usual approach to incorporating risk in production analysis in the economics and agricultural economics literatures is to reformulate the neoclassical production problem with random output price or output. It is assumed that the decision maker chooses inputs to maximize the mathematical expectation of utility, with utility a function of profit. The analytical framework is explicitly static, and the decision problem must be interpreted as a single period problem. This is true for virtually all models, whether they be theoretical, applied, programming, or econometric (see Roumasset, Boussard, and Singh; Anderson, Dillon, and Hardaker). In this section I discuss the properties of models based on this static decision framework.

A first important property of static models is that only means and covariances of output price p and output Q enter the model if decision makers are assumed to be risk neutral. All parameters of the probability distribution of p and Q generally enter a static model only if decision makers are risk averse. To demonstrate this fact, first assume risk neutrality so maximization of expected utility is equivalent to maximization of expected profit. We have

$$E[\pi] = E[pQ - wx] = E[p]E[Q] + COV[p,Q] - wx,$$

where COV[p,Q] is the covariance of p and Q and COV[p,Q] = E[pQ] - E[p]E[Q]. To introduce higher moments of p and Q into the decision problem, it is necessary to assume decision makers are risk averse, as above in (1).

This characteristic of static models is most significant for decision making. It implies that it is important to incorporate higher moments of prices and output in production analysis only if farmers are sufficiently risk averse for the higher moments to have a measurable impact on their objective functions. Therefore, it is possible to know whether or not risk affects farmers' optimal decisions only if it is possible to obtain accurate measurements of their utility functions; hence, the above quote of Binswanger is correct in the static framework.

A second important characteristic of static models is that only revenue is uncertain; cost is certain. This is because in the static decision model, all inputs are chosen before production begins, and therefore all input prices are known when decisions are made. Only output price and output can logically be argued to be unknown at the beginning of the production process. Thus, static models cannot logically exhibit cost uncertainty.[2] This is a serious shortcoming for models of agricultural production. Consider, for example, the traditional problem of planning for harvest labor with output and wage rate uncertainty; this cannot be analyzed with a static model. Static models also preclude analysis of the risks inherent in modern agriculture, such as the timing of irrigation, fertilization, and pest control.

In defense of static models one might argue that they provide useful comparative static results. For example, one can show that the demand for a risk increasing input (one that causes a mean-preserving spread of the output distribution--see Pope and Kramer) is greater for a risk neutral farmer than a risk averse farmer. But for whom is this a useful result? It may be useful to economists for testing theoretical constructs and rationalizing observed behavior, but it is questionable whether it is useful to farm managers.

THE RISK-EFFICIENCY HYPOTHESIS

The above analysis suggests that we need to carefully reconsider the nature of agricultural production to devise a more useful approach. It seems obvious that agricultural production, and therefore agricultural decision making, occur over time. Farmers face production risk due to natural phenomena occuring over time and they face economic risk due to market fluctuations and related economic phenomena occuring over time. If all relevant variables were known with certainty, farmers would face the classical maximization problem.

However, once decisions are made, natural and economic conditions change, and previously optimal decisions based on old information become suboptimal with new information.

These facts lead me to hypothesize that risk affects both productivity (technical efficiency) and optimal resource use (allocative efficiency), and hence economic efficiency (the combination of technical and economic efficiency). I refer to this proposition as the risk-efficiency hypothesis. It is remarkable that the risk-efficiency hypothesis is defined without reference to the Arrow-Pratt concept of risk aversion. Therefore, if this hypothesis is true, it is not necessary to argue that farmers are risk averse to justify why they are, or should be, concerned with risk. Moreover, farmers need not be risk averse to benefit from information about production or price risks. The risk-efficiency hypothesis does imply that it is necessary to model and measure the dynamic structure of agricultural production to be able to evaluate the effects risk has on agricultural production and income. In the next section I show that dynamic models validate the logic of the risk-efficiency hypothesis.

DYNAMIC PRODUCTION MODELS

Two kinds of dynamics may occur in production. I refer to them as output dynamics and input dynamics. Define Q_t, x_t and u_t as output, the input vector, and the random production shock in period t. The general form of the production function with output dynamics is

$$Q_t = f_t[x_t, Q_{t-1}, Q_{t-2}, \ldots, u_t].$$

Output dynamics means that Q_t depends on past rates of output and thus implicitly on past inputs and random shocks to production. Crop production in a single season is an important example of output dynamics. For example, define the production stages as planting, cultivation, and harvest. Then Q_1 represents the "product" of the planting stage; Q_2 represents the product of cultivation, the fully grown crop, and depends on Q_1; and Q_3 represents the harvested crop and depends on Q_2. A crop rotation is another example of output dynamics across seasons and crops. E.g., the productivity of corn grown in a given season depends on the crops grown in preceding seasons.

Processes which exhibit input dynamics depend directly on past inputs:

$$Q_t = f_t[x_t, x_{t-1},\ldots, u_t].$$

Generally, input dynamics occur when there is some form of capital investment. Besides the conventional problem of investment in machinery and structures, phenomena such as soil fertility, pest resistance, and learning by doing are examples of input dynamics. In all these cases, inputs chosen in previous periods affect production in future periods.

Dynamic production models can be formulated as multistage or multiperiod decision problems. The multistage model is typified by the crop production problem mentioned above, where there is a sequence of subprocesses or operations leading to a final output. Multiperiod models consist of a sequence of temporally related, but distinct, production processes; a typical agricultural example is a crop rotation. The same dynamic production functions can be used to represent multistage or multiperiod problems. However, the distinction between multiperiod and multistage models is important in specifying the farmer's objective function. A risk-neutral multistage model with T stages has the objective function

(2) $\max_{\{x_t\}} \quad E[p_T Q_T - \sum_{t=1}^{T} \gamma_t \, w_t x_t]$

where γ_t is a discount factor. The risk-neutral multiperiod objective function is

(3) $\max_{\{x_t\}} \quad E[\sum_{t=1}^{T} \pi_t \, \gamma_t]$ where $\pi_t = p_t Q_t - w_t x_t$.

In equations (2) and (3), the variables p_t, Q_t, w_t, x_t, and γ_t are defined over the time unit appropriate for each decision problem. If the decision maker is risk averse we replace profit with utility of profit in the above equations. Note that all multistage processes involve intermediate products and thus exhibit output dynamics. Multiperiod models may have output dynamics, input dynamics, or both.

These optimization problems are stochastic control problems, and their solutions can be obtained by applying the dynamic programming algorithm. Let E_t denote the expectations operator defined over the joint probability distribution of all random variables at time t. These variables are current and future output and future prices. Let μ_t be the parameter vector defining this distribution. Letting x_j* denote the optimal input vector in period j, the optimal solution of the multiperiod problem in the t-th period satisfies

(4) $\max_{x_t} \quad E_t[\pi_t] + E_t[\sum_{j=t+1}^{T} \gamma_j \pi_j | x_j = x_j^*]$

The solution to the multistage problem is defined analogously. The general solution in both cases has the form (Aoki)

(5) $x_t^* = x_t^*[x_{t-1}, x_{t-2}, \ldots , Q_{t-1}, Q_{t-2}, \ldots , w_t, \mu_t]$.

From equations (2) - (5) we can deduce some important facts about dynamic production models.

First, dynamic production models have a recursive structure. This is true of production functions with output dynamics and the input demand functions of all dynamic production models. This structure is important in specifying and estimating dynamic production models. Second, production functions are generally nonlinear, and therefore profit or the present value of profit is a nonlinear function of past outputs if the process exhibits output dynamics. This means that mean technical and economic efficiency generally depend on production risk. Third, costs are uncertain in dynamic models, in contrast to static models. With sequential decision making, profit is a nonlinear function of future periods' input and output prices and outputs. Therefore, optimal decisions depend on output and price risk, as shown by the presence of μ_t in equation 5.

A fundamental conclusion can be drawn from the general structure of dynamic production models. Risk-neutral dynamic production models show that risk affects expected productivity and optimal resource allocation, as predicted by the risk-efficiency hypothesis. Therefore, all farmers could gain from information about price and production risk, whether or not they are risk averse.

AN EXAMPLE

A two-input, two-stage model can illustrate the relationships that are possible in dynamic production models. The first-stage production function is

$$Q_1 = \alpha_0 + \alpha_1 x_1 + \varepsilon_1$$

where Q_1 is first-stage output, x_1 is the input, and ε_1 is a random shock to production realized after x_1 is chosen. The production function for stage 2 is quadratic,

$$Q_2 = \beta_0 + \beta_1 x_2 + 1/2\ \beta_2\ x_2^2 + \beta_3 Q_1 + \beta_4 x_2\ Q_1 + 1/2\ \beta_5 Q_1^2 + \varepsilon_2$$

where Q_2, x_2, and ε_2 are similarly defined. This model exhibits output dynamics because Q_2 depends on Q_1. Also note concavity of the stage 2 production function requires $\beta_2 < 0$, $\beta_5 < 0$, and $\beta_2\beta_5 > \beta_4^2$.

Profit is

$$\pi_2 = p_2 Q_2 - w_1 x_1 - w_2 x_2$$

where p_2 is the output price realized at the end of the production period.

The risk-neutral manager chooses x_1 and x_2 to maximize expected profit. Assume the farmer uses all information available at the beginning of each stage and solves the decision problem sequentially using the "open loop feedback" algorithm (Antle 1983b). In stage 2 the optimal value for x_2 is found by maximizing expected profit conditional on the information available at the beginning of stage 2. Output Q_1 and w_2 are known when x_2 is chosen so the problem is

$$\max_{x_2}\ E_2[\pi_2 | Q_1,\ w_2,\ w_1]$$

where E_2 denotes the expectation operator defined over the conditional joint probability distribution function of p_2 and Q_2 given Q_1, w_1, and w_2. Assuming p_2 and Q_2 are independently distributed, the solution x_2^* is linear in $w_2/E_2[p_2]$ and Q_1. The optimal value of x_1 is obtained by maximizing expected profit given w_1 and the knowledge that x_2^* is a function of Q_1 and hence x_1.

Thus, the optimal x_1 is found by solving

$$\max_{x_1} \; E_1[\pi_2 | w_1, \; x_2 = x_2^*]$$

where E_1 is defined over the joint probability distribution of w_2, p_2, Q_1 and Q_2 at time 1.

It can be shown, with some tedious algebra, that this expression depends on the means and variances of Q_1 and $w_2/E_1[p_2]$ (Antle 1983a). In fact in this model,

$$\frac{\partial E_1[\pi_2]}{\partial Var[Q_1]} = E_1[p_2] \, \frac{1}{2}(\beta_5 - \beta_4^2/\beta_2) < 0$$

$$\frac{\partial E_1[\pi_2]}{\partial Var[w_2/E_1[p_2]]} = -\frac{1}{2}\frac{1}{\beta_2} E_1[p_2] > 0.$$

Thus, in this model, expected profit is decreasing in output variance and increasing in the normalized input variance. Mathematically, this result is explained by the quadratic structure of the production function and the sequential solution of the decision problem. It is worth noting that output variance affects expected profit if and only if there is output dynamics in the model. If there is only input dynamics, x_2^* depends on x_1, not on Q_1, so that expected profit depends only on price risk, and not on production risk.

This model has an interesting economic interpretation. The optimal input choice in stage 1 depends on output uncertainty because the productivity of x_1 depends on x_2, and the productivity of x_2 in turn depends on Q_1. The "riskier" is first-stage production, the riskier is x_2 and hence the lower is the expected productivity of x_1. The result that expected profit is increasing in $Var[w_2/E_1[p_2]]$ is due to the effects input price variance has on cost and revenue. Cost is a concave function of w_2 so expected cost is decreasing in its variance. The input price variance lowers expected productivity, but by less than expected cost, so expected profit is increasing in the variance.

It is useful to compare the interpretation of this dynamic model to that of a static model. Suppose, for example, that x_1 is a risk-decreasing "insurance" input such as a pesticide. Then $\partial Var[Q_1]/\partial x_1 < 0$. With a static, risk-averse model the greater is the effect of the pesticide on output variance (in absolute terms), the higher is the rate of pesticide use, because expected utility is decreasing in output variance. In the dynamic, risk-neutral model the greater is the pesticide's effect on output variance, the higher the rate of pesticide use, because lower output variance in the first production stage (i.e., in plant growth) means higher expected productivity in the second production stage (i.e., in harvest). Of course, these two interpretations are not mutually exclusive; production may be dynamic and the farmer may be risk averse. However, note that we can directly observe that production is in fact dynamic and that farmers do in fact make input decisions sequentially. We can not directly observe, nor even readily infer, whether or not farmers are risk averse. Therefore, it seems reasonable to question the established paradigm for risk analysis based on static, risk averse models. Dynamic, risk-neutral models may be a better first-order approximation to the decision problems farmers actually face.

An important question is whether or not the qualitative properties of this two-stage model can be generalized. I show elsewhere (Antle 1983a) that this is not the case. In fact, in a model with three or more stages, it can be shown that the effect of higher moments on expected profit can not be determined *a priori*. The result depends on the functional structure of the model.

TOWARDS EMPIRICAL APPLICATION OF DYNAMIC RISK MODELS

As Leamer (p. 40) has noted, methodologies are like sex: "they are better demonstrated than discussed, but often better anticipated than experienced." One might suggest that this is the case with dynamic models, due to their inherent mathematical complexity (Burt, Zilberman) and substantial data requirements. Let us consider first the methodological problems one encounters if the needed data are available, and then the data issue. The following results on estimation are developed in Antle and Havenner.

As I pointed out above, dynamic production models are recursive systems of production functions and input demand equations. However, these input demand equations are difficult to derive analytically because they are the solution to an optimal stochastic control problem. It is well known that this is true except for models with quadratic objective functions. One interesting exception is the dynamic Cobb-Douglas production model. Although nonlinear, it has a solution which is a log-linear recursive analog of the usual static model. However, more complex functional forms are unlikely to have this property.

Fortunately, in some cases it is not necessary to solve the control problem to obtain estimates of dynamic production function parameters with desirable properties. In particular, when all outputs are observed (in all stages or periods) then it is only necessary to estimate the system of production functions. This means that any nonlinear production function can be utilized to model dynamic production problems when all output data are available. The associated estimation problems are easily solved with conventional linear or nonlinear estimation procedures. When all outputs are not observed, as is usually the case with intermediate production stages in agriculture, it is necessary to estimate the production functions and input demand equations jointly to obtain consistent and efficient estimates. In the case of models that are linear in parameters, such as the Cobb-Douglas, estimation requires only conventional procedures such as generalized least squares or three-stage least squares.

More difficult estimation problems arise when analytical solutions of the control problem cannot be obtained. In this case some approximation methods are available, but this problem remains at the frontiers of econometric methodology. Some recent research has made significant progress towards solving these estimation problems (Hansen and Singleton).

Measurement of production risk must preceed analysis of production risk in both static and dynamic models. The literature spans work by Day, Anderson, Just and Pope, and Antle (1983c). This latter study provides a general econometric methodology for estimating higher moments of output as functions of inputs. Thus, given input and output data for a production stage or period, there exist means of modeling and estimating the effects of alternative production practices and input combinations on production risk.

The recent developments in price expectations modeling also need to be used by agricultural economists to incorporate risk in production analysis. Both conventional time series models as well as the rational expectations approach provide means of modeling and estimating price expectations mechanisms and need to be incorporated in dynamic production models.

Let us return to the question of data. A major obstacle to implementation of dynamic production models is data limitations. In the case of single season crop production, input data by operation or production stage are virtually nonexistent. There is similarly a lack of consistent micro time series data which could be used to study multiperiod production problems. However, these difficulties are not insurmountable. With the advent of computerized farm data management, it will become increasingly feasible to obtain these kinds of detailed production data. Certain examples already exist. Some California dairy cooperatives have established on-going computerized production data management systems, and a large-scale data collection and management system on dairies is being established by the UC Davis Veterinary Medicine Teaching and Research Center in Tulare County. It seems fair to say that it will be possible in the near future to overcome the data limitations we now face. At present it is crucial to recognize the limitations of existing data bases so that future data collection systems can surmount past inadequacies.

CONCLUSION

My main points can be summed up as follows. Incorporating risk in production analysis means incorporating probability distribution parameters in decision models. Static models have serious limitations: they imply risk matters only if decision makers are risk averse, and they cannot be used to model cost uncertainty. Dynamic models, in contrast, support the risk-efficiency hypothesis, and show that input and output price risk and production risk generally affect productivity and optimal resource allocation, whether or not the decision maker is risk averse. An important implication for both research and extension is that all farmers, whether risk neutral or risk averse, can gain from information about price and production risk.

It seems evident that agricultural production is a dynamic phenomenon. However, it is not evident that farmers are risk averse, and risk preferences are difficult to measure. It is therefore remarkable, and somewhat paradoxical, that the static, risk-averse decision model is the predominant paradigm in the agricultural economics literature. The implication is that research needs to be redirected towards measurement and analysis of dynamic, risk-neutral models as a first step towards understanding the role risk plays in farm management.

Critics of this view will likely argue that I am simply calling for more mathematical complexity which will not prove to be any more useful than the simpler static approach. To the contrary, I am suggesting that researchers face tradeoffs in incorporating risk in production analysis. Static, risk-neutral models use a simpler production function but require specification and estimation of the utililty function parameters. Dynamic, risk-neutral models are based on more complex production functions and require more detailed production data, but do not require estimation of the utility function. Of course, risk aversion can be incorporated in dynamic models as well, but is not necessary to obtain models in which "risk matters." The important economic question facing the research community is which approach will yield the greatest social returns. This is an open empirical question. However, I submit that whatever are the costs of research based on static models, the benefits are limited because the results have little relevance to the decision problems farmers face. Considering the high rates of return usually attributed to agricultural research that had direct effects on farm-level productivity, my guess is that the returns to detailed research on agricultural decision making in a dynamic, uncertain environment would be high.

FOOTNOTES

John M. Antle is an assistant professor, Department of Agricultural Economics, University of California, Davis. I wish to thank Bob Collins, Steve Hatchett, Dale Heien, John Kushman, Alex McCalla, Bruce McCarl, and Ray Nelson for their comments on earlier drafts of this paper. This is Giannini Foundation Research Paper No. 678.

[1]Note that α can be interpreted as the moments of g, because the moments uniquely define the distribution. See Antle(1983c).

[2]Some static models have, nevertheless, been specified with unknown input prices. My point is that when such models are interpreted as single period decision models, costs cannot be uncertain. In a truly timeless static model, both revenues and costs can be uncertain, but the model then lacks any relation to the logical sequence of decisions faced by farm managers.

REFERENCES

Anderson, J. R. "Sparse Data, Climatic Variability, and Yield Uncertainty in Response Analysis." Amer. J. Agr. Econ. 56(1974): 77-83.

Anderson, J. R., J. L. Dillon and J. B. Hardaker. Agricultural Decision Analysis. Ames: The Iowa State University Press, 1977.

Antle, J. M. "Production Dynamics, Uncertainty, and Agricultural Decision Analysis." University of California, Davis, Department of Agricultural Economics, Working Paper No. 83-3, 1983a.

__________. "Sequential Decision Making in Production Models." Amer. J. Agr. Econ. 65(1983b): 282-290.

__________. "Testing the Stochastic Structure of Production: A Flexible Moment-Based Approach." J. Bus. and Econ. Statist. 3(1983c): 192-201.

Antle, J. M. and A. Havenner. "Formulating and Estimating Dynamic Production Models." University of California, Davis, Department of Agricultural Economics, Working Paper No. 83-7, 1983.

Aoki, M. Optimization of Stochastic Systems. New York: Academic Press, 1967.

Binswanger, H. P. "Risk and Uncertainty in Agricultural Development: An Overview." Risk, Uncertainty, and Agricultural Development, eds. J. A. Roumassett, J. Boussard, and I. Singh, pp. 383-398. New York: Agricultural Development Council, 1979.

Burt, O. R. "Dynamic Programming: Has Its Day Arrived?" West. J. Agr. Econ. 7(1982): 381-394.

Day, R. H. "Probability Distributions of Field Crops." J. Farm Econ. 47(1965): 713-741.

Hansen, L. P. and K. J. Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." Econometrica 50(1982): 1269-1286.

Just, R. E. and R. D. Pope. "Production Function Estimation and Related Risk Considerations." Amer. J. Agr. Econ. 61(May 1979): 277-284.

______. "Stochastic Specification of Production Functions and Economic Implications." J. Econometrics 7(1978): 67-86.

Leamer, E. E. "Let's Take the Con Out of Econometrics." Amer. Econ. Rev. 73(1983): 31-43.

Pope, R. D. and R. A. Kramer. "Production Uncertainty and Factor Demands for the Competitive Firm." South. Econ. J. 46(Oct. 1979): 489-501.

Rothschild, M. and J. E. Stiglitz. "Increasing Risk: I. A Definition." J. Econ. Theory 2(1970): 225-243.

Roumassett, J. A., J. Boussard, and I. Singh, ed. Risk, Uncertainty and Agricultural Development. New York: Agricultural Development Council, 1979.

Zilberman, D. "The Use and Potential of Optimal Control Models in Agricultural Economics." West. J. Agr. Econ. 7(1982): 395-406.